10·6-1

Ex 10.6

Q1 $\frac{d^2y}{dx^2}+4y=\sec 2x$ —— (1)

$(D^2+4)y=\sec 2x$

$D^2+4=0$

$D^2=-4$

$D^2=(i)^2 4$

$D=\pm 2i$

$\therefore y_c=c_1\cos 2x+c_2\sin 2x$ —— (2)

Replace c_1 by u_1 & c_2 by u_2 in y_c we get

Assumed $y_p=u_1\cos 2x+u_2\sin 2x$ —— (3)

from (2) $y_1=\cos 2x$ & $y_2=\sin 2x$

$y_1'=-2\sin 2x$ & $y_2'=2\cos 2x$

$W=y_1y_2'-y_2y_1'$

$=(\cos 2x)(2\cos 2x)-(\sin 2x)(-2\sin 2x)$

$=2\cos^2 2x+2\sin^2 2x$

$W=2(\cos^2 2x+\sin^2 2x)=2(1)=2$

$F(x)=\sec 2x$

$u_1=\int\frac{-y_2F(x)}{W}dx$

$=\int-\frac{\sin 2x\ \sec 2x}{2}dx=\int-\frac{1}{2}\frac{\sin 2x}{\cos 2x}dx$

$u_1=\frac{+1}{4}\int\frac{-2\sin 2x}{\cos 2x}dx=\frac{1}{4}\ln|\cos 2x|$

$u_2=\int\frac{y_1F(x)}{W}dx$

$u_2=\int\frac{\cos 2x\ \sec 2x}{2}dx=\frac{1}{2}\int dx=\frac{1}{2}x$

Put values of u_1 & u_2 in (3) i.e. Assumed y_p

we get $y_p=\frac{1}{4}\ln|\cos 2x|\cos 2x+\frac{x}{2}\sin 2x$

G.Sol $y=y_c+y_p$

$y=c_1\cos 2x+c_2\sin 2x+\frac{1}{4}(\ln|\cos 2x|)+\frac{x}{2}\sin 2x$ Ans.

Working Rule

For sol of $y''+Py'+Qy=F(x)$

(1) Find $y_c=c_1y_1+c_2y_2$

If $y_c=c_1\cos x+c_2\sin x$ then Replace c_1 by u_1 & c_2 by u_2 to get assumed $y_p=u_1\cos x+u_2\sin x$

(2) where u_1 & u_2 are fns of 'x'

Note on comparing y_c we get

(3) $y_1=\cos x$ & $y_2=\sin x$

$W=y_1y_2'-y_2y_1'$

$F(x)=$ R.H.S of (1)

(4) For finding u_1 & u_2 use formulae

$u_1=\int\frac{-y_2F(x)}{W}dx$

$u_2=\int\frac{y_1F(x)}{W}dx$

(5) Put values of u_1 & u_2 in Assumed y_p

(6) For General Sol

$y=y_c+y_p$.

Q2 $\frac{d^2y}{dx^2}+y=\tan x\sec x$ — (1)

$(D^2+1)y=\tan x\sec x$

$D^2+1=0$

$D=\pm i$

$y_c=c_1\cos x+c_2\sin x$ — (2)

$y_p=u_1\cos x+u_2\sin x$ (supposed) — (3)

from (2) $y_1=\cos x$ & $y_2=\sin x$

$W=y_1y_2'-y_1'y_2$

$W=\cos x\cos x+\sin x\sin x$

$W=\cos^2x+\sin^2x=\boxed{1}$

$u_1=\int\frac{-y_2\,F(x)}{W}dx$

$=\int-\frac{\sin x\tan x\sec x}{1}dx$

$=\int-\sin x\frac{\sin x}{\cos x}\frac{1}{\cos x}dx$

$=\int-\tan^2x\,dx$

$=\int(1-\sec^2x)dx$

$u_1=x-\tan x$

$u_2=\int\frac{y_1\,F(x)}{W}dx$

$=\int\frac{\cancel{\cos x}\tan x\cancel{\sec x}}{1}dx$

$=-\ln|\cos x|$

$u_2=\ln\sec x$. Put u_1 & u_2 in y_p in (3)

$\therefore y_p=(x-\tan x)\cos x+\ln\sec x\sin x$

G.Sol $y=y_c+y_p$

$y=c_1\cos x+c_2\sin x+(x-\tan x)\cos x+\ln\sec x\sin x$.

(4) $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+5y=e^{-2x}\sec x$ — (1)

$(D^2+4D+5)y=e^{-2x}\sec x$

$D^2+4D+5=0$

$D=\frac{-4\pm\sqrt{16-20}}{2}=\frac{-4\pm2i}{2}=-2\pm i$

$y_c=e^{-2x}(c_1\cos x+c_2\sin x)$ — (2)

$y_p=e^{-2x}(u_1\cos x+u_2\sin x)$ (supposed) (3)

from (2) $y_1=e^{-2x}\cos x$ & $y_2=e^{-2x}\sin x$

$W=y_1y_2'-y_2y_1'$

$=e^{-2x}\cos x[\cos x e^{-2x}-2e^{-2x}\sin x]-[e^{-2x}\sin x](-\sin x e^{-2x}-2\cos x e^{-2x})$

$=e^{-4x}[\cos^2x-2\cos x\sin x]+e^{-4x}[\sin^2x+2\sin x\cos x]$

$=e^{-4x}[\cos^2x-\cancel{2\cos x\sin x}+\sin^2x+\cancel{2\sin x\cos x}]$

$=e^{-4x}$

$u_1=\int\frac{-y_2F(x)}{W}dx$

$=\int\frac{-e^{-2x}\sin x\,e^{-2x}\sec x}{e^{-4x}}dx=\int-\frac{\sin x}{\cos x}dx$

$=\ln|\cos x|$

$u_2=\int\frac{y_1F(x)}{W}dx$

$u_2=\int\frac{\cos x\cancel{e^{-2x}}\cancel{e^{-2x}}\sec x}{\cancel{e^{-4x}}}dx=\int dx=x$

Put u_1,u_2 in (3)

