Mathematics Class 9th

Chapter No: 5 **Factorization**

Factorization:

If a polynomial $p(x)$ can be expressed as $p(x) = g(x).h(x)$, then each of the polynomials $g(x)$ and $h(x)$ is called a factor of $p(x)$. For instance, in the distributive property

$$ab + ac = a(b + c)$$

a and $(b + c)$ are factors of $(ab + ac)$.

The process of expressing an algebraic expression in term of its factor is called factorization.

1. **Factorization of the Expression of the type: $ka + kb + kc$**
   For Example: $5a - 5b + 5c$
   Solution:

   $$5a - 5b + 5c$$
   $$= 5(a - b + c) \quad \text{(Taking 5 as Common)}$$

2. **Factorization of the Expression of the type: $ac + ad + bc + bd$**
   For Example: $3x + xy - 3a - ay$
   Solution:

   $$3x + xy - 3a - ay$$
   $$= x(3 + y) - a(3 + y)$$
   $$= (x - a)(3 + y$$

3. **Factorization of the Expression of the type: $a^2 \pm 2ab + b^2$**
   i. $a^2 + 2ab + b^2 = (a + b)^2 = (a + b)(a + b)$
   ii. $a^2 - 2ab + b^2 = (a - b)^2 = (a - b)(a - b)$

   For Example 1: $25x^2 + 40x + 16$

   Solution:

   $$25x^2 + 40x + 16$$
   $$= (5x)^2 + 2(5x)(4) + (4)^2$$
   $$= (5x + 5)^2$$

$$= (5x+5)(5x+5)$$

For Example 2: $12x^2 - 36x + 27$

Solution:

$$12x^2 - 36x + 27$$
$$= 3(4x^2 - 12x + 9)$$
$$= 3\{(2x)^2 - 2(2x)(3) + (3)^2\}$$
$$= 3(2x-3)^2$$
$$= 3\{(2x-3)(2x-3)\}$$

4. **Factorization of the Expression of the type: $a^2 - b^2$**

i. $a^2 - b^2 = (a+b)(a-b)$

For Example: $4x^2 - (2y-z)^2$

Solution:

$$4x^2 - (2y-z)^2$$
$$= (2x)^2 - (2y-z)^2$$
$$= [2x + (2y-z)][2x - (2y-z)]$$
$$= (2x+2y-z)\,(2x-2y+z)$$

5. **Factorization of the Expression of the type: $a^2 \pm 2ab + b^2 - c^2$**

i. $a^2 + 2ab + b^2 - c^2 = (a+b)^2 - c^2 = (a+b+c)(a+b-c)$

ii. $a^2 - 2ab + b^2 - c^2 = (a-b)^2 - c^2 = (a-b+c)(a-b-c)$

For Example: $x^2 + 6x + 9 - 4y^2$

Solution:

$$x^2 + 6x + 9 - 4y^2$$
$$= (x)^2 + 2(x)(3) + (3)^2 - (2y)^2$$
$$= (x+3)^2 - (2y)^2$$
$$= (x+3+2y)(x+3-2y)$$

Exercise 5.1

1. **Factorize:**

i. $2abc - 4abx + 2abd$

Solution:

$2abc - 4abx + 2abd$

Taking "$2ab$" as a Common

$= 2ab(c - 2x + d)$

ii. $\mathbf{9xy - 12x^2y + 18y^2}$

Solution:

$9xy - 12x^2y + 18y^2$

Taking "$3y$" as a Common

$= 3y(3x - 4x^2 + 6y)$

iii. $\mathbf{-3x^2y - 3x + 9xy^2}$

Solution:

$-3x^2y - 3x + 9xy^2$

Taking "$-3x$" as a Common

$= -3x(xy + 1 - 3y^2)$

iv. $\mathbf{5ab^2c^3 - 10a^2b^3c - 20a^3bc^2}$

Solution:

$5ab^2c^3 - 10a^2b^3c - 20a^3bc^2$

Taking "$5abc$" as a Common

$= 5abc(bc^3 - 2ab^2 - 4a^2c)$

v. $\mathbf{3x^3y(x - 3y) - 7x^2y^2(x - 3y)}$

Solution:

$3x^3y(x - 3y) - 7x^2y^2(x - 3y)$

Taking "$(x - 3y)$" as a Common

$= (x - 3y)(3x^3y - 7x^2y^2)$

Again Taking "x^2y" as a Common

$= (x - 3y)x^2y(3x - 7y)$

$= x^2y(3x - 7y)(x - 3y)$

vi. $\mathbf{2xy^3(x^2 + 5) + 8xy^2(x^2 + 5)}$

Solution:

$2xy^3(x^2+5)+8xy^2(x^2+5)$

Taking "(x^2+5)" as a Common

$=(2xy^3+8xy^2)(x^2+5)$

Again Taking "$2xy^2$" as a Common

$=2xy^2(y+4)(x^2+5)$

2. Factorize:

i. $\boldsymbol{5ax-3ay-5bx+3by}$

Solution:

$5ax-3ay-5bx+3by$

Rearrange the Terms

$=5ax-5bx-3ay+3by$

$=5x(a-b)-3y(a-b)$

$=(5x-3y)(a-b)$

ii. $\boldsymbol{3xy+2y-12x-8}$

Solution:

$3xy+2y-12x-8$

$=y(3x+2)-4(3x+2)$

$=(3x+2)(y-4)$

iii. $\boldsymbol{x^3+3xy^2-2x^2y-6y^3}$

Solution:

$x^3+3xy^2-2x^2y-6y^3$

$=x(x^2+3y^2)-2y(x^2+3y)$

$=(x-2y)(x^2+3y^2)$

iv. $\boldsymbol{(x^2-y^2)z+(y^2-z^2)x}$

Solution:

$(x^2-y^2)z+(y^2-z^2)x$

$=x^2z-y^2z+xy^2-xz^2$

Rearrange the Terms

$= x^2z - xz^2 + xy^2 - y^2z$

$= xz(x - z) + y^2(x - z)$

$= (x - z)(xz + y^2)$

3. Factorize:

i. $\mathbf{144a^2 + 24 + 1}$

Solution:

$144a^2 + 24 + 1$

$= (12a)^2 + 2(12a)(1) + (1)^2$

$= (12a + 1)^2$

ii. $\mathbf{\frac{a^2}{b^2} - 2 + \frac{b^2}{a^2}}$

Solution:

$\frac{a^2}{b^2} - 2 + \frac{b^2}{a^2}$

$= \left(\frac{a}{b}\right)^2 - 2\left(\frac{a}{b}\right)\left(\frac{b}{a}\right) + \left(\frac{a}{b}\right)^2 = \left(\frac{q}{b} - \frac{b}{a}\right)^2$

iii. $\mathbf{(x + y)^2 - 14z(x + y) + 49z^2}$

Solution:

$(x + y)^2 - 14z(x + y) + 49z^2$

$= (x + y)^2 - 2(x + y)(7z) + (7z)^2$

$= (x + y - 7z)^2$

iv. $\mathbf{12x^2 - 36x + 27}$

Solution:

$12x^2 - 36x + 27$

$= 3(4x^2 - 12x + 9)$

$= 3[(2x)^2 - 2(2x)(3) + (3)^2]$

$= 3(2x - 3)^2$

4. Factorize:

i. $\mathbf{3x^2 - 75y^2}$

Solution:

$3x^2 - 75y^2$

Taking "3" as a Common

$= 3(x^2 - 25y^2)$

$= 3[(x)^2 - (5y)^2]$

$= 3(x + 5y)(x - 5y)$

ii. $\mathbf{x(x - 1) - y(y - 1)}$

Solution:

$x(x - 1) - y(y - 1)$

$= x^2 - x - y^2 + y$

Rearrange the Terms

$= x^2 - y^2 - x + y$

$= (x^2 - y^2) - (x - y)$

$= (x + y)(x - y) - (x - y)$

Taking "$(x - y)$" as a Common

$= (x - y)(x + y - 1)$

iii. $\mathbf{128am^2 - 242an^2}$

Solution:

$128am^2 - 242an^2$

Taking "$2a$" as a Common

$= 2a(64m^2 - 121n^2)$

$= 2a[(8m)^2 - (11n)^2]$

$= 2a(8m + 11n)(8m - 11n)$

iv. $\mathbf{3x - 243x^3}$

Solution:

$3x - 243x^3$

Taking "$3x$" as a Common

$= 3x(1 - 81x^2)$

$= 3x[(1)^2 - (9x)^2]$

$= 3x(1 + 9x)(1 - 9x)$

5. Factorize:

i. $\boldsymbol{x^2 - y^2 - 6y - 9}$

Solution:

$x^2 - y^2 - 6y - 9$

$= x^2 - [y^2 + 6y + 9]$

$= x^2 - [(y)^2 + 2(y)(3) + (3)^2]$

$= x^2 - (y + 3)^2$

$= (x)^2 - (y + 3)^2$

$= (x + y + 3)(x - (y + 3))$

$= (x + y + 3)(x - y - 3)$

ii. $\boldsymbol{x^2 - a^2 + 2a - 1}$

Solution:

$x^2 - a^2 + 2a - 1$

$= x^2 - [a^2 - 2a + 1]$

$= x^2 - [(a)^2 - 2(a)(1) + (1)^2]$

$= x^2 - (a - 1)^2$

$= (x)^2 - (a - 1)^2$

$= (x + a - 1)(x - (a - 1))$

$= (x + a - 1)(x - a + 1)$

iii. $\mathbf{4x^2 - y^2 - 2y - 1}$

Solution:

