This is a Title with Each Word Capitalized and Support α, β and Break of Line.

Final Year Project Report

Presented by

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In Partial Fulfillment

of the Requirment for the Degree of

Master in Mathematics

DEPARTMENT OF MATHEMATICS
COMSATS UNIVERSITY ISLAMABAD
Attock Campus

Spring 2016

COMSATS UNIVERSITY ISLAMABAD
ATTOCK CAMPUS

FINAL APPROVAL

This project titled

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submitted to the Department of Mathematics

by

Name	**Registration Number**
Umer Din Khan	CIIT/FA14-RMT-001/ATK

in partial fulfillment of the requirements for the award of the degree of Master in Mathematics has been accepted.

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DECLARATION OF THE STUDENT

I, **Umer Din Khan**, Registration Number **CIIT/FA14-RMT-001/ATK**, hereby solemnly declare that I have produced the work presented in this project, during the scheduled period of study.

Date: **June 14, 2016**

....
Signature of the Student

dedicated to my beloved parents

Contents

Abstract

A function is convex if the line segment joining two points on the graph lies above the graph. These functions have important properties and applications in mathematics. Specially, they are very important in optimization and minimization problems. Also these functions are used in statistic and functional analysis. A positive function f is logarithmic convex if log f is convex. It would seem that log convex functions unremarkable because they are so simply related to convex functions. But they have some surprising properties.

In the first chapter we generalize results for logarithmic convexity of Giaccardi's difference for classes of functions with the help of divided difference.

Acknowledgements

Praise is to ***Almighty Allah***, *WHO* is Lord of the world, the Answerer of prayers and the Source of peace, whose blessing and exaltation flourished to the scared wealth of knowledge.

Special praises and regards for His Last Messenger, Holy Prophet ***Hazrat Muhammad (PBUH)***. Holy Prophet said that I AM the light, whoever follows ME, will never be in the darkness.

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Student Name

Notations

The notation and concepts used in this monograph are more or less specified. The reader is assumed to be familiar with the elements of Mathematical Analysis, as well as General Algebra, Matrix Theory and Topology, and since the standard notation and concepts were used, it was believed unnecessary to define all of them.

We give some of the Notation used in the Monograph.

$\mathbb{Z}$ the set of integer

$\mathbb{N}$ the set of positive integer

$\mathbb{Q}$ the set of rational numbers

$\mathbb{R}$ the set of real numbers

Chapter 1

Introduction

1.1 Monotone functions

Let $x, y, z \in \mathbb{R}$

Definition 1.1.1. If a function f is either increasing or decreasing on I, then f is called a *monotone function* on I.

In practice we often use the following criterion for monotonicity:

Theorem 1.1.2. *If f is a differentiable function on I, it is monotone on I if and only if the sign of f' remains the same throughout I. In particular, if $f'(x) > 0$, except maybe on a set of points of I which does not contain any interval of I, then and only then f is an strictly increasing function; if $f'(x) \geq 0$, then f is increasing; if $f'(x) < 0$ on I, then f is a strictly decreasing function and for $f'(x) \leq 0$, is increasing.*

$$\infty \partial \pm \sum df d\sqrt{b^2 - 4ac} \tag{1.1}$$

$$\infty \partial \pm \sum \tag{1.2}$$

From equation 1.2, we have One can note that if f and g are monotone in same direction then $f \circ g$ is increasing and if f and g are monotone in opposite direction then $f \circ g$ is decreasing in their respective domains.

1.2 Convex functions

The fundamental work of Jensen in the years 1905, 1906 is the starting point of the systematic study of convex functions. Even before Jensen, the literature shows results which refer to convex functions.

In fact the roots of such functions can be found in the work of Hölder in 1889 and J. Hadamard in 1893, although these roots were not explicitly specified in their works. The general theory of convex functions is the origin of powerful tools for the study of problems in analysis. Inequalities involving convex functions are the most efficient tools in the development of several branches of mathematics and has been given considerable attention in the literature.

Chapter 2

Inequalities of Hadamard's Type for Lipschitzian Mappings.

In this chapter, we give some inequalities of Hadamard's type for M-Lipschitzian function. Some application which are connected with *log* functions, exponential functions etc., for two positive numbers are also given. It is given in [3].

Definition 2.0.1. A function $f : I \to \mathbb{R}$ defined on a closed interval $I = [a, b]$ is said to satisfy a Lipschitz condition if for any constant M and for points $x, y \in [a, b]$

$$|f(x) - f(y)| \leq M |x - y| .$$

2.1 Hadamard's type inequalities

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We will start with the following theorem containing two inequalities of Hadamard's type for M-Lipschitzian mapping. We need the following Lemma to prove the theorem.

