[(Exercise 8.6)]

Q1 Show that the shortest distance b/w the lines $x+a = 2y = -12z$ & $x = y+2a = 6(z-a)$ is $2a$.

Sol. Given lines are

$$\left.\begin{aligned} x+a &= 2y = -12z \\ x &= y+2a = 6(z-a) \end{aligned}\right]$$

or $$\left.\begin{aligned} \frac{x+a}{1} &= \frac{y}{\frac{1}{2}} = \frac{z}{-12} \\ x = y+2a, &\; x = 6z-6a \end{aligned}\right]$$

or $$\frac{x+a}{12} = \frac{y}{6} = \frac{z}{-1} \quad \text{——— (1)}$$

$$x-y-2a = 0 = x-6z+6a \quad \text{——— (2)}$$

Eq. of a plane through line (2) is

$$(x-y-2a) + K(x-6z+6a) = 0$$

$$(1+K)x - y - 6Kz - 2a + 6Ka = 0$$

Now d.rs. of normal to this plane are $1+K, -1, -6K$

If this plane is ∥ to line (1)

Then $12(1+K) + 6(-1) - 1(-6K) = 0$

$$12 + 12K - 6 + 6K = 0$$

$$18K + 6 = 0$$

$$3K + 1 = 0$$

$K = -\frac{1}{3}$

Put in eq. of plane

$(x-y-2a) - \frac{1}{3}(x-6z+6a) = 0$

$3x-3y-6a-x+6z-6a = 0$

$\boxed{2x-3y+6z-12a=0}$ is eq. of plane through line ② & ∥ to line ①.

let d be req. shortest distance then

d = Distance of pt. $(-a,0,0)$ from plane $2x-3y+6z-12a=0$

$= \frac{|2(-a)-0+0-12a|}{\sqrt{4+9+36}}$

$= \frac{|-14a|}{7}$

$= \frac{14a}{7}$

$d = 2a$

Q2 Find the shortest distance b/w the axis of x & the st. line $ax+by+cz+d=0=a'x+b'y+c'z+d'$

Sol. We know that eq. of x-axis in symmetric form is

$\frac{x}{1} = \frac{y}{0} = \frac{z}{0}$ ——— ①

Now given line is

$$ax+by+cz+d=0=a'x+b'y+c'z+d' \quad \text{——} (2)$$

Now eq. of a plane Containing this line is

$$(ax+by+cz+d)+k(a'x+b'y+c'z+d')=0$$

or $(a+ka')x+(b+kb')y+(c+kc')z+d+kd'=0$

If this plane is $\parallel$ to x-axis then

$$1(a+ka')=0$$

$$\Rightarrow \boxed{k=-\frac{a}{a'}}$$

Put in eq. of plane

$$(ax+by+cz+d)-\frac{a}{a'}(a'x+b'y+c'z+d')=0$$

$$\cancel{aa'x}+a'by+a'cz+a'd-\cancel{aa'x}-ab'y-ac'z-ad'=0$$

$$\boxed{(a'b-ab')y+(a'c-ac')z+(a'd-ad')=0}$$

is eq. of plane Containing line (2)

let d' be req. shortest distance then

d' = Distance of pt. $(0,0,0)$ from plane

$$=\frac{(a'b-ab')(0)+(a'c-ac')(0)+(a'd-ad')}{\sqrt{(a'b-ab')^2+(a'c-ac')^2}}$$

$$d'=\frac{a'd-ad'}{\sqrt{(a'b-ab')^2+(a'c-ac')^2}} \quad \text{——} \text{ Ans.}$$

Q3 Show that the shortest distance b/w the st. lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ & $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ 121

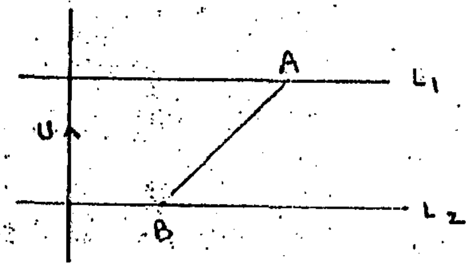
is $\frac{1}{\sqrt{6}}$ & eqs. of the st. line perpendicular to both are $11x+2y-7z+6=0=7x+y-5z+7$

Sol:- Given lines are

$$\left.\begin{array}{l}\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \quad \text{--- (1)} \\ \& \ \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5} \quad \text{--- (2)}\end{array}\right.$$