$\therefore y_p=e^{-2x}(\ln|\cos x|\cos x+x\sin x)$

G.Sol $y=y_c+y_p$

$=e^{-2x}(c_1\cos x+c_2\sin x)+e^{-x}[\cos x\ln\cos x+x\sin x]$ Ans

③ $\dfrac{d^2y}{dx^2} - 3\dfrac{dy}{dx} + 2y = (1+e^{-x})^{-1}$

$(D^2 - 3D + 2)y = 0$

$D^2 - 3D + 2 = 0$

$D^2 - D - 2D + 2 = 0$

$D[D-1] - 2[D-1] = 0$

$(D-2)(D-1) = 0.$

$D = 1, 2.$

$y_c = c_1 e^{x} + c_2 e^{2x}$

$y_p = u_1 e^{x} + u_2 e^{2x}$

$F(x) = \dfrac{1}{1+e^{-x}} \qquad y_1 = e^{x},\ y_2 = e^{2x}$

$W = y_1 y_2' - y_2 y_1'$

$= e^{x}\, 2e^{2x} - e^{2x} e^{x}$

$= 2e^{3x} - e^{3x} = \boxed{e^{3x}}$

$u_1 = \displaystyle\int -\frac{y_2 F(x)}{W}\, dx$

$= -\displaystyle\int \frac{e^{2x} \cdot 1}{e^{3x}(1+e^{-x})}\, dx$

$= -\displaystyle\int \frac{e^{-x}}{(1+e^{-x})}\, dx = \ln(1+e^{-x})$

$u_2 = \displaystyle\int \frac{y_1 F(x)}{W}\, dx$

$= \displaystyle\int \frac{e^{x} \cdot 1}{e^{3x}(1+e^{-x})}\, dx$

$= \displaystyle\int \frac{e^{-2x}}{1+e^{-x}}\, dx \qquad$ Put $e^{-x} = z$, $\ -e^{-x} dx = dz$, $\ dx = \dfrac{dz}{-z}$

$= \displaystyle\int \frac{-z^2 \cdot \frac{dz}{z}}{1+z}$

$= \displaystyle\int \frac{-z\, dz}{1+z} = \int \frac{(-z-1+1)\, dz}{1+z}$

$u_2 = -\displaystyle\int dz + \int \frac{dz}{1+z} = -z + \ln(z+1) = -e^{-x} + \ln(1+e^{-x})$

$y_p = \ln(1+e^{-x})\, e^{x} + (-e^{-x} + \ln(1+e^{-x}))\, e^{2x}$

General Sol $y = y_c + y_p$

⑤ $\dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 4y = \dfrac{e^{2x}}{1+x}$

$D^2 - 4D + 4 = 0$

$(D-2)^2 = 0 \qquad D = 2, 2$

$y_c = (c_1 + c_2 x) e^{2x}$

$y_p = u_1 e^{2x} + u_2 x e^{2x}$

$y_1 = e^{2x}, \quad y_2 = x e^{2x}, \quad F(x) = \dfrac{e^{2x}}{1+x}$

$W = y_1 y_2' - y_2 y_1' = e^{2x}(e^{2x} + 2x e^{2x}) - x e^{2x} \cdot 2e^{2x}$

$W = e^{4x}$

$u_1 = \displaystyle\int \frac{-y_2 F(x)\, dx}{W} = -\int x e^{2x} \frac{e^{2x}}{1+x} \frac{dx}{e^{4x}}$

$= -\displaystyle\int \frac{x\, dx}{1+x} = -\int \frac{(x+1-1)\, dx}{1+x}$

$= -x + \ln|1+x|$

$u_2 = \displaystyle\int \frac{y_1 F(x)\, dx}{W} = \int e^{2x} \frac{e^{2x}}{1+x} \frac{dx}{e^{4x}}$

$u_2 \Rightarrow \displaystyle\int \frac{dx}{1+x} = \ln|1+x|$

$y_p = [-x + \ln|1+x|]\, e^{2x} + \ln|1+x|\, x e^{2x}$

General Sol $y = y_c + y_p$

$y = (c_1 + c_2 x) e^{2x} + [-x + \ln|1+x|]\, e^{2x}$
$\qquad + \ln|1+x|\, x e^{2x}.$

x ——— x

$\boxed{10.6}$

⑧ $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=e^{-x}\ln x$

$(D^2+2D+1)y=0$

$D^2+2D+1=0$

$(D+1)^2=0 \Rightarrow D=-1,-1$

$y_c=(c_1+c_2x)e^{-x}$

$y_p=(u_1+u_2x)e^{-x}=u_1e^{-x}+u_2xe^{-x}$

$y_1=e^{-x}$, $y_2=xe^{-x}$, $F(x)=e^{-x}\ln x$

$W=y_1y_2'-y_1'y_2=e^{-x}(-xe^{-x}+e^{-x})-(-e^{-x})(xe^{-x})$
$=\cancel{-xe^{-2x}}+e^{-2x}+\cancel{xe^{-2x}}=\boxed{e^{-2x}}$

$u_1=\int -\frac{y_2F(x)}{W}dx=\int -\frac{xe^{-x}e^{-x}\ln x}{e^{-2x}}dx$

$=-\int \underset{II}{x}\,\underset{I}{\ln x}\,dx=-\left(\ln x\,\frac{x^2}{2}-\int \frac{1}{x}\cdot\frac{x^2}{2}dx\right)$

$=-\ln x\,\frac{x^2}{2}+\int \frac{x}{2}dx=-\ln x\,\frac{x^2}{2}+\frac{x^2}{4}$

$u_2=\int \frac{y_1F(x)}{W}dx=\int \frac{e^{-x}e^{-x}\ln x}{e^{-2x}}dx$

$=\int \ln x\,dx=\ln x\cdot x-\int \frac{1}{x}\cdot x\,dx$

$=x\ln x-\int dx=x\ln x-x$

$y_p=\left(-\ln x\,\frac{x^2}{2}+\frac{x^2}{4}\right)e^{-x}+(x\ln x-x)xe^{-x}$

$=\left[-\ln x\cdot\frac{x^2}{2}+\frac{x^2}{4}+x^2\ln x-x^2\right]e^{-x}$

$=\left[-2x^2\ln x+x^2+4x^2\ln x-4x^2\right]\frac{e^{-x}}{4}$

$y_p=(2x^2\ln x-3x^2)\frac{e^{-x}}{4}$

General Sol $y=y_c+y_p=(c_1+c_2x)e^{-x}+(2x^2\ln x-3x^2)\frac{e^{-x}}{4}$.

⑩ $x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=x^3e^x$

$\frac{d^2y}{dx^2}-\frac{2x}{x^2}\frac{dy}{dx}+\frac{2y}{x^2}=\frac{x^3e^x}{x^2}$

$\frac{d^2y}{dx^2}-\frac{2}{x}\frac{dy}{dx}+\frac{2y}{x^2}=xe^x$

$P=-\frac{2}{x}$ $Q=\frac{2}{x^2}$ $F(x)=xe^x$

$P+xQ=-\frac{2}{x}+x\cdot\frac{2}{x^2}=0$

$\therefore y_1=x$ is another sol of associated homogeneous eq.

$y_2=x^2$ is given sol.