Lemma 2.1.1.

$$\int_0^1 |2t-1|\, dt = \frac{1}{2}. \tag{2.1}$$

Proof. As

$$|2t-1| = \begin{cases} +(2t-1) & \text{if } t \geq \frac{1}{2} \\ -(2t-1) & \text{if } t < \frac{1}{2} \end{cases}$$

so

$\int_0^1 |2t-1|\, dt = \int_0^{\frac{1}{2}} (-(2t-1))dt + \int_{\frac{1}{2}}^1 (2t-1)dt.$

Now as

$\int_0^{\frac{1}{2}} (-(2t-1))dt = \frac{1}{4}$ and $\int_{\frac{1}{2}}^1 (2t-1)dt = \frac{1}{4}.$

Therefore

$\int_0^1 |2t-1|\, dt = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}.$ □

Theorem 2.1.2. *Let $f : I \subseteq \mathbb{R} \to \mathbb{R}$ be an M-Lipschitzian mapping on I and $a, b \in I$ with $a < b$. Then we have the inequalities*

$$\left| f\left(\frac{a+b}{2}\right) - \frac{1}{b-a}\int_a^b f(x)dx \right| \leq \frac{M}{4}(b-a) \tag{2.2}$$

and

$$\left| \frac{f(a)+f(b)}{2} - \frac{1}{b-a}\int_a^b f(x)dx \right| \leq \frac{M}{3}(b-a). \tag{2.3}$$

Proof. Let $t \in [0,1]$. Then we have, for all $a, b \in I$,

$$\begin{aligned} & |tf(a) + (1-t)f(b) - f(ta + (1-t)b)| \\ & = |t(f(a) - f(ta + (1-t)b)) + (1-t)(f(b) - f(ta + (1-t)b))| \\ & \leq t\,|f(a) - f(ta + (1-t)b)| + (1-t)\,|f(b) - f(ta + (1-t)b)| \\ & \leq tM\,|a - (ta + (1-t)b)| + (1-t)M\,|b - (ta + (1-t)b)| \\ & = 2t(1-t)M\,|b-a|. \end{aligned} \tag{2.4}$$

If we choose

$$t = \frac{1}{2},$$

we have

$$\left| \frac{f(a)+f(b)}{2} - f\left(\frac{a+b}{2}\right) \right| \leq \frac{M}{2}\,|b-a|. \tag{2.5}$$

If we put $ta+(1-t)b$ instead of a and $(1-t)a+tb$ instead of b in (2.5) respectively, then we have

$$\left|\frac{f(ta+(1-t)b)+f((1-t)a+tb)}{2}-f\left(\frac{a+b}{2}\right)\right| \leq \frac{M\,|2t-1|}{2}\,|b-a|\,. \tag{2.6}$$

for all $t \in [0,1]$.

If we integrate the inequality (2.6) on [0,1], we have

$$\left|\frac{1}{2}\left[\int_0^1 f(ta+(1-t)b)dt+\int_0^1 f((1-t)a+tb)dt\right]-f\left(\frac{a+b}{2}\right)\right| \leq \frac{M\,|b-a|}{2}\int_0^1 |2t-1|\,dt. \tag{2.7}$$

Also we have

$$\int_0^1 f(ta+(1-t)b)dt=\int_0^1 f((1-t)a+tb)dt=\frac{1}{b-a}\int_a^b f(x)dx. \tag{2.8}$$

Now using equation (2.8) and from Lemma 2.1.1 in equation (2.7), we have

$$\left|\frac{1}{b-a}\int_a^b f(x)dx-f\left(\frac{a+b}{2}\right)\right| \leq \frac{M\,|b-a|}{4} \tag{2.9}$$

$$\left|f\left(\frac{a+b}{2}\right)-\frac{1}{b-a}\int_a^b f(x)dx\right| \leq \frac{M\,|b-a|}{4}, \tag{2.10}$$

which is required inequality.

From equation (2.4), we have

$|tf(a)+(1-t)f(b)-f(ta+(1-t)b)| \leq 2t(1-t)M(b-a)$

for all $t \in [0,1]$ and $a, b \in I$ with $a < b$. Integrating on [0,1], we have

$$\left|f(a)\int_0^1 tdt+f(b)\int_0^1 (1-t)dt-\int_0^1 f(ta+(1-t)b)dt\right| \leq 2M(b-a)\int_0^1 t(1-t)dt$$

Hence, from

$$\int_0^1 tdt=\int_0^1 (1-t)dt=\frac{1}{2}$$

and

$$\int_0^1 f\left(ta+(1-t)b\right)dt=\frac{1}{b-a}\int_a^b f(x)dx,$$

we have

$$\left|\frac{f(a)+f(b)}{2}-\frac{1}{b-a}\int_0^1 f(x)dt\right| \leq \frac{M(b-a)}{3},$$

$\square$

which is our required equation.

From [**?**] and [**?**].

From [1] and [2]. $\frac{\partial z}{\partial x}$

$$\sum_{i=1}^{n}\frac{1}{n}$$

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