A pt. on line (1) is $A(1,2,3)$

A pt. on line (2) is $B(2,4,5)$



$\overrightarrow{AB} = (2-1)\hat{i} + (4-2)\hat{j} + (5-3)\hat{k}$

$\overrightarrow{AB} = \hat{i} + 2\hat{j} + 2\hat{k}$

Here d.rs. of (1) are 2, 3, 4

d.rs. of (2) are 3, 4, 5

Let $\vec{U}$ be a vector ⊥ to both given lines

then

$$\vec{U} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$$

$$= (15-16)\hat{i} - (10-12)\hat{j} + (8-9)\hat{k}$$

$$\vec{U} = -\hat{i} + 2\hat{j} - \hat{k}$$

Let d be req. shortest distance b/w lines then

$$d = \frac{\vec{AB}\cdot\vec{u}}{|\vec{u}|}$$

$$= \frac{(\hat{i}+2\hat{j}+2\hat{k})\cdot(-\hat{i}+2\hat{j}-\hat{k})}{\sqrt{1+4+1}}$$

$$= \frac{(1)(-1)+(2)(2)+(2)(-1)}{\sqrt{6}}$$

$$= \frac{-1+4-2}{\sqrt{6}}$$

$\boxed{d = \frac{1}{\sqrt{6}}}$ is req. distance.

Now eq. of line ⊥ to both given lines is

$$\begin{vmatrix} x-1 & y-2 & z-3 \\ 2 & 3 & 4 \\ -1 & 2 & -1 \end{vmatrix} = 0 = \begin{vmatrix} x-2 & y-4 & z-5 \\ 3 & 4 & 5 \\ -1 & 2 & -1 \end{vmatrix}$$

$$(x-1)(-3-8)-(y-2)(-2+4)+(z-3)(4+3) = 0 = (x-2)(-4-10)-(y-4)(-3+5) + (z-5)(6+4)$$

$$(x-1)(-11)-(y-2)(2)+(z-3)(7) = 0 = (x-2)(-14)-(y-4)(2)+(z-5)(10)$$

$$-11x-2y+7z+11+4-21 = 0 = -14x-2y+10z+28+8-50$$

$$-11x-2y+7z-6 = 0 = -14x-2y+10z-14$$

$$\underline{\ \ +2y-7z+6 = 0 = 7x+y-5z+7\ }$$

is req. eq.

$$= (-2+6)\hat{i} - (1-7)\hat{j} + (-6+14)\hat{k}$$

$$\boxed{\vec{u} = 4\hat{i} + 6\hat{j} + 8\hat{k}}$$

let d be req. shortest distance b/w lines then

$$d = \frac{\vec{AB}\cdot\vec{u}}{|\vec{u}|}$$

$$= \frac{(-4\hat{i} - 6\hat{j} - 8\hat{k})\cdot(4\hat{i} + 6\hat{j} + 8\hat{k})}{\sqrt{16+36+64}}$$

$$= \frac{-16-36-64}{\sqrt{116}}$$

$$= -\frac{116}{\sqrt{116}}$$

$$= -\sqrt{116}$$

$$= -\sqrt{4\times 29}$$

$$= -2\sqrt{29}$$

$$\boxed{d = 2\sqrt{29}}$$ (In magnitude)

Now given lines are

$$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1} = t \quad \text{——} (1)$$

$$\& \quad \frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1} = s \quad \text{——} (2)$$

Parametric eqs. of given lines are

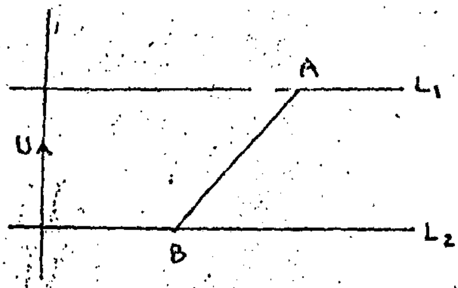
$$\left.\begin{array}{l} x = 3+t \\ y = 5-2t \\ z = 7+t \end{array}\right\} \text{——} (1) \quad \& \quad \left.\begin{array}{l} x = -1+7s \\ y = -1-6s \\ z = -1+s \end{array}\right\} \text{——} (2)$$

Q4 Find the shortest distance b/w the lines 124

$$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1} \quad \& \quad \frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}.$$

Find eq. of the st. line ⊥ to both the given st. lines & also its pt. of intersection with the given st. lines.