$4\mathrm{x}^2 - \mathrm{y}^2 - 2\mathrm{y} - 1$

$= 4x^2 - [y^2 + 2y + 1]$

$= 4x^2 - [(y)^2 + 2(y)(1) + (1)^2]$

$= 4x^2 - (y+1)^2$

$= (2x)^2 - (y+1)^2$

$= (2x+y+1)(2x-(y+1))$

$= (2x+y+1)(2x-y-1)$

iv. $\boldsymbol{x^2 - y^2 - 4x - 2y + 3}$

Solution:

Note:

$\boldsymbol{3 = 4 - 1}$

$x^2 - y^2 - 4x - 2y + 3$

$= x^2 - y^2 - 4x - 2y + 4 - 1$

$= x^2 - 4x + 4 - y^2 - 2y - 1$

$= (x^2 - 4x + 4) - (y^2 + 2y + 1)$

$= [(x)^2 - 2(x)(2) + (2)^2] - [(y)^2 + 2(y)(1) + (1)^2]$

$= (x-2)^2 - (y+1)^2$

$= (x-2+y+1)\big(x-2-(y+1)\big)$

$= (x-2+y+1)(x-2-y-1)$

$= (x+y-1)(x-y-3)$

v. $\boldsymbol{25x^2 - 10x + 1 - 36z^2}$

Solution:

$25x^2 - 10x + 1 - 36z^2$

$= (25x^2 - 10x + 1) - (36z^2)$

$= [(5x)^2 - 2(5x)(1) + (1)^2] - (6z)^2$

$= (5x-1)^2 - (6z)^2$

$= (5x-1+6z)(5x-1-6z)$

vi. $\boldsymbol{x^2 - y^2 - 4xy + 4z^2}$

Solution:

$x^2 - y^2 - 4xz + 4z^2$

$= (x^2 - 4xz + 4z^2) - y^2$

$= [(x)^2 - 2(x)(2z) + (2z)^2] - (y)^2$

$= (x-2z)^2 - (y)$

$$= (x - 2z + y)(x - 2z - y)$$

1. **Factorization of the Expression of the type: $a^4 + a^2b^2 + b^4$ or $a^4 + 4b^4$**
   For Example: $81x^4 + 36x^2y^2 + 16y^4$

   Solution:

$$81x^4 + 36x^2y^2 + 16y^4$$

$$= (9x^2)^2 + 72x^2y^2 + (4y^2)^2 - 36x^2y^2$$

$$= (9x^2)^2 + 2(9x^2)(4y)^2 + (4y^2)^2 - 36x^2y^2$$

$$= (9x^2 + 4y^2)^2 - (6xy)^2$$

$$= (9x^2 + 4y^2 + 6xy)(9x^2 + 4y^2 - 6xy)$$

2. **Factorization of the Expression of the type: $x^2 + px + q$**

   For Example: $x^2 - 7x + 12$

   Solution:

$$x^2 - 7x + 12$$
$$= x^2 - 3x - 4x + 12$$
$$= x(x - 3) - 4(x - 3)$$
$$= (x - 3)(x - 4)$$

3. **Factorization of the Expression of the type: $ax^2 + bx + c. a \neq 0$**

   For Example: $9x^2 + 21x - 8$

   Solution:

$$9x^2 + 21x - 8$$
$$= 9x^2 + 24x - 3x - 8$$
$$= 3x(3x + 8) - 1(3x + 8)$$
$$= (3x - 1)(3x + 8)$$

4. **Factorization of the Expression of the type:**

   i. $(ax^2 + bx + c)(ax^2 + bx + d) + k$

   ii. $(x + a)(x + b)(x + c)(x + d) + k$

   iii. $(x + a)(x + b)(x + c)(x + d) + kx^2$

For Example: $\boldsymbol{(x^2-4x-5)(x^2-4x-12)-144}$

Solution:

$(x^2-4x-5)(x^2-4x-12)-144$

Suppose that $y=x^2-4x$

$=(y-5)(y-12)-144$

$=y(y-12)-5(y-12)-144$

$=y^2-12y-5y+60-144$

$=y^2-17y-84$

$=y^2-21y+4y-84$

$=y(y-21)+4(y-21)$

$=(y-21)(y+4)$

Replace $y=x^2-4x$

$=(x^2-4x-21)(x^2-4x+4)$

$=(x^2-7x+3x-21)[(x)^2-2(x)(2)+(2)^2]$

$=[x(x-7)+3(x-7)](x-2)^2$

$=(x-7)(x+3)(x-2)(x-2)$

Note

We find the pair of numbers:

- ✓ If we multiply then it become $-84=4\times-21$
- ✓ And if we add them then it will be $-17=4-21$

5. **Factorization of the Expression of the type:**

i. $(a+b)^3=a^3+3a^2b+3a^2+b^3$

ii. $(a-b)^3=a^3-3a^2b+3ab^2-b^3$

For Example: $\boldsymbol{x^3-8y^3-6x^2y+12xy^2}$

Solution:

$x^3-8y^3-6x^2y+12xy^2$

$=(x)^3-(2y)^2-3(x)^2(2y)+3(x)(2y)^2$

$=(x)^3-3(x)^2(2y)+3(x)(2y)^2-(2y)^2=(x-2y)^3$

6. **Factorization of the Expression of the type: $a^3 \pm b^3$**

i. $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

ii. $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

For Example: $27x^3 + 64y^3$

Solution:

$$27x^3 + 64y^3$$

$$= (3x)^3 + (4y)^3$$

$$= (3x + 4y)[(3x)^2 - (3x)(4y) + (4y)^2]$$

$$= (3x + 4y)(9x^2 - 12xy + 16y^2)$$

Exercise 5.2

1. **Factorize:**

i. $x^4 + \frac{1}{x^4} - 3$

Solution:

$$x^4 + \frac{1}{x^4} - 3 = x^4 + \frac{1}{x^4} - 2 - 1$$

$$= x^4 - 2 + \frac{1}{x^4} - 1$$

$$= \left[(x^2)^2 - 2(x^2)\left(\frac{1}{x^2}\right) + \left(\frac{1}{x^2}\right)^2\right] - 1$$

$$= \left(x^2 - \frac{1}{x^2}\right)^2 - (1)^2$$

$$= \left(x^2 - \frac{1}{x^2} + 1\right)\left(x^2 - \frac{1}{x^2} - 1\right)$$

Remember!

$a^2 - b^2 = (a+b)(a-b)$

ii. $3x^4 + 12y^4$

Solution:

$3x^4 + 12y^4 = 3(x^4 + 4y^4)$

$= 3[x^4 + 4y^4 + 4x^2y^2 - 4x^2y^2]$

$= 3[\{(x^2)^2 + (2y^2)^2 + 2(x^2)(2y^2)\} - 4x^2y^2]$

$= 3[(x^2 + 2y^2)^2 - (2xy)^2]$

$= 3(x^2 + 2y^2 + 2xy)(x^2 + 2y^2 - 2xy)$

iii. $\boldsymbol{a^4 + 3a^2b^2 + 4b^4}$

Solution:

$a^4 + 3a^2b^2 + 4b^4 = a^4 + 4a^2b^2 - a^2b^2 + 4b^4$

Rearrange the terms

$= a^4 + 4a^2b^2 + 4b^4 - a^2b^2$

$= [(a^2)^2 + 2(a^2)(2b^2) + (2b^2)^2] - (ab)^2$

$= (a^2 + 2b^2)^2 - (ab)^2$

$= (a^2 + 2b^2 + ab)(a^2 + 2b^2 - ab)$

iv. $\boldsymbol{4x^4 + 81}$

Solution:

$4x^4 + 81 = 4x^4 + 81 + 36x^2 - 36x^2$

$= [(2x^2)^2 + (9)^2 + 2(2x^2)(9)] - (6x)^2$

$= (2x^2 + 9) - (6x)^2$

$= (2x^2 + 9 + 6x)(2x^2 + 9 - 6x)$

v. $\boldsymbol{x^4 + x^2 + 25}$

Solution:

$x^4 + x^2 + 25 = x^4 + x^2 + 25 + 10x^2 - 10x^2$

Rearrange the terms

$= x^4 + 10x^2 + 25 - 10x^2 - x^2$

$= [(x^2)^2 + 2(x^2)(5) + (5)^2] - 10x^2 - x^2$

$= (x^2 + 5)^2 - 9x^2$

$= (x^2 + 5)^2 - (3x)^2$

$= (x^2 + 5 + 3x)(x^2 + 5 - 3x)$

$= (x^2 + 3x + 5)(z^2 - 3x + 5)$

vi. $\boldsymbol{x^4 + 4x^2 + 16}$

Solution:

$x^4 + 4x^2 + 16 = x^4 + 4x^2 + 16 + 8x^2 - 8x^2$

Rearrange the terms

$= x^4 + 8x^2 + 16 - 8x^2 + 4x^2$

$= [(x^2)^2 + 2(x^2)(4) + (4)^2] - 8x^2 + 4x^2$

$= (x^2 + 4)^2 - 4x^2$

$= (x^2 + 4)^2 - (2x)^2$

$= (x^2 + 4 + 2x)(x^2 + 4 - 2x)$

$= (x^2 + 2x + 4)(x^2 - 2x + 4)$

2. Factorize:

i. $\boldsymbol{x^2 + 14x + 48}$

Solution:

$x^2 + 14x + 48$

$= x^2 + 8x + 6x + 48$

$= x(x+8) + 6(x+8)$

$= (x+8)(x+6)$

ii. $\boldsymbol{x^2 - 21x + 108}$

Solution:

$x^2 - 21x + 108$

$= x^2 - 12x - 9x + 108$

$= x(x-12) - 9(x-12)$

$= (x-12)(x-9)$

iii. $\boldsymbol{x^2 - 11x - 42}$

Solution:

$x^2 - 11x - 42$

$= x^2 - 14x + 3x - 42$

$= x(x-14) + 3(x-14)$

$= (x-14)(x+3)$

iv. $\boldsymbol{x^2 + x - 132}$

Solution:

$x^2 + x - 132$

$= x^2 + 12x - 11x - 132$

$= x(x+12) - 11(x+12)$

$= (x+12)(x-11)$

3. Factorize:

i. $\boldsymbol{4x^2 + 12x + 5}$

Solution:

$4x^2 + 12x + 5$

$= 4x^2 + 10x + 2x + 5$

$= 2x(2x + 5) + 1(2x + 5)$

$= (2x + 5)(2x + 1)$

ii. $\mathbf{30x^2 + 7x - 15}$

Solution:

$30x^2 + 7x - 15$

$= 30x^2 + 25x - 18x - 15$

$= 5x(6x + 5) - 3(6x + 5)$

$= (6x + 5)(5x - 3)$

iii. $\mathbf{2x^2 - 65x + 21}$

Solution:

$2x^2 - 65x + 21$

$= 24x^2 - 56x - 9x + 21$

$= 8x(3x - 7) - 3(3x - 7)$

$= (3x - 7)(8x - 3)$

iv. $\mathbf{5x^2 - 16x - 21}$

Solution:

$5x^2 - 16x - 21$

$= 5x^2 - 21x + 5x - 21$

$= x(5x - 21) + 1(5x - 21) = (5x - 21)(x + 1)$

v. $\boldsymbol{4x^2 - 17xy + 4y^2}$

Solution:

$$4x^2 - 17xy + 4y^2$$

$$= 4x^2 - 16xy - xy + 4y^2$$

$$= 4x(x - 4y) - y(x - 4y)$$

$$= (x - 4y)(4x - y)$$

vi. $\boldsymbol{3x^2 - 38xy - 13y^2}$

Solution:

$$3x^2 - 38xy - 13y^2$$

$$= 3x^2 - 39xy + xy - 13y^2$$

$$= 3x(x - 13y) + y(x - 13y)$$

$$= (x - 13)(3x + y)$$

vii. $\boldsymbol{5x^2 + 33xy - 14y^2}$

Solution:

$$5x^2 + 33xy - 14y^2$$

$$= 5x^2 + 35xy - 2xy - 14y^2$$

$$= 5x(x + 7y) - 2y(x + 7y)$$

$$= (x + 7y)(5x - 2y)$$

viii. $\boldsymbol{\left(5x - \frac{1}{x}\right)^2 + 4\left(5x - \frac{1}{x}\right) + 4, x \neq 0}$

Solution:

$$\left(5x - \frac{1}{x}\right)^2 + 4\left(5x - \frac{1}{x}\right) + 4$$

$$= \left(5x - \frac{1}{x}\right)^2 + 2\left(5x - \frac{1}{x}\right)(2) + (2)^2$$

$$= \left(5x - \frac{1}{x} + 2\right)^2$$

$$= \left(5x - \frac{1}{x} + 2\right)\left(5x - \frac{1}{x} + 2\right)$$

4. Factorize:

   i. $\boldsymbol{(x^2 + 5x + 4)(x^2 + 5x + 6) - 3}$

   Solution:

   $(x^2 + 5x + 4)(x^2 + 5x + 6) - 3$

   Suppose that $y = x^2 + 5x$

   $= (y + 4)(y + 6) - 3$

   $= y(y + 6) + 4(y + 6) - 3$

   $= y^2 + 6y + 4y + 24 - 3$

   $= y^2 + 10y + 21$

   $= y^2 + 3y + 7y + 21$

   $= y(y + 3) + 7(y + 3)$

   $= (y + 3)(y + 7)$

   Replace the value of y which is $y = x^2 + 5x$

   $= (x^2 + 5x + 3)(x^2 + 5x + 7)$

   ii. $\boldsymbol{(x^2 - 4x)(x^2 - 4x - 1) - 20}$

   Solution:

   $(x^2 - 4x)(x^2 - 4x - 1) - 20$

   Suppose that $y = x^2 - 4x$

$= (y)(y-1) - 20$

$= y(y-1) - 20$

$= y^2 - y - 20$

$= y^2 - 5y + 4y - 20$

$= y(y-5) + 4(y-5)$

$= (y-5)(y+4)$

Replace the value of y which is $y = x^2 - 4x$

$= (x^2 - 4x - 5)(x^2 - 4x + 4)$

$= (x^2 - 5x + x - 5)[(x)^2 - 2(x)(2) + (2)^2]$

$= [x(x-5) + 1(x-5)](x-2)^2$

$= (x-5)(x+1)(x-2)^2$

$= (x-5)(x+1)(x-2)(x-2)$

iii. $\boldsymbol{(x+2)(x+3)(x+4)(x+5) - 15}$

Solution:

$(x+2)(x+3)(x+4)(x+5) - 15$

$= [(x+2)(x+5)]\,[(x+3)(x+4)] - 15$

$= [x(x+5) + 2(x+5)]\,[x(x+4) + 3(x+4)] - 15$

$= (x^2 + 5x + 2x + 10)(x^2 + 4x + 3x + 12) - 15$

$= (x^2 + 7x + 10)(x^2 + 7x + 12) - 15$

Suppose that $y = x^2 + 7x$

$= (y+10)(y+12) - 15$

$= y(y+12) + 10(y+12) - 15$

$= y^2 + 12y + 10y + 120 - 15$

$= y^2 + 22y + 105$

$= y^2 + 15y + 7y + 105$

$= y(y + 15) + 7(y + 15)$

$= (y + 15)(y + 7)$

Replace the value of y which is $y = x^2 + 7x$

$= (x^2 + 7x + 15)(x^2 + 7x + 7)$

iv. $(\boldsymbol{x} + \mathbf{4})(\boldsymbol{x} - \mathbf{5})(\boldsymbol{x} + \mathbf{6})(\boldsymbol{x} - \mathbf{7}) - \mathbf{504}$

Solution:

$(x + 4)(x - 5)(x + 6)(x - 7) - 504$

$= [(x + 4)(x - 5)][(x + 6)(x - 7)] - 504$

$= [x(x - 5) + 4(x - 5)]\,[x(x - 7) + 6(x - 7)] - 504$

$= (x^2 - 5x + 4x - 20)(x^2 - 7x + 6x - 42) - 504$

$= (x^2 - x - 20)(x^2 - x - 42) - 504$

Suppose that $y = x^2 - x$

$= (y - 20)(y - 42) - 504$

$= y(y - 42) - 20(y - 42) - 504$

$= y^2 - 42y - 20y + 840 - 504$

$= y^2 - 62y + 336$

$= y^2 - 56y - 6y + 336$

$= y(y - 56) - 6(y - 56)$

$= (y - 56)(y - 6)$

Replace the value of y which is $y = x^2 - x$

$= (x^2 - x - 56)(x^2 - x - 6)$

$= (x^2 - 8x + 7x - 56)(x^2 - 3x + 2x - 6)$

$= [x(x - 8) + 7(x - 8)][x(x - 3) + 2(x - 3)]$

$= (x - 8)(x + 7)(x - 3)(x + 2)$

v. $\boldsymbol{(x + 1)(x + 2)(x + 3)(x + 6) - 3x^2}$

Solution:

$(x + 1)(x + 2)(x + 3)(x + 6) - 3x^2$

$= [(x + 1)(x + 6)][(x + 2)(x + 3)] - 3x^2$

$= [x(x + 6) + 1(x + 6)][x(x + 3) + 2(x + 3)] - 3x^2$

$= (x^2 + 6x + x + 6)(x^2 + 3x + 2x + 6) - 3x^2$

$= (x^2 + 7x + 6)(x^2 + 5x + 6) - 3x^2$

$= x\left(x + \frac{6}{x} + 7\right)x\left(x + \frac{6}{x} + 5\right) - 3x^2$ $\therefore$ Taking 'x' as a Common

Suppose that $y = x + \frac{6}{x}$

$= x^2(y + 7)(y + 5) - 3x^2$

$= x^2[(y + 7)(y + 5) - 3]$ $\therefore$ Taking 'x^2' as a Common

$= x^2[y(y + 5) + 7(y + 5) - 3]$

$= x^2[y^2 + 5y + 7y + 35 - 3]$

$= x^2(y^2 + 12y + 32)$

$= x^2(y^2 + 8y + 4y + 32)$

$= x^2[y(y + 8) + 4(y + 8)]$

$= x^2(y + 8)(y + 4)$

Replace the value of y which is $y = x + \frac{6}{x}$

$$= x^2\left[\left(x + \frac{6}{x} + 8\right)\left(x + \frac{6}{x} + 4\right)\right]$$