$\therefore y_c=c_1x+c_2x^2$

$y_p=u_1x+u_2x^2$ $y_1=x, y_2=x^2$

$W=y_1y_2'-y_1'y_2$ $F(x)=xe^x$

$W=2x^2-x^2=\boxed{x^2}$

$u_1=\int -\frac{y_2F(x)}{W}dx$

$=\int -\frac{x^2\cdot xe^x}{x^2}dx=-\int \underset{I}{x}\,\underset{II}{e^x}dx$

$u_1=-xe^x+\int 1\cdot e^x dx=-xe^x+e^x$

$u_2=\int \frac{y_1F(x)}{W}dx=\int \frac{x\,xe^x}{x^2}$

$u_2=\int e^x=e^x$

$y_p=(-xe^x+e^x)x+e^xx^2$

$=\cancel{-x^2e^x}+xe^x+\cancel{e^xx^2}$

$y_p=xe^x$

General Sol $y=y_c+y_p$

$y=c_1x+c_2x^2+xe^x$

⑥ $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + y = e^x \sin^{-1}x$ —①

$(D^2 - 2D + 1)y = e^x \sin^{-1} x$

$D^2 - 2D + 1 = 0$

$(D-1)^2 = 0$

$D = 1, 1$

$y_c = (c_1 + c_2 x)e^x$ —②

$y_p = (u_1 + u_2 x)e^x$ (Supposed) —③

from ② $y_1 = e^x$ & $y_2 = xe^x$

$W = y_1 y_2' - y_1' y_2$

$= e^x(1 \cdot e^x + xe^x) - e^x xe^x$

$= e^{2x} + \cancel{xe^{2x}} - \cancel{xe^{2x}}$

$W = e^{2x}$

$u_1 = \int \dfrac{-y_2 F(x)}{W} dx$

$= \int \dfrac{-xe^x e^x \sin^{-1}x}{e^{2x}} dx$

$= \int -x \sin^{-1}x \, dx$ IBP

$= -\dfrac{x^2}{2} \sin^{-1}x + \int \dfrac{x^2}{2} \dfrac{dx}{\sqrt{1-x^2}}$

$= -\dfrac{x^2}{2} \sin^{-1}x - \dfrac{1}{2}\int \dfrac{1-x^2 - 1}{\sqrt{1-x^2}} dx$

$= -\dfrac{x^2}{2} \sin^{-1}x - \dfrac{1}{2}\int \sqrt{1-x^2}\,dx + \dfrac{1}{2}\int \dfrac{dx}{\sqrt{1-x^2}}$

$= -\dfrac{x^2}{2} \sin^{-1}x - \dfrac{1}{2}\left[\dfrac{x\sqrt{1-x^2}}{2} + \dfrac{1}{2}\sin^{-1}x\right] + \dfrac{\sin^{-1}x}{2}$

$= -\dfrac{x^2}{2} \sin^{-1}x - \dfrac{x\sqrt{1-x^2}}{4} - \dfrac{\sin^{-1}x}{4} + \dfrac{\sin^{-1}x}{2}$

$u_1 = -\dfrac{x^2}{2} \sin^{-1}x - \dfrac{x\sqrt{1-x^2}}{4} + \dfrac{\sin^{-1}x}{4}$

$u_2 = \int \dfrac{y_1 F(x)}{W} dx =$

$= \int \dfrac{\cancel{e^x}\, \cancel{e^x} \sin^{-1}x}{\cancel{e^{2x}}} dx = \int 1 \cdot \sin^{-1}x \, dx$ IBP

$u_2 = x \sin^{-1}x - \int x \cdot \dfrac{1}{\sqrt{1-x^2}} dx$

$u_2 = x \sin^{-1}x + \dfrac{1}{2}\int (1-x^2)^{-\frac{1}{2}}(-2x)\,dx$

$= x \sin^{-1}x + \dfrac{1}{2} \dfrac{(1-x^2)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}$

$u_2 = x \sin^{-1}x + \dfrac{\cancel{1}}{\cancel{2}} \dfrac{\sqrt{1-x^2}}{\cancel{\frac{1}{2}}}$

Put u_1, u_2 in ③

$y_p = e^x\left[\left(-\dfrac{x^2}{2}\sin^{-1}x - \dfrac{x\sqrt{1-x^2} + \sin^{-1}x}{4}\right) + \left(x\sin^{-1}x + \sqrt{1-x^2}\right)x\right]$

$y_p = e^x\left[-\dfrac{x^2}{2}\sin^{-1}x - \dfrac{x\sqrt{1-x^2}}{4} + \sin^{-1}x + x^2\sin^{-1}x + x\sqrt{1-x^2}\right]$

G.Sol $y = y_c + y_p$

$= (c_1 + c_2 x)e^x + e^x\left(-\dfrac{x^2}{2}\sin^{-1}x - \dfrac{x\sqrt{1-x^2}}{4} + \sin^{-1}x + x^2\sin^{-1}x + x\sqrt{1-x^2}\right)$

Q7 $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = e^x \tan 2x$ — ①

$(D^2-2D+5)y = e^x \tan 2x$

$D^2-2D+5 = 0$

$D = \frac{2 \pm \sqrt{4-20}}{2} = \frac{2 \pm i4}{2} = 1 \pm i2$

$y_c = e^x(c_1 \cos 2x + c_2 \sin 2x)$ — ②

$y_p = e^x(u_1 \cos 2x + u_2 \sin 2x)$ supposed ③

from ② $y_1 = e^x \cos 2x$ & $y_2 = e^x \sin 2x$

$y_1' = e^x \cos 2x + e^x(-2\sin 2x)$

$y_2' = e^x \sin 2x + e^x \cos 2x(2)$

$W = y_1 y_2' - y_1' y_2$

$= (e^x \cos 2x)\, e^x(\sin 2x + 2\cos 2x)$

$\quad - e^x(\cos 2x - 2\sin 2x)(e^x \sin 2x)$

$= e^{2x}(\cancel{\cos 2x \sin 2x} + 2\cos^2 2x - \cancel{\sin 2x \cos 2x} + 2\sin^2 2x)$

$W = e^{2x}\, 2(\cos^2 2x + \sin^2 2x) = \boxed{2e^{2x}}$

$u_1 = \int -\frac{y_2 F(x)}{W} dx = \int -\frac{\cancel{e^x} \sin 2x\, \cancel{e^x} \tan 2x\, dx}{2\cancel{e^{2x}}}$

$= -\frac{1}{2}\int \frac{\sin^2 2x}{\cos 2x} dx$

Let $\sin 2x = t$
$2\cos 2x\, dx = dt$
$dx = \frac{dt}{2\cos 2x}$

$= -\frac{1}{2}\int \frac{t^2}{\cos 2x} \frac{dt}{2\cos 2x}$

$\because \cos^2 2x = 1 - \sin^2 2x = 1 - t^2$

$= -\frac{1}{4}\int \frac{t^2\, dt}{1-t^2}$

$= \frac{1}{4}\int \left[\frac{(1-t^2)}{(1-t^2)} - 1\right] dt$

$= \frac{1}{4}\int dt - \frac{1}{4}\int \frac{dt}{1-t^2}$

$\because \int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left[\frac{a+x}{a-x}\right]$