Sol:- Given lines are



$$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1} \quad \text{—(1)}$$

$$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1} \quad \text{—(2)}$$

A pt. on line (1) is $A(3,5,7)$

A pt. on line (2) is $B(-1,-1,-1)$

$$\overrightarrow{AB} = (-1-3)\hat{i} + (-1-5)\hat{j} + (-1-7)\hat{k}$$

$$\boxed{\overrightarrow{AB} = -4\hat{i} - 6\hat{j} - 8\hat{k}}$$

Here d.rs. of line (1) are $1, -2, 1$

& d.rs. of line (2) are $7, -6, 1$

Let $\vec{u}$ be a vector perpendicular to both lines then

$$\vec{u} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 1 \\ 7 & -6 & 1 \end{vmatrix}$$

Expanding from R_1

Any pt. on ① is $P(3+t, 5-2t, 7+t)$

Any pt. on ② is $Q(-1+7s, -1-6s, -1+s)$

D.rs. of line PQ are $3+t+1-7s, 5-2t+1+6s, 7+t+1-s$

$$= t-7s+4, -2t+6s+6, t-s+8$$

If PQ is the line of shortest distance then then PQ is perp. to both the lines, so

$$\left.\begin{aligned} 1(t-7s+4)-2(-2t+6s+6)+1(t-s+8)&=0 \\ 7(t-7s+4)-6(-2t+6s+6)+1(t-s+8)&=0 \end{aligned}\right]$$

$$\left.\begin{aligned} t-7s+\cancel{4}+4t-12s-\cancel{12}+t-s+\cancel{8} &= 0 \\ 7t-49s+\cancel{28}+12t-36s-\cancel{36}+t-s+\cancel{8} &= 0 \end{aligned}\right)$$

$$\left.\begin{aligned} 6t-20s &= 0 \\ 20t-86s &= 0 \end{aligned}\right]$$

$$\Rightarrow \boxed{t=0} \ \& \ \boxed{s=0}$$

Hence Co-ords. of pts. P & Q are

$P(3,5,7)$ & $Q(-1,-1,-1)$

Now eq. of line of shortest distance is

$$\frac{x-3}{3+1} = \frac{y-5}{5+1} = \frac{z-7}{7+1}$$

$$\frac{x-3}{4} = \frac{y-5}{6} = \frac{z-7}{8}$$

or $\boxed{\frac{x-3}{2} = \frac{y-5}{3} = \frac{z-7}{4}}$ is req. line.

Q5 Find the co-ords. of the pt. on the join of $(-3,7,-13)$ & $(-6,1,-10)$ which is nearest to the intersection of the planes

$2x-y-3z+32=0$ & $3x+2y-15z-8=0$

Sol. Eq. of the line through $(-3,7,-13)$ & $(-6,1,-10)$ is

$$\frac{x+3}{-3+6} = \frac{y-7}{7-1} = \frac{z+13}{-13+10}$$

$$\frac{x+3}{3} = \frac{y-7}{6} = \frac{z+13}{-3}$$

or $$\frac{x+3}{1} = \frac{y-7}{2} = \frac{z+13}{-1} = t \quad \text{——— (1)}$$

$$\Rightarrow \left.\begin{aligned} x &= -3+t \\ y &= 7+2t \\ z &= -13-t \end{aligned}\right]$$

Any pt. on line (1) is $P(-3+t, 7+2t, -13-t)$

Also given eq. of line is

$$2x-y-3z+32=0=3x+2y-15z-8=0$$

Let l, m, n be d.c.s. of this line then since it lies on both planes so by condition of perpendicularity

$$\left.\begin{aligned} 2l-m-3n &= 0 \\ 3l+2m-15n &= 0 \end{aligned}\right]$$

$$\frac{l}{15+6} = \frac{-m}{-30+9} = \frac{n}{4+3}$$

$$\frac{l}{21} = \frac{m}{21} = \frac{n}{7}$$

$$\frac{l}{3} = \frac{m}{3} = \frac{n}{1}$$

So d.rs. of given line are 3, 3, 1

To find a pt. on line put $z = 0$ in above eqs.

$$\left.\begin{array}{l} 2x - y + 3z = 0 \\ 3x + 2y - 8 = 0 \end{array}\right]$$

$$\frac{x}{8-64} = \frac{-y}{-16-96} = \frac{1}{4+3}$$

$$\frac{x}{-56} = \frac{-y}{-112} = \frac{1}{7}$$

$$\boxed{x = -8}, \boxed{y = 16}, \boxed{z = 0}$$

So $(-8, 16, 0)$ is a ~~pt on given~~ line.