5. Factorize:

i. $\boldsymbol{x^3 + 48x - 12x^2 - 64}$

Solution:

> **Remember!**
>
> i. $(a+b)^3 = a^3 + 3a^2b + 3a^2 + b^3$
>
> ii. $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$$x^3 + 48x - 12x^2 - 64$$

Rearrange the terms

$$= x^3 - 12x^2 + 48x - 64$$

$$= (x)^3 - 3(x)^2(4) + 3(x)(x)^2 - (4)^2$$

$$= (x-4)^3$$

ii. $\boldsymbol{8x^3 + 60x^2 + 150x + 125}$

Solution:

$$8x^3 + 60x^2 + 150x + 125$$

$$= (2x)^3 + 3(2x)^2(5) + 3(2x)(5)^2 + (5)^3$$

$$= (2x+5)^3$$

iii. $\boldsymbol{x^3 - 18x^2 + 108x - 216}$

Solution:

$$x^3 - 18x^2 + 108x - 216$$

$$= (x)^3 - 3(x)^2(6) + 3(x)(6)^2 - (6)^3$$

$$= (x-6)^3$$

iv. $\boldsymbol{8x^3 - 125y^3 - 60x^2y + 150xy^2}$

Solution:

$8x^3 - 125y^3 - 60x^2y + 150xy^2$

Rearrange the terms

$= 8x^3 - 60x^2y + 150xy^2 - 125y^3$

$= (2x)^3 - 3(2x)^2(5y) + 3(2x)(5y)^2 - (5y)^3$

$= (2x - 5y)^3$

6. Factorize:

i. $\mathbf{27 + 8x^3}$

Remember!

i. $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

ii. $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Solution:

$27 + 8x^3$

$= (3)^3 + (2x)^3$

$= (3 + 2x)[(3)^2 - (3)(2x) + (2x)^2]$

$= (3 + 2x)(9 - 6x + 4x^2)$

ii. $\mathbf{125x^3 - 216y^3}$

Solution:

$125x^3 - 216y^3$

$= (5x - 6y)[(5x)^2 + (5x)(6y) + (6y)^2]$

$= (5x - 6y)(25x^2 + 30xy + 36y^2)$

iii. $\mathbf{64x^3 + 27y^3}$

Solution:

$64x^3 + 27y^3$

$= (4x + 3y)[(4x)^2 - (4x)(3y) + (3y)^2]$

$= (4x + 3y)(16x^2 - 12xy + 9y^2)$

iv. $8x^3 + 125y^3$

Solution:

$$8x^3 + 125y^3$$

$$= (2x + 5y)[(2x)^2 - (2x)(5y) + (5y)^2]$$

$$= (2x + 5y)(4x^2 - 10xy + 25y^2)$$

Remainder Theorem:

If a polynomial $\mathrm{p(x)}$ is a divided by a linear divisor $(x - a)$, then the remainder is $\mathrm{p(a)}$.

Example: 1 Find the remainder when $9x^2 - 6x + 2$ is divided by

i. $x - 3$

Solution:

Let $\mathrm{p(x)} = 9x^2 - 6x + 2$ ………………………. (1)

When $\mathrm{p(x)}$ is divided by $x - 3$, then by Remainder Theorem, the remainder is:

$x - 3 = 0$

$x = 3$

Put the value of x in eq. (1)

$\mathrm{R} = \mathrm{p}(3) = 9(3)^2 - 6(3) + 2$

$\mathrm{R} = \mathrm{p}(3) = 9 \times 9 - 18 + 2$

$\mathrm{R} = \mathrm{p}(3) = 83 - 18 = 65$

ii. $x + 3$

Solution:

Let $\mathrm{p(x)} = 9x^2 - 6x + 2$ ………………………. (1)

When $\mathrm{p(x)}$ is divided by $x + 3$, then by Remainder Theorem, the remainder is:

$x + 3 = 0$

$x = -3$

Put the value of x in eq. (1)

$R = p(-3) = 9(-3)^2 - 6(-3) + 2$

$R = p(-3) = 9 \times 9 + 18 + 2$

$R = p(-3) = 101$

iii. $3x + 1$

Solution:

Let $p(x) = 9x^2 - 6x + 2$ .... (1)

When $p(x)$ is divided by $3x + 1$, then by Remainder Theorem, the remainder is:

$3x + 1 = 0$

$3x = -1$

$x = \frac{-1}{3}$

Put the value of x in eq. (1)

$R = p\left(\frac{-1}{3}\right) = 9\left(\frac{-1}{3}\right)^2 - 6\left(\frac{-1}{3}\right) + 2$

$R = p\left(\frac{-1}{3}\right) = 9\left(\frac{1}{9}\right) + 2 + 2$

$R = p\left(\frac{-1}{3}\right) = 1 + 2 + 2$

$R = p\left(\frac{-1}{3}\right) = 5$

iv. x

Solution:

Let $p(x) = 9x^2 - 6x + 2$ .... (1)

When $p(x)$ is divided by x, then by Remainder Theorem, the remainder is:

$x = 0$

Put the value of x in eq. (1)

$R = p(0) = 9(0)^2 - 6(0) + 2$

$R = p(0) = 0 - 0 + 2$

$R = p(0) = 2$

Example: 2 Find the value of k if the expression $x^3 + kx^2 + 3x - 4$ leaves a remainder of -2 when divided by $x + 2$.

Solution:

Let $\mathrm{p(x)} = x^3 + kx^2 + 3x - 4$ …………………… (A)

When $\mathrm{p(x)}$ is divided by $x + 2$, then by Remainder Theorem, the remainder is:

$x + 2 = 0$

$x = -2$

Put the value of x in eq. (A)

$\mathrm{p}(-2) = (-2)^3 + k(-2)^2 + 3(-2) - 4$

$\mathrm{p}(-2) = -8 + 4k - 6 - 4$

$\mathrm{p}(-2) = 4k - 18$ ……………………… (1)

According to the given condition we have,

$\mathrm{p}(-2) = -2$ …………………………… (2)

By Comparing eq. (1) and (2) we get,

$4k - 18 = -2$

$4k = -2 + 18$

$4k = 16$

$k = \frac{16}{4}$

$k = 4$

Zero of a Polynomial:

If a specific number $x = a$ is substituted for the variable x in a polynomial $\mathrm{p(x)}$ so that the value. $\mathrm{p(a)}$ is zero then $x = a$ is called zero of polynomial $\mathrm{p(x)}$.

Factor Theorem:

The polynomial $(x - a)$ is a factor of the polynomial $\mathrm{p(x)}$ if and only if $\mathrm{p(a)} = 0$.

Example: 1 Determine if $(x - 1)$ is a factor of $x^3 - 4x^2 + 3x + 2$.

Solution:

Let $\mathrm{p(x)} = x^3 - 4x^2 + 3x + 2$ ……………………… (1)

Then the remainder for $x - 2$ is

$x - 2 = 0$

$x = 2$

Put the value of x in eq. (1)

$p(2) = (2)^3 - 4(2)^2 + 3(2) + 2$

$p(2) = 8 - 16 + 6 + 2$

$p(2) = 16 - 16 = 0$

Hence by Factor theorem, $x - 2$ is a factor of give polynomial $p(x)$.

Exercise 5.3

1. Use the remainder theorem to find the remainder when:

i. $3x^3 - 10x^2 + 13x - 6$ is divided by $(x - 2)$

Solution:

Let $p(x) = 3x^3 - 10x^2 + 13x - 6$ .... (1)

When $p(x)$ is divided by $x - 2$, then by Remainder Theorem, the remainder is:

$x - 2 = 0$

$x = 2$

Put the value of x in eq. (1)

$R = p(2) = 3(2)^3 - 10(2)^2 + 13(2) - 6$

$R = p(2) = 3 \times 8 - 10 \times 4 + 26 - 6$

$R = p(2) = 24 - 40 + 26 - 6$

$R = p(2) = 50 - 46 = 4$

ii. $4x^3 - 4x + 3$ is divided by $(2x - 1)$.