$= \frac{1}{4}t - \frac{1}{4}\left[\frac{1}{2}\ln\frac{(1+t)}{(1-t)}\right]$

$u_1 = \frac{\sin 2x}{4} - \frac{1}{8}\ln\left(\frac{1+\sin 2x}{1-\sin 2x}\right)$

$u_1 = \frac{\sin 2x}{4} - \frac{1}{8}\ln\left(\frac{\sin^2 x + \cos^2 x + 2\sin x \cos x}{\sin^2 x + \cos^2 x - 2\sin x \cos x}\right)$

$= \frac{\sin 2x}{4} - \frac{1}{8}\ln\left(\frac{\cos x + \sin x}{\cos x - \sin x}\right)^2$

$= \frac{\sin 2x}{4} - \frac{1}{8}\ln\left(\frac{1+\tan x}{1-\tan x}\right)^2$

$= \frac{\sin 2x}{4} - \frac{2}{8}\ln\left(\tan\left(\frac{\pi}{4}+x\right)\right)$

$\because \tan\frac{\pi}{4} = 1$

$\because \tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \tan\beta}$

$u_1 = \frac{\sin 2x}{4} - \frac{1}{4}\ln\left(\tan\left(\frac{\pi}{4}+x\right)\right)$

$u_2 = \int \frac{y_1 F(x)}{W} dx$

$= \int \frac{\cancel{e^x} \cos 2x \cdot \cancel{e^x} \tan 2x}{2\cancel{e^{2x}}} dx$

$= \frac{1}{2}\int \cancel{\cos 2x} \frac{\sin 2x}{\cancel{\cos 2x}} dx$

$= \frac{1}{2}\left(-\frac{\cos 2x}{2}\right)$

$= -\frac{1}{4}\cos 2x$

Put u_1, u_2 in y_p

$\therefore y_p = \frac{e^x}{4}\left[\sin 2x - \ln\left(\tan\left(\frac{\pi}{4}+x\right)\right)\cos 2x - (\cos 2x)\sin 2x\right]$

G.Sol $y = y_c + y_p$

$y = e^x(c_1 \cos 2x + c_2 \sin 2x) + \frac{e^x}{4}\left[\sin 2x - \ln\left(\tan\left(\frac{\pi}{4}+x\right)\right)\cos 2x - (\cos 2x)\sin 2x\right]$

Ans.

(9) $\dfrac{d^2y}{dx^2}+2\dfrac{dy}{dx}+2y=2e^{-x}\tan^2 x$

$(D^2+2D+2)y=0$

$D^2+2D+2=0$

$D=\dfrac{-2\pm\sqrt{4-4.1.2}}{2}$

$D=\dfrac{-2\pm 2i\sqrt{-4}}{2}=-1\pm i$

$y_c=e^{-x}(c_1\cos x+c_2\sin x)$

$y_p=e^{-x}(u_1\cos x+u_2\sin x)$

$y_p=u_1e^{-x}\cos x+u_2e^{-x}\sin x$

$y_1=e^{-x}\cos x,\quad y_2=e^{-x}\sin x,\ F(x)=2e^{-x}\tan^2 x$

$W=y_1y_2'-y_1'y_2$

$=e^{-x}\cos x(-e^{-x}\sin x+e^{-x}\cos x)-(-e^{-x}\cos x-e^{-x}\sin x)(e^{-x}\sin x)$

$=-e^{-2x}\cancel{\cos x\sin x}+e^{-2x}\cos^2 x+e^{-2x}\cancel{\cos x\sin x}+e^{-2x}\sin^2 x$

$=e^{-2x}(\cos^2 x+\sin^2 x)$

$\boxed{W=e^{-2x}}$

$u_1=\displaystyle\int\frac{-y_2F(x)}{W}dx$

$=\displaystyle\int\frac{-e^{-x}\sin x(2e^{-x}\tan^2 x)}{e^{-2x}}dx$

$=-2\displaystyle\int\sin x\tan^2 x\,dx$

$=-2\displaystyle\int\frac{\sin^3 x}{\cos^2 x}dx$ Put $\cos x=z$, $-\sin x\,dx=dz$

$=2\displaystyle\int\frac{\sin^2 x}{\cos^2 x}(-\sin x\,dx)$ Also $1-\cos^2 x=\sin^2 x$, $1-z^2=\sin^2 x$.

$=2\displaystyle\int\frac{(1-z^2)dz}{z^2}$

$=2\displaystyle\int\frac{dz}{z^2}-2\int z\,dz$

$=\dfrac{-2}{z}-z^2=-2\left[\dfrac{1}{\cos x}+\cos x\right]$

$-2(\cos^2 x+1)$

$u_2=\displaystyle\int\frac{y_1F(x)dx}{W}=\int\frac{e^{-x}\cos x\,2e^{-x}\tan^2 x\,dx}{e^{-2x}}$

$u_2=2\displaystyle\int\cos x\tan^2 x\,dx$

$=2\displaystyle\int\frac{\sin^2 x}{\cos x}dx$ Put $\sin x=z$, $\cos x\,dx=dz$

$=2\displaystyle\int\frac{z^2\cos x\,dx}{\cos^2 x}$

$=2\displaystyle\int\frac{z^2\,dz}{1-z^2}=-2\int\frac{-z^2}{1-z^2}dz$

$=-2\displaystyle\int\frac{1-z^2-1}{1-z^2}dz$

$=-2\displaystyle\int\left(1-\frac{1}{1-z^2}\right)dz$

$=-2\displaystyle\int dz+2\int\frac{dz}{1-z^2}$

$=-2z+2\cdot\dfrac{1}{2}\ln\dfrac{(1+z)}{1-z}$

$=-2\sin x+\ln\left|\dfrac{1+\sin x}{1-\sin x}\right|$

$u_2=-2\sin x+\ln\left|\tan\left(\dfrac{x}{2}+\dfrac{\pi}{4}\right)\right|$

$y_p=-2(\cos^2 x+1)e^{-x}\cos x+\left[-2\sin x+\ln\left(\tan\frac{x}{2}+\frac{\pi}{4}\right)\right](e^{-x}\sin x)$

General Sol $y=y_c+y_p$

$=e^{-x}(c_1\cos x+c_2\sin x)+\left[-2(\cos^2 x+1)e^{-x}\cos x+\left(-2\sin x+\ln\left|\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right|\right)e^{-x}\sin x\right]$ Ans.

$\boxed{10.6}$ $\boxed{10.6-8}$

(11) $x^2\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} - y = \dfrac{1}{1+x}$ $y_1 = \frac{1}{x}$ is a sol (given)

$$\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} - \frac{y}{x^2} = \frac{1}{x^2(1+x)} \quad \text{— (1)}$$

$P = \frac{1}{x}$ $Q = \frac{-1}{x^2}$

$P + Qx = \frac{1}{x} + \left(\frac{-1}{x^2}\right)x = 0$

$\therefore y_2 = x$ is another sol of $\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} - \frac{y}{x^2} = 0$.