Now eq. of given line through $(-8, 16, 0)$ & with d.rs. 3, 3, 1 is

$$\frac{x+8}{3} = \frac{y-16}{3} = \frac{z}{1} = s \quad \text{——— (2)}$$

$$\Rightarrow \left.\begin{array}{l} x = -8 + 3s \\ y = 16 + 3s \\ z = s \end{array}\right]$$

Any pt. on this line is $Q(-8+3s, 16+3s, s)$

Now d.rs. of line PQ are $-3+t+8-3s,\ 7+2t-16-3s,\ -13-t-s$

$$= t-3s+5,\ 2t-3s-9,\ -13-t-s$$

If PQ is perp. to both lines (1) & (2)

then

$$\left.\begin{aligned} 1(t-3s+5)+2(2t-3s-9)-1(13-t-s) &= 0 \\ 3(t-3s+5)+3(2t-3s-9)+1(13-t-s) &= 0 \end{aligned}\right]$$

$$\left.\begin{aligned} 6t-8s &= 0 \\ 8t-19s-25 &= 0 \end{aligned}\right] \begin{aligned} &\text{——— I} \\ &\text{——— II} \end{aligned}$$

Multiplying I by 4 & II by 3

$$\cancel{24t}-32s = 0 \quad \text{——— I}$$

$$\underline{\cancel{24t}-57s = 75} \quad \text{——— II}$$

$$\;\;- \quad + \quad\; -$$

$$25s = -75$$

$$\boxed{s=-3}$$

Put in I

$$6t-8(-3)=0$$

$$6t+24=0$$

$$t+4=0$$

$$\boxed{t=-4}$$

Put $t=-4$ in Co-ords of P

$$P(-3-4,\ 7-8,\ -13+4)$$

or $P(-7,-1,-9)$ is the req. pt.

Q6 Find the length & eq. of the common perpendicular of the lines

$L: 6x+8y+3z-11=0, \quad x+2y+z-3=0$

$M: 3x-9y+3z=0, \quad x+y-z=0$

Sol.

Given lines are

$L : 6x+8y+3z-13 = 0 = x+2y+z-3$

$M : 3x-9y+5z = 0 = x+y-z$

We will write both eqs. in symmetric form.

Let l_1, m_1, n_1 be d.cs. of line L, since it lies on both planes, so by condition of perpendicularity

$$\left.\begin{array}{l} 6l_1+8m_1+3n_1 = 0 \\ l_1+2m_1+n_1 = 0 \end{array}\right]$$

$$\frac{l_1}{8-6} = \frac{-m_1}{6-3} = \frac{n_1}{12-8}$$

$$\frac{l_1}{2} = \frac{m_1}{-3} = \frac{n_1}{4}$$

So d.rs. of L are $2, -3, 4$

To find a pt. on L Put $z = 0$

$$\left.\begin{array}{l} 6x+8y-13 = 0 \\ x+2y-3 = 0 \end{array}\right]$$

$$\frac{x}{-24+26} = \frac{-y}{-18+13} = \frac{1}{12-8}$$

$$\frac{x}{2} = \frac{y}{5} = \frac{1}{4}$$

$$x = \frac{1}{2}, \quad y = 5/4$$

So a pt. on L is $\left(\frac{1}{2}, \frac{5}{4}, 0\right)$

Now eq. of L through $\left(\frac{1}{2}, \frac{5}{4}, 0\right)$ & having d.rs. $2, -3, 4$ is

$$\frac{x-\frac{1}{2}}{2} = \frac{y-\frac{5}{4}}{-3} = \frac{z}{4} \quad \text{———} \quad (1)$$