Solution:

Let $p(x) = 4x^3 - 4x + 3$ .... (1)

When $p(x)$ is divided by $2x - 1$, then by Remainder Theorem, the remainder is:

$2x - 1 = 0$

$2x = 1$

$x = \frac{1}{2}$

Put the value of x in eq. (1)

$$R = p\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^3 - 4\left(\frac{1}{2}\right) + 3$$

$$R = p\left(\frac{1}{2}\right) = 4\left(\frac{1}{8}\right) - 2 + 3$$

$$R = p\left(\frac{1}{2}\right) = \frac{1}{2} + 1$$

$$R = p\left(\frac{1}{2}\right) = \frac{1+2}{2}$$

$$R = p\left(\frac{1}{2}\right) = \frac{3}{2}$$

iii. $6x^4 + 2x^3 - x + 2$ is divided by $(x + 2)$

Solution:

Let $p(x) = 6x^4 + 2x^3 - x + 2$ …………………………… (1)

When $p(x)$ is divided by $x + 2$, then by Remainder Theorem, the remainder is:

$x + 2 = 0$

$x = -2$

Put the value of x in eq. (1)

$$R = p(-2) = 6(-2)^4 + 2(-2)^3 - (-2) + 2$$

$$R = p(-2) = 6 \times 16 + 2 \times (-8) + 2 + 2$$

$$R = p(-2) = 96 - 16 + 4$$

$$R = p(-2) = 100 - 16$$

$$R = p(-2) = 84$$

iv. $(2x - 1)^3 + 6(3 + 4x)^2 - 10$ is divided by $(2x + 1)$

Solution:

Let $p(x) = (2x - 1)^3 + 6(3 + 4x)^2 - 10$ ……………………… (1)

When $p(x)$ is divided by $2x + 1$, then by Remainder Theorem, the remainder is:

$2x + 1 = 0$

$2x = -1$

$$x = \frac{-1}{2}$$

Put the value of x in eq. (1)

$$\mathrm{R} = \mathrm{p}\left(\frac{-1}{2}\right) = \left[2\left(\frac{-1}{2}\right) - 1\right]^3 + 6\left[3 + 4\left(\frac{-1}{2}\right)\right]^2 - 10$$

$$\mathrm{R} = \mathrm{p}\left(\frac{-1}{2}\right) = (-1-1)^3 + 6[3 + 2(-1)]^2 - 10$$

$$\mathrm{R} = \mathrm{p}\left(\frac{-1}{2}\right) = (-2)^3 + 6(3-2)^2 - 10$$

$$\mathrm{R} = \mathrm{p}\left(\frac{-1}{2}\right) = -8 + 6(1)^2 - 10$$

$$\mathrm{R} = \mathrm{p}\left(\frac{-1}{2}\right) = -8 + 6 - 10$$

$$\mathrm{R} = \mathrm{p}\left(\frac{-1}{2}\right) = -18 + 6$$

$$\mathrm{R} = \mathrm{p}\left(\frac{-1}{2}\right) = -12$$

v. **$x^3 - 3x^2 + 4x - 14$ is divided by $(x+2)$**

Solution:

Let $\mathrm{p(x)} = x^3 - 3x^2 + 4x - 14$ ……………………… (1)

When $\mathrm{p(x)}$ is divided by $x+2$, then by Remainder Theorem, the remainder is:

$$x + 2 = 0$$

$$x = -2$$

Put the value of x in eq. (1)

$$\mathrm{R} = \mathrm{p}(-2) = (-2)^3 - 3(-2)^2 + 4(-2) - 14$$

$$\mathrm{R} = \mathrm{p}(-2) = -8 - 12 - 8 - 14$$

$$\mathrm{R} = \mathrm{p}(-2) = -42$$

2.

i. **If $(x+2)$ is a factor of $3x^2 - 4kx - 4k^2$, then find the value of k.**

Solution:

Let $\mathrm{p(x)} = 3x^2 - 4kx - 4k^2$ …………………………… (1)

When $\mathrm{p(x)}$ is divided by $x+2$, then by Remainder Theorem, the remainder is:

$x + 2 = 0$

$x = -2$

Put the value of x in eq. (1)

$\mathrm{p}(-2) = 3(-2)^2 - 4\mathrm{k}(-2) - 4\mathrm{k}^2$

$\mathrm{p}(-2) = 3(4) + 8\mathrm{k} - 4\mathrm{k}^2$

$\mathrm{p}(-2) = 12 + 8\mathrm{k} - 4\mathrm{k}^2$ ………………………… (A)

According to the given condition,

$\mathrm{p}(-2) = 0$ …………………………… (B)

By Comparing Eq. (A) and (B), we get

$12 + 8\mathrm{k} - 4\mathrm{k}^2 = 0$

Rearrange the terms

$-4k^2 + 8k + 12 = 0$

Taking '-4' as a Common

$-4(k^2 - 2k - 3) = 0$

$k^2 - 2k - 3 = \frac{0}{-4}$

$k^2 - 2k - 3 = 0$

$k^2 - 3k + k - 3 = 0$

$k(k - 3) + 1(k - 3) = 0$

$(k - 3)(k + 1) = 0$

$\Rightarrow$ $k - 3 = 0$ and $k + 1 = 0$

$k = 3$ $k = -1$

ii. If $(x - 1)$ is a factor of $x^3 - kx^2 + 11x - 6$, then find the value of k.

Solution:

Let $\mathrm{p(x)} = x^3 - kx^2 + 11x - 6$ ……………………………… (1)

When $\mathrm{p(x)}$ is divided by $x - 1$, then by Remainder Theorem, the remainder is:

$x - 1 = 0$

$x = 1$

Put the value of x in eq. (1)

$p(1) = (1)^3 - k(1)^2 + 11(1) - 6$

$p(1) = 1 - k + 11 - 6$

$p(1) = 6 - k$ ………………………………. (A)

According to given condition

$p(1) = 0$ ………………………………… (B)

By Comparing Eq. (A) and (B)

$6 - k = 0$

$k = 6$

3. Without actual long division determine whether

i. $(x-2)$ and $(x-3)$ are factors of $p(x) = x^3 - 12x^2 + 44x - 48$.

Solution:

$p(x) = x^3 - 12x^2 + 44x - 48$

Then the remainder for $x - 2$ is:

$p(2) = (2)^3 - 12(2)^2 + 44(2) - 48$

$p(2) = 8 - 12(4) + 88 - 48$

$p(2) = 96 - 48 - 48$

$p(2) = 96 - 96$

$p(2) = 0$

Hence by Factor Theorem $(x - 2)$ is a factor of given polynomial $p(x)$.

Again $p(x) = x^3 - 12x^2 + 44x - 48$

Then the remainder for $x - 3$ is:

$p(3) = (3)^3 - 12(3)^2 + 44(3) - 48$

$p(3) = 27 - 12(9) + 132 - 48$

$p(3) = 27 - 108 + 132 - 48$

$$\mathrm{p}(3) = 159 - 156 = 3 \neq 0$$

Hence by Factor Theorem $(x-3)$ is not a factor of given polynomial $\mathrm{p}(\mathrm{x})$.

ii. $\boldsymbol{(x-2)(x+3)}$ **and** $\boldsymbol{(x-4)}$ **are factors of** $\boldsymbol{q(x) = x^3 + 2x^2 - 5x - 6}$**.**

Solution:

$$\mathrm{q}(\mathrm{x}) = x^3 + 2x^2 - 5x - 6$$

Then the remainder for $x-2$ is:

$$\mathrm{q}(2) = (2)^3 + 2(2)^2 - 5(2) - 6$$

$$\mathrm{q}(2) = 8 + 8 - 10 - 6$$

$$\mathrm{q}(2) = 16 - 16 = 0$$

Hence by Factor Theorem $(x-2)$ is a factor of given polynomial $\mathrm{q}(\mathrm{x})$.

Again $\mathrm{q}(\mathrm{x}) = x^3 + 2x^2 - 5x - 6$

Then the remainder for $x+3$ is:

$$\mathrm{q}(-3) = (-3)^3 + 2(-3)^2 - 5(-3) - 6$$

$$\mathrm{q}(-3) = -27 + 2(9) + 15 - 6$$

$$\mathrm{q}(-3) = -27 + 18 + 15 - 6$$

$$\mathrm{q}(-3) = 33 - 33 = 0$$

Hence by Factor Theorem $(x+3)$ is a factor of given polynomial $\mathrm{q}(\mathrm{x})$.

Now, Again $\mathrm{q}(\mathrm{x}) = x^3 + 2x^2 - 5x - 6$

Then the remainder for $x-4$ is:

$$\mathrm{q}(4) = (4)^3 + 2(4)^2 - 5(4) - 6$$

$$\mathrm{q}(4) = 64 + 2(16) - 20 - 6$$

$$\mathrm{q}(4) = 54 + 32 - 20 - 6$$

$$\mathrm{q}(4) = 96 - 26 = 70 \neq 0$$

Hence by Factor Theorem $(x-4)$ is not a factor of given polynomial $\mathrm{q}(\mathrm{x})$.

4. **For what value of** $\boldsymbol{m}$ **is the polynomial** $\mathbf{p}(\mathrm{x}) = \boldsymbol{4x^3 - 7x^2 + 6x - 3m}$ **exactly divisible by** $\boldsymbol{x+2}$?

Solution:

$p(x) = 4x^3 - 7x^2 + 6x - 3m$ ………………………… (1)

When $p(x)$ is divided by $x + 2$, then by Remainder Theorem, the remainder is:

$x + 2 = 0$

$x = -2$

Put the value of x in eq. (1)

$p(-2) = 4(-2)^3 - 7(-2)^2 + 6(-2) - 3m$

$p(-2) = 4(-8) - 7(4) - 12 - 3m$

$p(-2) = -32 - 28 - 12 - 3m$

$p(-2) = -72 - 3m$ ……………………… (A)

According to give condition,

$p(-2) = 0$ ………………………………. (B)

By Comparing Eq. (A) and (B), we get,

$-72 - 3m = 0$

$-3m = 72$

$m = \frac{72}{-3}$

$m = -24$

5. Determine the value of k if $p(x) = kx^3 + 4x^2 + 3x - 4$ and $q(x) = x^3 - 4x + k$ leaves the same remainder when divided by $(x - 3)$.