$y_c = c_1\frac{1}{x} + c_2 x$

$y_p = u_1\cdot\frac{1}{x} + u_2\cdot x$

$y_1 = \frac{1}{x}$, $y_2 = x$, $F(x) = \frac{1}{x^2(1+x)}$

$W = y_1y_2' - y_1'y_2 = \frac{1}{x}\cdot 1 - \left(\frac{-1}{x^2}\right)x$

$W = \frac{1}{x} + \frac{1}{x} = \frac{2}{x}$

$u_1 = \int \frac{-y_2F(x)}{W}dx = -\int x\cdot\frac{1}{x^2(1+x)}\cdot\frac{x}{2}dx$

$u_1 = -\frac{1}{2}\int\frac{dx}{1+x} = -\frac{1}{2}\ln(1+x)$

$u_2 = \int\frac{y_1F(x)}{W}dx$

$= \int\frac{1}{x}\cdot\frac{1}{x^2(1+x)}\cdot\frac{x}{2}dx$

$= \frac{1}{2}\int\frac{dx}{x^2(1+x)}$ —— (A)

$\frac{1}{x^2(1+x)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{1+x}$

$1 = Ax(1+x) + B(1+x) + Cx^2$

$x = 0 \Rightarrow \boxed{1 = B}$

$x = -1 \Rightarrow \boxed{1 = C}$

Comparing coeff of x^2

$0 = A + C$

$0 = A + 1 \Rightarrow \boxed{A = -1}$

$\therefore \frac{1}{x^2(1+x)} = \frac{-1}{x} + \frac{1}{x^2} + \frac{1}{1+x}$

$\therefore$ (A) becomes $\frac{1}{2}\int\left(\frac{-1}{x} + \frac{1}{x^2} + \frac{1}{1+x}\right)dx$

$u_2 = -\frac{1}{2}\ln x - \frac{1}{2x} + \frac{1}{2}\ln|1+x|$

$y_p = \left[\frac{-1}{2}\ln(1+x)\cdot\frac{1}{x}\right] + \left[-\frac{1}{2}\ln x - \frac{1}{2x} + \frac{1}{2}\ln(1+x)\right](x)$

$y_p =$

General Sol is

$y = y_c + y_p$

$= c_1\frac{1}{x} + c_2 x - \frac{1}{2x}\ln(1+x) + x\left(-\frac{1}{2}\ln x - \frac{1}{2x} + \frac{1}{2}\ln(1+x)\right)$

$\boxed{10.6-9}$

(12) ~~Find~~ the general sol of [illegible]

$$x(x-2)\frac{d^2y}{dx^2}-(x^2-2)\frac{dy}{dx}+2(x-1)y=3x^2(x-2)^2e^x \quad \text{—(1)}$$

given that $y_1=x^2$ is a sol of associated Homo Eq.

Sol from (1) $\dfrac{d^2y}{dx^2}-\dfrac{(x^2-2)}{x(x-2)}\dfrac{dy}{dx}+\dfrac{2(x-1)}{x(x-2)}y=\dfrac{3x^2(x-2)^2e^x}{x(x-2)}$

$$y''-\frac{(x^2-2)}{x(x-2)}y'+\frac{2(x-1)y}{x(x-2)}=3x(x-2)e^x$$

Compare with $y''+py'+qy=F(x)$

$$p=-\frac{(x^2-2)}{x(x-2)},\quad q=\frac{2(x-1)}{x(x-2)},\quad F(x)=3x(x-2)e^x$$

$$\because p+qx=-\frac{(x^2-2)}{x(x-2)}+\frac{2(x-1)}{x(x-2)}x\neq 0$$

Now check

$$1+p+q=1+\left[-\frac{(x^2-2)}{x(x-2)}\right]+\frac{2(x-1)}{x(x-2)}$$

$$=\frac{x(x-2)-x^2+\cancel{2}+2x-\cancel{2}}{x(x-2)}=\frac{\cancel{x^2}-\cancel{2x}-\cancel{x^2}+\cancel{2x}}{x(x-2)}=0$$

thus $y_2=e^x$ is another sol of associated Homogeneous eq.

So $y_c=c_1y_1+c_2y_2$

$y_c=c_1x^2+c_2e^x$

$y_p=u_1x^2+u_2e^x$ (supposed)

$W=y_1y_2'-y_1'y_2$

$=x^2e^x-2xe^x$

$W=xe^x(x-2)$

$$\because u_1=\int-\frac{y_2F(x)}{W}dx$$

$$=-\int\frac{\cancel{e^x}\,3\cancel{x}\,\cancel{(x-2)}e^x}{\cancel{x}\cancel{e^x}\cancel{(x-2)}}dx$$

$$=-3\int e^x dx$$

$u_1=-3e^x$

$$u_2=\int\frac{y_1F(x)}{W}dx$$

$$=\int\frac{x^2\cdot 3\cancel{x}\cancel{(x-2)}\cancel{e^x}}{\cancel{x}\cancel{e^x}\cancel{(x-2)}}dx$$

$$=3\int x^2dx$$

$u_2=3\dfrac{x^3}{3}=x^3$

Put u_1,u_2 in y_p

$y_p=(-3e^x)x^2+(x^3)e^x$

G.Sol is $y=y_c+y_p$

$y=c_1x^2+c_2e^x+x^3e^x-3e^xx^2$ Ans

$\boxed{10\cdot 6-10}$

(13) Find sol of $(\sin^2 x)\dfrac{d^2y}{dx^2} - (\sin 2x)\dfrac{dy}{dx} + (1+\cos^2 x)\,y = \sin^3 x$ given that $y_1 = \sin x$ & $y_2 = x\sin x$ are linearly independent sol of associated homogeneous eq.