Let l_2, m_2, n_2 be d.cs. of line M, since it lies on both planes, so by condition of perpendicularity

$$\left.\begin{aligned} 3l_2 - 9m_2 + 5n_2 &= 0 \\ l_2 + m_2 - n_2 &= 0 \end{aligned}\right]$$

$$\frac{l_2}{9-5} = \frac{-m_2}{-3-5} = \frac{n_2}{3+9}$$

$$\frac{l_2}{4} = \frac{m_2}{8} = \frac{n_2}{12}$$

$$\frac{l_2}{1} = \frac{m_2}{2} = \frac{n_2}{3}$$

So d.r.s. of line M are 1, 2, 3

To find a pt. on line M Put $z = 0$

$$\left.\begin{aligned} 3x - 9y &= 0 \\ x + y &= 0 \end{aligned}\right]$$

$$\text{or} \quad \left.\begin{aligned} x - 3y &= 0 \\ x + y &= 0 \end{aligned}\right] \quad \begin{aligned} &\text{——— I} \\ &\text{——— II} \end{aligned}$$

Subt. we get

$$-4y = 0$$

$$\boxed{y = 0}$$

Put in I

$$x - 0 = 0$$

$$\boxed{x = 0}$$

So a pt. on line M is $(0, 0, 0)$

Hence eq. of M through $(0,0,0)$ having d.r.s. 1, 2, 3 is

$$\frac{x}{1} = \frac{y}{2} = \frac{z}{3} \quad \text{———} \ (2)$$

Now we want to find shortest distance b/w (1) & (2)

A pt. on line (1) is $A\left(\frac{1}{2}, \frac{5}{4}, 0\right)$

A pt. on line (2) is $B(0, 0, 0)$

$$\overrightarrow{AB} = -\frac{1}{2}\hat{i} - \frac{5}{4}\hat{j} + 0\hat{k}$$

Let $\vec{u}$ be a vector perp. to both lines, then

$$\vec{u} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 4 \\ 1 & 2 & 3 \end{vmatrix}$$

$$= (-9-8)\hat{i} - (6-4)\hat{j} + (4+3)\hat{k}$$

$$\vec{u} = -17\hat{i} - 2\hat{j} + 7\hat{k}$$

Let d be req. shortest distance b/w lines then

$$d = \frac{\vec{AB}\cdot\vec{u}}{|\vec{u}|}$$

$$= \frac{\left(-\frac{1}{2}\hat{i} - \frac{5}{4}\hat{j} + 0\hat{k}\right)\cdot\left(-17\hat{i} - 2\hat{j} + 7\hat{k}\right)}{\sqrt{289+4+49}}$$

$$= \frac{\frac{17}{2} + \frac{5}{2} + 0}{\sqrt{342}}$$

$$= \frac{\frac{22}{2}}{\sqrt{342}}$$

$$\boxed{d = \frac{11}{\sqrt{342}}}$$

Now eq. of Common perpendicular is

$$\begin{vmatrix} x-\frac{1}{2} & y-\frac{5}{4} & z \\ 2 & -3 & 4 \\ -17 & -2 & 7 \end{vmatrix} = 0 = \begin{vmatrix} x & y & z \\ 1 & 2 & 3 \\ -17 & -2 & 7 \end{vmatrix}$$

$$\left(x-\frac{1}{2}\right)(-21+8) - \left(y-\frac{5}{4}\right)(14+68) + z(-4+51) = 0 = x(14+6) - y(7+51) + z(-2+34)$$

$$\left(x-\frac{1}{2}\right)(-13) - \left(y-\frac{5}{4}\right)(82) + z(-55) = 0 = 20x - 58y + 32z$$

$$-13x - 82y - 55z + \frac{13}{2} + \frac{205}{2} = 0 = 20x - 58y + 32z$$

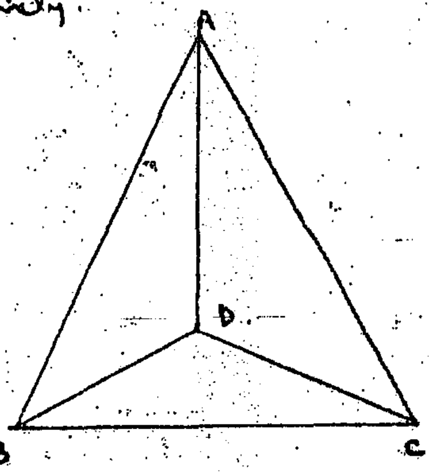
$$-13x - 82y - 55z + 109 = 0 = 10x - 29y + 16z$$

or $13x+82y+55z-109=0 = 10x-29y+16z$ is req. eq.