Solution:

$p(x) = kx^3 + 4x^2 + 3x - 4$ ………………………………. (1)

$q(x) = x^3 - 4x + k$ ……………………………………… (2)

When $p(x)$ is divided by $x - 3$, then by Remainder Theorem, the remainder is:

$x - 3 = 0$

$x = 3$

Put the value of x in both eq. (1) and eq. (2)

$p(3) = k(3)^3 + 4(3)^2 + 3(3) - 4$

$$\text{p}(3) = 27k + 4(9) + 9 - 4$$

$$\text{p}(3) = 27k + 36 + 9 - 4$$

$\text{p}(3) = 27k + 41$ .... (A)

Now put in eq. (2)

$$\text{q}(3) = (3)^3 - 4(3) + k$$

$$\text{q}(3) = 27 - 12 + k$$

$\text{q}(3) = 15 + k$ .... (B)

According to given condition,

$$\text{q}(3) = \text{q}(3)$$

Now by Comparing Eq. (A) and Eq. (B)

$$27k + 41 = 15 + k$$

$$27k - k = 15 - 41$$

$$26k = -26$$

$$k = \frac{-26}{26}$$

$$k = -1$$

6. **The remainder after dividing the polynomial $\mathbf{p(x)} = \boldsymbol{x^3 + ax^2 + 7}$ by $\boldsymbol{(x+1)}$ is $\boldsymbol{2b}$. Calculate the value of $\boldsymbol{a}$ and $\boldsymbol{b}$ if this expression leaves a remainder of $\boldsymbol{(b+5)}$ on being divided by $\boldsymbol{(x-2)}$.**

Solution:

$$\text{p(x)} = x^3 + ax^2 + 7$$

When p(x) is divided by $x + 1$, then by Remainder Theorem, the remainder is $2b$.

$\text{p}(-1) = 2b$ .... (A)

$$\text{p}(-1) = (-1)^3 + a(-1)^2 + 7$$

$$\text{p}(-1) = -1 + a + 7$$

$\text{p}(-1) = a + 6$ .... (B)

Now by comparing eq. (A) and (B)

$a + 6 = 2b$

$a = 2b - 6$ ………………………. (1)

When $\mathrm{p(x)}$ is divided by $x - 2$, then by Remainder Theorem, the remainder is $b + 5$.

$\mathrm{p}(2) = b + 5$ ……………………… (C)

$\mathrm{p}(2) = (2)^3 + a(2)^2 + 7$

$\mathrm{p}(2) = 8 + 4a + 7$

$\mathrm{p}(2) = 4a + 15$ ………………….. (D)

By Comparing eq. (C) and (D)

$b + 5 = 4a + 15$

$4a = b + 5 - 15$

$4a = b - 10$

$a = \frac{b-10}{4}$ …………………………. (2)

Now by Comparing Eq. (1) and Eq. (2)

$2b - 6 = \frac{b-10}{4}$

$4(2b - 6) = b - 10$

$8b - 24 = b - 10$

$8b - b = -10 + 24$

$7b = 14$

$b = 2$

Put the value of b in eq. (1)

$a = 2(2) - 6$

$a = 4 - 6 = -2,$ Hence $a = -2, b = 2$

7. The polynomial $x^2 + lx^2 + mx + 24$ has a factor $(x + 4)$ and it leaves a remainder of 36 when divided by $(x - 2)$. Find the values of l and m.

Solution:

Let $\mathrm{p(x)} = x^2 + lx^2 + mx + 24$

When $\mathrm{p(x)}$ is divided by $x+4$, then by Remainder Theorem, the remainder is 0.

$$\mathrm{p}(-4)=0$$

$$(-4)^2+l(-4)^2+m(-4)+24=0$$

$$-64+16l-4m+24=0$$

$$16l-4m-40=0$$

Taking $'4'$ as a Common

$$4(4l-m-10)=0$$

$$4l-m-10=\frac{0}{4}$$

$$4l-m-10=0$$

$4l-m=10$ ………………………………. (1)

When $\mathrm{p(x)}$ is divided by $x-2$, then by Remainder Theorem, the remainder is 36.

$$\mathrm{p}(2)=36$$

$$(2)^2+l(2)^2+m(2)+24=36$$

$$8+4l+2m+24=36$$

$$4l+2m+32=36$$

$$4l+2m=36-32$$

$$4l+2m=4$$

Taking '2' as a Common

$$2(2l+2m)=4$$

$$2l+m=\frac{4}{2}$$

$2l+m=2$ ……………………………. (2)

By Adding Eq. (1) and Eq. (2), we get,

$$4l-m=10$$

$$2l+m=2$$

____

$6l = 12$

$l = \frac{12}{6}$

$l = 2$

Put $l = 2$ in Eq. (1) we get,

$4(2) - m = 10$

$8 - m = 10$

$m = 8 - 10$

$m = -2$

Hence $l = 2$, and $m = -2$

8. The expression $lx^3 + mx^2 - 4$ leaves remainder of -3 and 12 when divided by $(x - 1)$ and $(x + 2)$ respectively. Calculate the values of l and m.

Solution:

Let $p(x) = lx^3 + mx^2 - 4$

When $p(x)$ is divided by $x - 1$, then by Remainder Theorem, the remainder is -3.

$p(1) = -3$

$l(1)^3 + m(1)^2 - 4 = -3$

$l + m - 4 = -3$

$l + m = -3 + 4$

$l + m = 1$ ………………………………………. (1)

When $p(x)$ is divided by $x + 2$, then by Remainder Theorem, the remainder is 12.

$p(-2) = 12$

$l(-2)^3 + m(-2)^2 - 4 = 12$

$-8l + 4m - 4 = 12$

$-8l + 4m = 12 + 4$

$-8l + 4m = 16$

Taking '4' as a Common

$4(-2l+m)=16$

$-2l+m=\frac{16}{4}$

$-2l+m=4$ ………………………….. (2)

Subtract eq. (2) from eq. (1), we get

$l+m=1$

$-2l+m=4$

$+ \quad - \quad -$

$3l=-3$

$l=-1$

Put $l=-1$ in eq. (1), we get

$-1+m=1$

$m=1+1=2$

Hence $m=2$ and $l=-1$.

9. The expression $ax^3-9x^2+bx+3a$ is exactly divisible by x^2-5x+6. Find the values of a and b.

Solution:

Let $\mathrm{p(x)}=ax^3-9x^2+bx+3a$

As $x^2-5x+6=0$

$x^2-3x-2x+6=0$

$x(x-3)-2(x-3)=0$

$(x-3)(x-2)=0$

When $\mathrm{p(x)}$ is divided by $x-3$, then by Remainder Theorem, the remainder is:

$\mathrm{p}(3)=0$

$a(3)^3-9(3)^2+b(3)+3a=0$

$27a-9(9)+3b+3a=0$

$27a-81+3b+3a=0$

$30a + 3b = 81$ ……………………………. (1)

When p(x) is divided by $x - 2$, then by Remainder Theorem, the remainder is:

$\mathrm{p}(2) = 0$

$a(2)^3 - 9(2)^2 + b(2) + 3a = 0$

$8a - 36 + 2b + 3a = 0$

$11a + 2b = 36$ ………………………………. (2)

Multiply eq. (1) by '2' and eq. (2) by '3'

$60a + 6b = 162$ ……………………………. (3)

$33a + 6b = 108$ ……………………………… (4)

Subtract eq. (4) from eq. (3), we get

$60a + 6b = 162$

$\pm 33a \pm 6b = \pm 108$

$27a = 54$

$a = 2$

Put $a = 2$ in eq. (3), we get

$60(2) + 6b = 162$

$120 + 6b = 162$

$6b = 162 - 120$

$6b = 42$

$b = 7$

Hence $a = 2$, and $b = 7$.

Exercise 5.4

Factorize of the following cubic polynomials by factor theorem.

1. $\boldsymbol{x^3 - 2x^2 - x + 2}$

Solution:

Let $\mathrm{p}(\mathrm{x}) = x^3 - 2x^2 - x + 2$ ……………………………. (1)

Let $x = 1$, Put in eq. (1), we get

$\mathrm{p}(1) = (1)^3 - 2(1)^2 - (1) + 2$

$\mathrm{p}(1) = 1 - 2 - 1 + 2$

$\mathrm{p}(1) = 0$

Hence $x - 1$ is a factor of $\mathrm{p}(\mathrm{x})$.

Let $x = 2$, Put in eq. (1), we get

$\mathrm{p}(2) = (2)^3 - 2(2)^2 - (2) + 2$

$\mathrm{p}(2) = 8 - 8 - 2 + 2$

$\mathrm{p}(2) = 0$

Hence $x - 2$ is also a factor of $\mathrm{p}(\mathrm{x})$.

Let $x = -1$, Put in eq. (1), we get

$\mathrm{p}(-1) = (-1)^3 - 2(-1)^2 - (-1) + 2$

$\mathrm{p}(-1) = -1 - 2 + 1 + 2$

$\mathrm{p}(-1) = 0$

Hence $x + 1$ is also a factor of $\mathrm{p}(\mathrm{x})$.

Thus the factorize from of $\mathrm{p}(\mathrm{x}) = x^3 - 2x^2 - x + 2$ is $(x + 1)(x - 1)(2 - 2)$.

2. $\boldsymbol{x^3 - x^2 - 22x + 40}$

Solution:

Let $\mathrm{p}(\mathrm{x}) = x^3 - x^2 - 22x + 40$ ………………………………. (1)

Let $x = 2$, Put in eq. (1), we get

$\mathrm{p}(2) = (2)^3 - (2)^2 - 22(2) + 40$

$\mathrm{p}(2) = 8 - 4 - 44 + 40$

$\mathrm{p}(2) = 48 - 48 = 0$

Hence $x-2$ is a factor of $\mathrm{p(x)}$.