Sol $\because y_1 = \sin x$ & $y_2 = x\sin x$ (Given)

Note from (1) $\dfrac{d^2y}{dx^2} - \dfrac{\sin 2x}{\sin^2 x}\dfrac{dy}{dx} + \dfrac{(1+\cos^2 x)y}{\sin^2 x} = \dfrac{\sin^3 x}{\sin^2 x}$

$\Rightarrow \dfrac{d^2y}{dx^2} = \dfrac{\sin 2x}{\sin^2 x}\dfrac{dy}{dx} + \dfrac{1+\cos^2 x}{\sin^2 x}y = \sin$ (1)

$\therefore F(x) = \sin x$

$$\text{Hence } Y_c = c_1 y_1 + c_2 y_2$$
$$= c_1 \sin x + c_2 x\sin x$$
$$y_p = u_1 \sin x + u_2 x\sin x$$
$$W = y_1 y_2' - y_1' y_2$$
$$= \sin x(\sin x + x\cos x) - \cos x\, x\sin x$$
$$= \sin^2 x + \cancel{x\sin x\cos x} - \cancel{x\cos x\sin x}$$
$$W = \sin^2 x$$
$$u_1 = \int \frac{-y_2 F(x)}{W}$$
$$= -\int \frac{x\cancel{\sin x}\,\cancel{\sin x}}{\cancel{\sin^2 x}}dx$$
$$= -\int x\,dx = \boxed{-\frac{x^2}{2}}$$
$$u_2 = \int \frac{y_1 F(x)}{W}$$
$$= \int \frac{\sin x \sin x}{\sin^2 x}dx$$
$$= \int dx = \boxed{x}$$
$$y_p = -\frac{x^2}{2}\sin x + x^2 \sin x$$
$$\text{G.Sol } y = y_c + y_p$$
$$= c_1 \sin x + c_2 x\sin x - \frac{x^2}{2}\sin x + x^2 \sin x$$
$$= \left(c_1 + c_2 x - \frac{x^2}{2} + x^2\right)\sin x$$
$$= \left(c_1 + c_2 x + \frac{x^2}{2}\right)\sin x \quad \text{Ans.}$$

10.6

10.6 – 11

(14) $$\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - y = \frac{2e^x}{x^2} \quad \text{—— (1)}$$

The characteristic eq is $D^3 - 3D^2 + 3D - 1 = 0$

$(D-1)^3 = 0$ therefore $D = 1,1,1$

C.F of (1) is $\quad y_c = c_1 e^x + c_2 x e^x + c_3 x^2 e^x$

Let $\quad y_p = u_1 e^x + u_2 x e^x + u_3 x^2 e^x$

Here $y_1 = e^x \qquad (\because y_p = u_1 y_1 + u_2 y_2 + u_3 y_3$ assumed$)$

$y_2 = x e^x$

$y_3 = x^2 e^x \quad , \; F(x) = \frac{2e^x}{x^2}$

Substituting values in $u_1' y_1 + u_2' y_2 + u_3' y_3 = 0 \Rightarrow u_1' e^x + u_2' x e^x + u_3' x^2 e^x = 0$

Substituting values in $u_1' y_1' + u_2' y_2' + u_3' y_3' = 0 \Rightarrow u_1' e^x + u_2'(e^x + x e^x) + u_3'(2x e^x + x^2 e^x) = 0$

Substituting values in $u_1' y_1'' + u_2' y_2'' + u_3' y_3'' = F(x)$

$$\Rightarrow u_1' e^x + u_2'(2e^x + x e^x) + u_3'(2e^x + 4x e^x + x^2 e^x) = \frac{2e^x}{x^2}$$

Solving these eqs for u_1', u_2', u_3' by Cramer's Rule.

$$u_1' = \frac{\begin{vmatrix} 0 & xe^x & x^2e^x \\ 0 & e^x + xe^x & 2xe^x + x^2e^x \\ \frac{2e^x}{x^2} & 2e^x + xe^x & 2e^x + 4xe^x + x^2e^x \end{vmatrix}}{\begin{vmatrix} e^x & xe^x & x^2e^x \\ e^x & e^x + xe^x & 2xe^x + x^2e^x \\ e^x & 2e^x + xe^x & 2e^x + 4xe^x + x^2e^x \end{vmatrix}} = \frac{\frac{2e^x}{x^2}\begin{vmatrix} xe^x & x^2e^x \\ e^x + xe^x & 2xe^x + x^2e^x \end{vmatrix}}{\begin{vmatrix} e^x & xe^x & x^2e^x \\ e^x & e^x + xe^x & 2xe^x + x^2e^x \\ e^x & 2e^x + xe^x & 2e^x + 4xe^x + x^2e^x \end{vmatrix}}$$

$$u_1' = \frac{\frac{2e^x}{x^2}\left[xe^x(2xe^x + x^2e^x) - x^2e^x(e^x + xe^x)\right]}{\begin{vmatrix} e^x & xe^x & x^2e^x \\ 0 & e^x & 2xe^x \\ 0 & 2e^x & 2e^x + 4xe^x \end{vmatrix}} \qquad \begin{matrix} -R_1 + R_2 \\ -R_1 + R_3 \end{matrix}$$

$\boxed{10.6-12}$

$$= \frac{\frac{2e^x}{x^2}\left(2x^2e^{2x} + \cancel{x^3e^{2x}} - x^2e^{2x} - \cancel{x^3e^{2x}}\right)}{e^x\left[e^x(2e^x+4xe^x) - 2e^x(2xe^x)\right]}$$

$$= \frac{\frac{2e^x}{x^2}\left(x^2e^{2x}\right)}{e^x\left[2e^{2x} + \cancel{4xe^{2x}} - \cancel{4xe^{2x}}\right]} = \frac{2e^{3x}}{2e^{3x}} = 1$$

$$u_2' = \frac{\begin{vmatrix} e^x & 0 & x^2e^x \\ e^x & 0 & 2xe^x+x^2e^x \\ e^x & \frac{2e^x}{x^2} & 2e^x+4xe^x+x^2e^x \end{vmatrix}}{2e^{3x}} \text{ (as solved above)} = \frac{-\frac{2e^x}{x^2}\begin{vmatrix} e^x & x^2e^x \\ e^x & 2xe^x+x^2e^x \end{vmatrix}}{2e^{3x}}$$

$$= \frac{-\frac{2e^x}{x^2}\left[e^x(2xe^x+x^2e^x) - e^x(x^2e^x)\right]}{2e^{3x}} = \frac{-\frac{2e^{2x}}{x^2}\left(2xe^x + \cancel{x^2e^x} - \cancel{x^2e^x}\right)}{2e^{3x}}$$

$$= \frac{-4x\cancel{e^{3x}}}{x^2}\cdot\frac{1}{\cancel{2e^{3x}}} = -\frac{2}{x}$$

$$u_3' = \frac{\begin{vmatrix} e^x & xe^x & 0 \\ e^x & e^x+xe^x & 0 \\ e^x & 2e^x+xe^x & \frac{2e^x}{x^2} \end{vmatrix}}{2e^{3x}} = \frac{\frac{2e^x}{x^2}\begin{vmatrix} e^x & xe^x \\ e^x & e^x+xe^x \end{vmatrix}}{2e^{3x}}$$

$$= \frac{\frac{2e^x}{x^2}\left[e^x(e^x+xe^x) - e^x(xe^x)\right]}{2e^{3x}} = \frac{\frac{2e^x}{x^2}\left(e^{2x} + \cancel{xe^{2x}} - \cancel{xe^{2x}}\right)}{2e^{3x}}$$

$$= \frac{\cancel{2e^{3x}}}{x^2\,\cancel{2e^{3x}}} = \frac{1}{x^2}$$

$$\therefore\ u_1 = \int dx = x$$

$$u_2 = \int -\frac{2}{x}\,dx = -2\ln|x|$$

$$u_3 = \int \frac{dx}{x^2} = -\frac{1}{x}$$

$$y_p = u_1y_1 + u_2y_2 + u_3y_3$$

$$= xe^x + xe^x(-2\ln|x|) + x^2e^x\left(-\tfrac{1}{x}\right)$$

$$= \cancel{xe^x} - 2xe^x\ln|x| - \cancel{xe^x}$$

$$y_p = -2xe^x\ln|x|$$

$$y = c_1e^x + c_2xe^x + c_3x^2e^x - 2xe^x\ln|x|$$

$\boxed{10.6}$

(15) $\dfrac{d^3y}{dx^3} - 2\dfrac{dy}{dx} - 4y = e^{-x}\tan x$ $\boxed{10.6.13}$