<u>Q7</u> Show that the shortest distance b/w any two opposite edges of the tetrahedron formed by the planes $y+z=0$, $z+x=0$, $x+y=0$ & $x+y+z=a$ is $\frac{2a}{\sqrt{6}}$ & that the three st. lines of the shortest distance intersect at the pt. $(-a,-a,-a)$.

<u>Sol:-</u>

Let the planes $y+z=0$, $z+x=0$, $x+y=0$ & $x+y+z=a$ be ABC, ACD, ABD & BCD respectively.



then eq. of line AC is

$$\left.\begin{array}{l} y+z=0 \\ z+x=0 \end{array}\right\}$$

or $y=-z$

$x=-z$

or $\frac{x}{1}=\frac{y}{1}=\frac{z}{-1}$ ——— ①

is symmetric form of line AC

Now the eq. of opposite edge BD is

$$\left.\begin{array}{l} x+y=0 \\ x+y+z=a \end{array}\right\}$$

Let l, m, n be d.cs. of this line

Since it lies on both planes so by condition of perpendicularity

$$\left.\begin{array}{l} l+m+0n=0 \\ l+m+n=0 \end{array}\right\}$$

$$\frac{l}{1-0} = \frac{-m}{1-0} = \frac{n}{1-1}$$

$$\frac{l}{1} = \frac{m}{-1} = \frac{n}{0}$$

So d.rs. of line is $1, -1, 0$

To find a pt. on this line put $x = 0$ in above eqs.

$$\left.\begin{array}{r} 0 + y = 0 \\ 0 + y + z = a \end{array}\right\} \begin{array}{l} \Rightarrow \boxed{y = 0} \\ \Rightarrow y + z = a \quad \text{or} \boxed{z = a} \end{array}$$

So a pt. on this line BD is $(0, 0, a)$

Hence eq. of this line is

$$\frac{x}{1} = \frac{y}{-1} = \frac{z-a}{0} \quad \text{———} \quad (2)$$

Now we will find shortest distance b/w (1) & (2)

A pt. on line (1) is $A(0,0,0)$

A pt. on line (2) is $B(0,0,a)$

Now $\overrightarrow{AB} = 0\hat{i} + 0\hat{j} + a\hat{k}$

let $\vec{U}$ be a vector perp. to both lines then

$$\vec{U} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 1 & -1 & 0 \end{vmatrix}$$

Expanding from R_1

$$= (0-1)\hat{i} - (0+1)\hat{j} + (-1-1)\hat{k}$$

$$\vec{U} = -\hat{i} - \hat{j} - 2\hat{k}$$

let d be the shortest distance b/w lines

then $d = \dfrac{\overrightarrow{AB} \cdot \vec{U}}{|\vec{U}|}$

$$d = \frac{(0\hat{i}+0\hat{j}+a\hat{k})\cdot(-\hat{i}-\hat{j}-2\hat{k})}{\sqrt{1+1+4}}$$

$$= \frac{0+0-2a}{\sqrt{6}}$$

$\boxed{d = \frac{2a}{\sqrt{6}}}$ is req. distance.

Similarly we can show that the shortest distance b/w opposite edges AB, CD & BC, AD is also $\frac{2a}{\sqrt{6}}$

Now eq. of line of shortest distance b/w opposite edges AC & BD is

$$\begin{vmatrix} x & y & z \\ 1 & 1 & -1 \\ -1 & -1 & -2 \end{vmatrix} = 0 = \begin{vmatrix} x & y & z-a \\ 1 & -1 & 0 \\ -1 & -1 & -2 \end{vmatrix}$$

$$(-2-1)x-(-2-1)y+(-1+1)z = 0 = (2-0)x-(-2+0)y+(-1-1)(z-a)$$

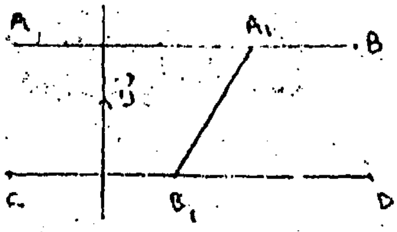
$$-3x+3y+0z = 0 = 2x+2y-2z+2a$$

$$\boxed{x-y = 0 = x+y-z+a}$$

we see that the pt. $(-a,-a,-a)$ satisfies this eq. So this pt. lies on the line of shortest distance b/w AC & BD. Similarly $(-a,-a,-a)$ also lies on the other two lines of shortest distance. Hence it lies on the intersection of all three lines of shortest distances.