Let $x=4$, Put in eq. (1), we get

$$\mathrm{p}(4)=(4)^3-(4)^2-22(4)+40$$

$$\mathrm{p}(4)=64-16-88+40$$

$$\mathrm{p}(4)=104-104=0$$

Hence $x-4$ is also a factor of $\mathrm{p(x)}$.

Let $x=-5$, Put in eq. (1), we get

$$\mathrm{p}(-5)=(-5)^3-(-5)^2-22(-5)+40$$

$$\mathrm{p}(-5)=-125-25+110+40$$

$$\mathrm{p}(-5)=-150+150=0$$

Hence $x+5$ is also a factor of $\mathrm{p(x)}$.

Thus the factorize from of $\mathrm{p(x)}=x^3-x^2-22x+40$ is$(x+5)(x-2)(x-4)$.

3. $\boldsymbol{x^3-6x^2+3x+10}$

Solution:

Let $\mathrm{p(x)}=\ x^3-6x^2+3x+10$ ……………………………… (1)

Let $x=-1$, Put in eq. (1), we get

$$\mathrm{p}(-1)=(-1)^3-6(-1)^2+3(-1)+10$$

$$\mathrm{p}(-1)=-1-6-3+10$$

$$\mathrm{p}(-1)-10+10=0$$

Hence $x+1$ is a factor of $\mathrm{p(x)}$.

Let $x=2$, Put in eq. (1), we get

$$\mathrm{p}(2)=(2)^3-6(2)^2+3(2)+10$$

$$\mathrm{p}(2)=8-24+6+10$$

$$\mathrm{p}(2)=24-24=0$$

Hence $x-2$ is also a factor of $\mathrm{p(x)}$.

Let $x=5$, Put in eq. (1), we get

$\text{p}(5) = (5)^3 - 6(5)^2 + 3(5) + 10$

$\text{p}(5) = 125 - 150 + 15 + 10$

$\text{p}(5) = 150 - 150 = 0$

Hence $x - 5$ is also a factor of p(x).

Thus the factorize from of $\text{p(x)} = x^3 - 6x^2 + 3x + 10$ is $(x + 1)(x - 2)(x - 5)$.

4. $\boldsymbol{x^3 + x^2 - 10x + 8}$

Solution:

Let $\text{p(x)} = x^3 + x^2 - 10x + 8$ .... (1)

Let $x = 1$, Put in eq. (1), we get

$\text{p}(1) = (1)^3 + (1)^2 - 10(1) + 8$

$\text{p}(1) = 1 + 1 - 10 + 8$

$\text{p}(1)10 - 10 = 0$

Hence $x - 1$ is a factor of p(x).

Let $x = 2$, Put in eq. (1), we get

$\text{p}(2) = (2)^3 + (2)^2 - 10(2) + 8$

$\text{p}(2) = 8 + 4 - 20 + 8$

$\text{p}(2) = 20 - 20 = 0$

Hence $x - 2$ is also a factor of p(x).

Let $x = -4$, Put in eq. (1), we get

$\text{p}(-4) = (-4)^3 + (-4)^2 - 10(-4) + 8$

$\text{p}(-4) = -64 + 16 + 40 + 8$

$\text{p}(-4) = -64 + 64 = 0$

Hence $x + 4$ is also a factor of p(x).

Thus the factorize from of $\text{p(x)} = x^3 + x^2 - 10x + 8$ is $(x - 1)(x - 2)(x + 4)$.

5. $\boldsymbol{x^3 - 2x^2 - 5x + 6}$

Solution:

Let $\mathrm{p(x)} = x^3 - 2x^2 - 5x + 6$ .... (1)

Let $x = 1$, Put in eq. (1), we get

$$\mathrm{p}(1) = (1)^3 - 2(1)^2 - 5(1) + 6$$

$$\mathrm{p}(1) = 1 - 2 - 5 + 6$$

$$\mathrm{p}(1) = 7 - 7 = 0$$

Hence $x - 1$ is a factor of $\mathrm{p(x)}$.

Let $x = -2$, Put in eq. (1), we get

$$\mathrm{p}(-2) = (-2)^3 - 2(-2)^2 - 5(-2) + 6$$

$$\mathrm{p}(-2) = -8 - 8 + 10 + 6$$

$$\mathrm{p}(-2) = -16 + 16 = 0$$

Hence $x + 2$ is also a factor of $\mathrm{p(x)}$.

Let $x = 3$, Put in eq. (1), we get

$$\mathrm{p}(3) = (3)^3 - 2(3)^2 - 5(3) + 6$$

$$\mathrm{p}(3) = 27 - 18 - 15 + 6$$

$$\mathrm{p}(3) = 33 - 33 = 0$$

Hence $x - 3$ is also a factor of $\mathrm{p(x)}$.

Thus the factorize from of $\mathrm{p(x)} = x^3 - 2x^2 - 5x + 6$ is $(x-1)(x+2)(x-3)$.

6. $\boldsymbol{x^3 + 5x^2 - 2x - 24}$

Solution:

Let $\mathrm{p(x)} = x^3 + 5x^2 - 2x - 24$ .... (1)

Let $x = 2$, Put in eq. (1), we get

$$\mathrm{p}(2) = (2)^3 + 5(2)^2 - 2(2) - 24$$

$$\mathrm{p}(2) = 8 + 20 - 4 - 24$$

$$\mathrm{p}(2) = 28 - 28 = 0$$

Hence $x - 2$ is a factor of $\mathrm{p(x)}$.

Let $x = -3$, Put in eq. (1), we get

$$p(-3) = (-3)^3 + 5(-3)^2 - 2(-3) + 24$$

$$p(-3) = -27 + 45 + 6 - 24$$

$$p(-3) = -51 + 51 = 0$$

Hence $x + 3$ is also a factor of $\mathrm{p(x)}$.

Let $x = -4$, Put in eq. (1), we get

$$p(-4) = (-4)^3 + 5(-4)^2 - 2(-4) - 24$$

$$p(-4) = -64 + 80 + 8 - 24$$

$$p(-4) = -88 + 88 = 0$$

Hence $x + 4$ is also a factor of $\mathrm{p(x)}$.

Thus the factorize from of $\mathrm{p(x)} = x^3 + 5x^2 - 2x - 24$ is $(x-2)(x+3)(x+4)$.

7. $\boldsymbol{3x^3 - x^2 - 12x + 4}$

Solution:

Let $\mathrm{p(x)} = 3x^3 - x^2 - 12x + 4$ .... (1)

Let $x = 2$, Put in eq. (1), we get

$$p(2) = 3(2)^3 - (2)^2 - 12(2) + 4$$

$$p(2) = 24 - 4 - 24 + 4$$

$$p(2) = 28 - 28 = 0$$

Hence $x - 2$ is a factor of $\mathrm{p(x)}$.

Let $x = -2$, Put in eq. (1), we get

$$p(-2) = 3(-2)^3 - (-2)^2 - 12(-2) + 4$$

$$p(-2) = -24 - 4 + 24 + 4$$

$$p(-2) = -28 + 28 = 0$$

Hence $x + 2$ is also a factor of $\mathrm{p(x)}$.

Let $x = \frac{1}{3}$ Put in eq. (1), we get

$$p\left(\frac{1}{3}\right) = 3\left(\frac{1}{3}\right)^3 - \left(\frac{1}{3}\right)^2 - 12\left(\frac{1}{3}\right) + 4$$

Note:

$$3x - 1 = 0$$

$$3x = 1$$

$$x = \frac{1}{3}$$

$$p\left(\frac{1}{3}\right) = 3\left(\frac{1}{27}\right) - \frac{1}{9} - 4 + 4$$

$$p\left(\frac{1}{3}\right) = \frac{1}{9} - \frac{1}{9} - 4 + 4 = 0$$

Hence $3x - 1$ is also a factor of $\mathrm{p(x)}$.

Thus the factorize from of $\mathrm{p(x)} = 3x^3 - x^2 - 12x + 4$ is $(x-2)(x+2)(3x-1)$.

8. $\boldsymbol{2x^3 + x^2 - 2x - 1}$

Solution:

Let $\mathrm{p(x)} = 2x^3 + x^2 - 2x - 1$ …………………………. (1)

Let $x = 1$, Put in eq. (1), we get

$$p(1) = 2(1)^3 + (1)^2 - 2(1) - 1$$

$$p(1) = 2 + 1 - 2 - 1$$

$$p(1) = 3 - 3 = 0$$

Hence $x - 1$ is a factor of $\mathrm{p(x)}$.

Let $x = -1$, Put in eq. (1), we get

$$P(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1$$

$$P(-1) = -2 + 1 + 2 - 1$$

$$P(-1) = -3 + 3 = 0$$

Hence $x + 1$ is also a factor of $\mathrm{p(x)}$.

Let $x = \frac{-1}{2}$, Put in eq. (1), we get

$$P\left(\frac{-1}{2}\right) = 2\left(\frac{-1}{2}\right)^3 + \left(\frac{-1}{2}\right)^2 - 2\left(\frac{-1}{2}\right) - 1$$

$$P\left(\frac{-1}{2}\right) = 2\left(\frac{-1}{8}\right) + \frac{1}{4} + \frac{2}{2} - 1$$

$$P\left(\frac{-1}{2}\right) = -\frac{1}{4} + \frac{1}{4} + 1 - 1$$

$$P\left(\frac{-1}{2}\right) = 0$$

Hence $2x + 1$ is also a factor of $\mathrm{p(x)}$.