Sol $D^3 - 2D - 4 = 0$

By synthetic division

$$\begin{array}{c|cccc} 2 & 1 & 0 & -2 & -4 \\ & \downarrow & 2 & 4 & 4 \\ \hline & 1 & 2 & 2 & \llcorner 0 \end{array}$$

$\therefore 2$ is Root $D = 2$

$(D-2)(D^2+2D+2) = 0$

$D^2 + 2D + 2 = 0$

$D = \dfrac{-2 \pm \sqrt{4 - 4\cdot1\cdot2}}{2(1)} = \dfrac{-2 \pm \sqrt{4-8}}{2}$

$= \dfrac{-2 \pm \sqrt{-4}}{2} = \dfrac{\cancel{2}(-1 \pm i)}{\cancel{2}}$

$D = 2, -1 \pm i$

$y_c = c_1 e^{2x} + (c_2 \cos x + c_3 \sin x)e^{-x}$

Let $y_p = u_1 y_1 + u_2 y_2 + u_3 y_3$

where $y_1 = e^{2x}$, $y_2 = e^{-x}\cos x$, $y_3 = e^{-x}\sin x$

$y_1' = 2e^{2x}$ $\quad y_1'' = 4e^{2x}$

$y_2' = -e^{-x}\cos x - e^{-x}\sin x$

$y_2'' = e^{-x}\cancel{\cos x} + e^{-x}\sin x + e^{-x}\sin x - \cancel{e^{-x}\cos x}$

$y_2'' = 2e^{-x}\sin x$

$y_3' = -e^{-x}\sin x + e^{-x}\cos x$

$y_3'' = e^{-x}\cancel{\sin x} - e^{-x}\cos x - e^{-x}\cos x - \cancel{e^{-x}\sin x}$

$y_3'' = -2e^{-x}\cos x$

Putting values in $u_1'y_1 + u_2'y_2 + u_3'y_3 = 0$: $\quad u_1'e^{2x} + u_2'e^{-x}\cos x + u_3'e^{-x}\sin x = 0$

Putting values in $u_1'y_1' + u_2'y_2' + u_3'y_3' = 0$: $\quad u_1'2e^{2x} + u_2'(-e^{-x}\cos x - e^{-x}\sin x) + u_3'(-e^{-x}\sin x + e^{-x}\cos x) = 0$

Putting values in $u_1'y_1'' + u_2'y_2'' + u_3'y_3'' = f(x)$: $\quad u_1'4e^{2x} + u_2'2e^{-x}\sin x + u_3'(-2e^{-x}\cos x) = e^{-x}\tan x$

Solving by Cramers Rule

$$u_1' = \dfrac{\begin{vmatrix} 0 & e^{-x}\cos x & e^{-x}\sin x \\ 0 & -e^{-x}\cos x - e^{-x}\sin x & -e^{-x}\sin x + e^{-x}\cos x \\ e^{-x}\tan x & 2e^{-x}\sin x & -2e^{-x}\cos x \end{vmatrix}}{\begin{vmatrix} e^{2x} & e^{-x}\cos x & e^{-x}\sin x \\ 2e^{2x} & -e^{-x}\cos x - e^{-x}\sin x & -e^{-x}\sin x + e^{-x}\cos x \\ 4e^{2x} & 2e^{-x}\sin x & -2e^{-x}\cos x \end{vmatrix}} = \cancel{e^{-x}\tan x}$$

$$= \dfrac{e^{-x}\tan x \begin{vmatrix} e^{-x}\cos x & e^{-x}\sin x \\ -e^{-x}\cos x - e^{-x}\sin x & -e^{-x}\sin x + e^{-x}\cos x \end{vmatrix}}{e^{2x}\cdot e^{-x}\cdot e^{-x}\begin{vmatrix} 1 & \cos x & \sin x \\ 2 & -\cos x - \sin x & -\sin x + \cos x \\ \end{vmatrix}}$$

Taking e^{2x} Common from C_1
e^{-x} Common from C_2
e^{-x} Common from C_3

$\boxed{10.6\text{-}14}$

Taking e^{-x} common from C_1 & C_2

$$U_1' = \frac{e^{-3x}\operatorname{Tan}x \begin{vmatrix} \operatorname{Cos}x & \operatorname{Sin}x \\ -\operatorname{Cos}x-\operatorname{Sin}x & -\operatorname{Sin}x+\operatorname{Cos}x \end{vmatrix}}{1\left[2\operatorname{Cos}^2x+2\cancel{\operatorname{Sin}x\operatorname{Cos}x}+2\operatorname{Sin}^2x-2\cancel{\operatorname{Sin}x\operatorname{Cos}x}\right]-\operatorname{Cos}x(-4\operatorname{Cos}x+4\operatorname{Sin}x-4\operatorname{Cos}x)+\operatorname{Sin}x(4\operatorname{Sin}x+4\operatorname{Cos}x+4\operatorname{Sin}x)}$$

$$= \frac{e^{-3x}\operatorname{Tan}x\left[-\operatorname{Sin}x\cancel{\operatorname{Cos}x}+\operatorname{Cos}^2x+\operatorname{Sin}x\cancel{\operatorname{Cos}x}+\operatorname{Sin}^2x\right]}{2(\operatorname{Cos}^2x+\operatorname{Sin}^2x)+8\operatorname{Cos}^2x-4\operatorname{Sin}x\cancel{\operatorname{Cos}x}+8\operatorname{Sin}^2x+4\operatorname{Sin}x\cancel{\operatorname{Cos}x}} = \frac{e^{-3x}\operatorname{Tan}x}{2+8(\operatorname{Sin}^2x+\operatorname{Cos}^2x)}$$

$$\boxed{U_1' = \frac{e^{-3x}\operatorname{Tan}x}{10}}$$

$$U_2' = \frac{\begin{vmatrix} e^{2x} & 0 & e^{-x}\operatorname{Sin}x \\ 2e^{2x} & 0 & -e^{-x}\operatorname{Sin}x+e^{-x}\operatorname{Cos}x \\ 4e^{2x} & e^{-x}\operatorname{Tan}x & -2e^{-x}\operatorname{Cos}x \end{vmatrix}}{10}$$