Q8 Find the shortest distance b/w the st. lines joining the pts. $A(3,2,-4)$ & $B(1,6,-6)$ & the st. line joining the pts. $C(-1,1,-2)$ & $D(-3,1,-6)$. Also find eq. of the line of shortest distance & Co-ords. of the feet of common perpendicular. 135

Solr Eq. of line AB is



$$\frac{x-3}{1-3} = \frac{y-2}{6-2} = \frac{z+4}{-6+4}$$

$$\frac{x-3}{-2} = \frac{y-2}{4} = \frac{z+4}{-2}$$

or $$\frac{x-3}{1} = \frac{y-2}{-2} = \frac{z+4}{1} \quad \text{———} \; ①$$

& eq. of line CD is

$$\frac{x+1}{-3+1} = \frac{y-1}{1-1} = \frac{z+2}{-6+2}$$

or $$\frac{x+1}{-2} = \frac{y-1}{0} = \frac{z+2}{-4}$$

$$\frac{x+1}{1} = \frac{y-1}{0} = \frac{z+2}{2} \quad \text{———} \; ②$$

A pt. on line ① is $A_1(3,2,-4)$

A pt. on line ② is $B_1(-1,1,-2)$

$$\overrightarrow{A_1B_1} = (-1-3)\hat{i} + (1-2)\hat{j} + (-2+4)\hat{k}$$

$$\overrightarrow{A_1B_1} = -4\hat{i} - \hat{j} + 2\hat{k}$$

Let $\vec{U}$ be a vector perp. to both lines ① & ②

then
$$\vec{U} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 1 \\ 1 & 0 & 2 \end{vmatrix}$$

$$\vec{U} = (-4-0)\hat{i} - (2-1)\hat{j} + (0+2)\hat{k}$$ 136

or $\vec{U} = -4\hat{i} - \hat{j} + 2\hat{k}$

let d be req. shortest distance b/w lines then

$$d = \frac{\vec{AB_1} \cdot \vec{U}}{|\vec{U}|}$$

$$= \frac{(-4\hat{i} - \hat{j} + 2\hat{k}) \cdot (-4\hat{i} - \hat{j} + 2\hat{k})}{\sqrt{16+1+4}}$$

$$= \frac{16+1+4}{\sqrt{21}}$$

$$= \frac{21}{\sqrt{21}}$$

$$\boxed{d = \sqrt{21}}$$

As lines are

$$\frac{x-3}{1} = \frac{y-2}{-2} = \frac{z+4}{1} = t \quad \text{——— (1)}$$

$$\& \quad \frac{x+1}{1} = \frac{y-1}{0} = \frac{z+2}{2} = s \quad \text{——— (2)}$$

Any pt. on line (1) is $P(3+t, 2-2t, -4+t)$

Any pt. on line (2) is $Q(-1+s, 1, -2+2s)$

D.rs. of PQ are $3+t+1-s,\ 2-2t-1,\ -4+t+2-2s$

or $t-s+4,\ -2t+1,\ t-2s-2$

Suppose PQ is line of shortest distance then PQ is perp. to both lines (1) + (2)

So
$$\left.\begin{aligned} 1(t-s+4) - 2(-2t+1) + 1(t-2s-2) &= 0 \\ 1(t-s+4) + 0(-2t+1) + 2(t-2s-2) &= 0 \end{aligned}\right]$$

$$\left.\begin{aligned} 6t - 5s &= 0 \\ 3t - 5s &= 0 \end{aligned}\right\}$$

$$\Rightarrow \boxed{t = s = 0}$$

So Co-ords. of feet of perpendiculars P & Q are :
P(3, 2, -4) & Q(-1, 1, -2)

Now eq. of Common perp. PQ is

$$\frac{x-3}{-1-3} = \frac{y-2}{1-2} = \frac{z+4}{-2+4}$$

$$\frac{x-3}{-4} = \frac{y-2}{-1} = \frac{z+4}{2}$$

or $$\frac{x-3}{4} = \frac{y-2}{1} = \frac{z+4}{-2}$$