$$U_2' = -e^{-x}\operatorname{Tan}x\frac{\begin{vmatrix} e^{2x} & e^{-x}\operatorname{Sin}x \\ 2e^{2x} & -e^{-x}\operatorname{Sin}x+e^{-x}\operatorname{Cos}x \end{vmatrix}}{10} = -\frac{\cancel{e^{-x}}\operatorname{Tan}x\,\cancel{e^{2x}}\,\cancel{e^{-x}}\begin{vmatrix} 1 & \operatorname{Sin}x \\ 2 & -\operatorname{Sin}x+\operatorname{Cos}x \end{vmatrix}}{10}$$

$$U_2' = -\frac{\operatorname{Tan}x(-\operatorname{Sin}x+\operatorname{Cos}x-2\operatorname{Sin}x)}{10} = +\frac{\operatorname{Tan}x}{10}(3\operatorname{Sin}x-\operatorname{Cos}x)$$

$$U_3' = \frac{\begin{vmatrix} e^{2x} & e^{-x}\operatorname{Cos}x & 0 \\ 2e^{2x} & -e^{-x}\operatorname{Cos}x-e^{-x}\operatorname{Sin}x & 0 \\ 4e^{2x} & 2e^{-x}\operatorname{Sin}x & e^{-x}\operatorname{Tan}x \end{vmatrix}}{10} = e^{-x}\operatorname{Tan}x\frac{\begin{vmatrix} e^{2x} & e^{-x}\operatorname{Cos}x \\ 2e^{2x} & -e^{-x}\operatorname{Cos}x-e^{-x}\operatorname{Sin}x \end{vmatrix}}{10}$$

$$= e^{-x}\operatorname{Tan}x\cdot e^{2x}\cdot e^{-x}\frac{\begin{vmatrix} 1 & \operatorname{Cos}x \\ 2 & -\operatorname{Cos}x-\operatorname{Sin}x \end{vmatrix}}{10} = \frac{\operatorname{Tan}x(-\operatorname{Cos}x-\operatorname{Sin}x-2\operatorname{Cos}x)}{10}$$

$$U_3' = \frac{\operatorname{Tan}x}{10}(-3\operatorname{Cos}x-\operatorname{Sin}x)$$

$$U_1 = \frac{1}{10}\int e^{-3x}\operatorname{Tan}x\,dx$$

$$U_2 = \frac{1}{10}\int(3\operatorname{Tan}x\operatorname{Sin}x \mp \operatorname{Tan}x\operatorname{Cos}x)dx = \frac{1}{10}\int\left(3\frac{\operatorname{Sin}^2x}{\operatorname{Cos}x} \mp \frac{\operatorname{Sin}x}{\operatorname{Cos}x}\operatorname{Cos}x\right)dx$$

$$= \frac{1}{10}\int\left(\frac{3(1-\operatorname{Cos}^2x)}{\operatorname{Cos}x} \mp \operatorname{Sin}x\right)dx = \frac{1}{10}\int(3\operatorname{Sec}x-3\operatorname{Cos}x \mp \operatorname{Sin}x)dx$$

$$= \frac{1}{10}\left[3\ln|\operatorname{Sec}x+\operatorname{Tan}x|-3\operatorname{Sin}x+\operatorname{Cos}x\right]$$

$U_3 =$

$$U_3 = \frac{1}{10}\int(-3\operatorname{Tan}x\operatorname{Cos}x - \operatorname{Tan}x\operatorname{Sin}x)\,dx$$

$$= \frac{1}{10}\int\left(-3\frac{\operatorname{Sin}x}{\cancel{\operatorname{Cos}x}}\cancel{\operatorname{Cos}x} - \frac{\operatorname{Sin}^2 x}{\operatorname{Cos}x}\right)dx$$

$$= \frac{1}{10}\int\left\{-3\operatorname{Sin}x - \frac{(1-\operatorname{Cos}^2 x)}{\operatorname{cos}x}\right\}dx$$

$$= \frac{1}{10}\int(-3\operatorname{Sin}x - \operatorname{Sec}x + \operatorname{Cos}x)\,dx$$

$$= \frac{1}{10}\left[3\operatorname{Cos}x - \ln|\operatorname{Sec}x + \operatorname{Tan}x| + \operatorname{Sin}x\right]$$

$$Y_p = U_1Y_1 + U_2Y_2 + U_3Y_3$$

$$= \frac{e^{2x}}{10}\int e^{-3x}\tan x\,dx + \frac{e^{-x}\operatorname{Cos}x}{10}\left[3\ln|\operatorname{Sec}x + \operatorname{Tan}x| - 3\operatorname{Sin}x + \operatorname{Cos}x\right]$$
$$+ \frac{e^{-x}\operatorname{Sin}x}{10}\left[3\operatorname{Cos}x - \ln|\operatorname{Sec}x + \operatorname{Tan}x| + \operatorname{Sin}x\right]$$

$$= \frac{e^{2x}}{10}\int e^{-3x}\tan x\,dx + \frac{e^{-x}}{10}\left[3\operatorname{Cos}x\ln|\operatorname{Sec}x + \operatorname{Tan}x| - 3\operatorname{Sin}x\operatorname{Cos}x + \operatorname{Cos}^2 x\right]$$
$$+ \frac{e^{-x}}{10}\left[3\operatorname{Cos}x\operatorname{Sin}x - \operatorname{Sin}x\ln|\operatorname{Sec}x + \operatorname{Tan}x| + \operatorname{Sin}^2 x\right]$$

$$= \frac{e^{2x}}{10}\int e^{-3x}\tan x\,dx + \frac{e^{-x}}{10}\left[(\ln|\operatorname{Sec}x + \operatorname{Tan}x|)\{3\operatorname{Cos}x - \operatorname{Sin}x\} + \operatorname{Cos}^2 x + \operatorname{Sin}^2 x \cancel{- 3\operatorname{Sin}x\operatorname{Cos}x + 3\operatorname{Sin}x\operatorname{Cos}x}\right]$$

$$= \frac{e^{2x}}{10}\int e^{-3x}\tan x\,dx + \frac{e^{-x}}{10}\left[(3\operatorname{Cos}x - \operatorname{Sin}x)\ln|\operatorname{Sec}x + \operatorname{Tan}x| + 1\right]$$

$$Y = Y_c + Y_p$$

$$= c_1e^{2x} + (C_2\operatorname{Cos}x + C_3\operatorname{Sin}x)e^{-x} + \frac{e^{2x}}{10}\int e^{-3x}\tan x\,dx$$
$$+ \frac{e^{-x}}{10}\left[(3\operatorname{Cos}x - \operatorname{Sin}x)\ln|\operatorname{Sec}x + \operatorname{Tan}x| + 1\right]$$