Thus the factorize from of $\mathrm{p(x)} = 2x^3 + x^2 - 2x - 1$ is $(x-1)(x+1)(2x+1)$.

Review Exercise

1. Multiple Choice Questions. Choose the correct answer.

i. The factors of $x^2 - 5x + 6$ are ……………

a) $x+1, x-6$ b) $x-2, x-3$ c) $x+6, x-1$ d) $x+2, x+3$

Correct Answer is: b

Explanation:

$$x^2 - 5x + 6 = x^2 - 2x - 3x + 6 = x(x-2) - 3(x-2) = (x-2)(x-3)$$

ii. Factors of $8x^3 + 27y^3$ are ………………

a) $(2x+3y), (4x^2+9y^2)$
b) $(2x-3y), (4x^2-9y^2)$
c) $(2x+3y), (4x^2-6xy+9y^2)$
d) $(2x-3y), (4x^2+6xy+9y^2)$

Correct Answer is: c

Explanation:

$$8x^3 + 27y^3 = (2x)^3 + (3y)^3 = (2x+3y)[(2x)^2 - (2x)(3y) + (3y)^2]$$
$$= (2x+3y)(4x^2 - 6xy + 9y^2)$$

iii. Factors of $3x^2 - x - 2$ are ………………

a) $(x+1), (3x-2)$

b) $(x+1), (3x+2)$

c) $(x-1), (3x-2)$

d) $(x-1), (3x+2)$

Correct Answer is: d

Explanation:

Let $\mathrm{p(x)} = 3x^2 - x - 2$ …………………….. (1)

Let $x = 1$, Put in eq. (1), we get

$$\mathrm{p}(1) = 3(1)^2 - (1) - 2$$

$$\mathrm{p}(1) = 3 - 1 - 2$$

$$\mathrm{p}(1) = 3 - 3 = 0$$

Hence $x-1$ is a factor of $\mathrm{p(x)}$.

Let $x=\frac{-2}{3}$ Put in eq. (1), we get

$$\mathrm{p}\left(\frac{-2}{3}\right)=3\left(\frac{-2}{3}\right)^2-\left(\frac{-2}{3}\right)-2$$

$$\mathrm{p}\left(\frac{-2}{3}\right)=3\left(\frac{4}{9}\right)+\frac{2}{3}-2$$

$$\mathrm{p}\left(\frac{-2}{3}\right)=\frac{4}{3}+\frac{2}{3}-2$$

$$\mathrm{p}\left(\frac{-2}{3}\right)=\frac{4+2-6}{3}$$

$$\mathrm{p}\left(\frac{-2}{3}\right)=\frac{0}{3}=0$$

Note:

$3x+2=0$

$3x=-2$

$x=\frac{-2}{3}$

Hence $3x+2$ is also a factor of $\mathrm{p(x)}$.

Thus the factorize from of $\mathrm{p(x)}=3x^2-x-2$ is $(x-1)(3x+2)$.

iv. **Factors of a^4-4b^4 are ……………..**

a) $(a-b),(a+b),(a^2+4b^2)$

b) $(a^2-2b^2),(a^2+2b^2)$

c) $(a-b),(a+b),(a^2-4b^2)$

d) $(a-2b),(a^2+2b^2)$

Correct Answer is: b

Explanation:

$$a^4-4b^4=(a^2)^2-(2b^2)^2=(a^2-2b^2)(a^2+2b^2)$$

v. **What will be added to complete to square of $9a^2-12ab$? …………………..**

a) $16b^2$ b) $16b^2$ c) $4b^2$ d) $-4b^2$

Correct Answer is: c

Explanation:

$$9a^2-12ab=(3a)^2-2(3a)(2b)+(2b)^2=(3a-2b)^2$$

vi. **Find m so that x^2+4x+m is a complete square ………………**

a) 8 b) -8 c) 4 d) 16

Correct Answer is: c

Explanation:

$$x^2 + 4x + m = x^2 + 4x + 4 = (x)^2 + 2(x)(2) + (2)^2 = (x+2)^2$$

vii. **Factors of $5x^2 - 17xy - 12y^2$ are ....**

a) $(x+4y)(5x+3y)$
b) $(x-4y)(5x-3y)$
c) $(x-4y)(5x+3y)$
d) $(5x-4y(x+3y)$

Correct Answer is: c

Explanation:

$$5x^2 - 17xy - 12y^2 = 5x^2 - 20xy + 3xy - 12y^2 = 5x(x-4y) + 3y(x-4y)$$
$$= (x-4y)(5x+3y)$$

viii. **Factors of $27x^3 - \frac{1}{x^3}$ ....**

a) $\left(3x - \frac{1}{x}\right), \left(9x^2 + 3 + \frac{1}{x^2}\right)$
b) $\left(3x + \frac{1}{x}\right), \left(9x^2 + 3 + \frac{1}{x^2}\right)$
c) $\left(3x - \frac{1}{x}\right), \left(9x^2 - 3 + \frac{1}{x^2}\right)$
d) $\left(3x + \frac{1}{x}\right), \left(9x^2 - 3 + \frac{1}{x^2}\right)$

Correct Answer is: a

Explanation:

$$27x^3 - \frac{1}{x^3} = (3x)^3 - \left(\frac{1}{x}\right)^3 = \left(3x - \frac{1}{x}\right)\left[(3x)^2 + (3x)\left(\frac{1}{x}\right) + \left(\frac{1}{x}\right)^2\right]$$
$$= \left(3x - \frac{1}{x}\right), \left(9x^2 + 3 + \frac{1}{x^2}\right)$$

2. **Completion Items. Fill in the blanks.**

i. $x^2 + 5x + 6 =$ ______________ **Answer:** $(x+2)(x+3)$

Explanation:

$$x^2 + 5x + 6 = x^2 + 2x + 3x + 2x + 6 = x(x+2) + 3(x+2) = (x+2)(x+3)$$

ii. $4a^2 - 16 =$ ________________ **Answer:** $4(a-2)(a+2)$

Explanation:

$$4a^2 - 16 = 4[a^2 - 4] = 4[(a)^2 - (2)^2] = 4(a-2)(a+2)$$

iii. $4a^2 + 4ab + ($ __________ $)$ is a complete square. **Answer:** b^2

Explanation:

$$4a^2 + 4ab + b^2 = (2a)^2 + 2(2a)(b) + (b)^2 = (2a+b)^2$$

iv. $\frac{x^2}{y^2} - 2 + \frac{y^2}{x^2} =$ _____________ **Answer:** $\left(\frac{x}{y} - \frac{y}{x}\right)^2$

Explanation:

$$\frac{x^2}{y^2} - 2 + \frac{y^2}{x^2} = \left(\frac{x}{y}\right)^2 - 2\left(\frac{x}{y}\right)\left(\frac{y}{x}\right) + \left(\frac{y}{x}\right)^2 = \left(\frac{x}{y} - \frac{y}{x}\right)^2$$

v. $(x+y)(x^2 - xy + y^2) =$ ______________ **Answer:** $x^3 + y^3$

vi. Factorized form of $a^4 - 16$ is ____________ **Answer:** $(a^2+4)(a^2-4)$

Explanation:

$$a^4 - 16 = (a^2)^2 - (4)^2 = (a^2+4)(a^2-4)$$

vii. If $x - 2$ is a factor of $\text{p(x)} = x^2 + 2kx + 8$, then k= __________ **Answer:** -3

Explanation:

$$\text{p(x)} = x^2 + 2kx + 8$$

$$\text{p}(2) = (2)^2 + 2k(2) + 8$$

$$\text{p}(2) = 4 + 4k + 8$$

$$\text{p}(2) = 12 + 4k \text{ .... (1)}$$

According to given condition

$$\text{p}(2) = 0 \text{ .... (2)}$$

By Comparing eq. (1) and eq. (2)

$$12 + 4k = 0$$

$$4k = -12$$

$$k = \frac{-12}{4}$$

$$k = -3$$

3. Factorize the following.

i. $\boldsymbol{x^2 + 8x + 16 - 4y^2}$

Solution:

$$x^2 + 8x + 16 - 4y^2$$

$$= (x^2 + 8x + 16) - 4y^2$$

$$= [(x)^2 + 2(x)(4) + (4)^2] - (2y)^2$$

$$= (x + 4)^2 - (2y)^2$$

$$= (x + 4 + 2y)(x + 4 - 2y)$$

$$= (x + 2y + 4)(x - 2y + 4)$$

ii. $\boldsymbol{4x^2 - 16y^2}$

Solution:

$$4x^2 - 16y^2$$

$$= 4[x^2 - 4y^2]$$

$$= 4[(x)^2 - (2y)^2]$$

$$= 4(x - 2y)(x + 2y)$$

iii. $\boldsymbol{9x^2 + 27x + 8}$

Solution:

$$9x^2 + 27x + 8$$

$$= 9x^2 + 24x + 3x + 8$$

$$= 3x(x + 8) + 1(3x + 8)$$

$$= (x+8)(3x+1)$$

iv. $\mathbf{1 - 64z^3}$

Solution:

$$1 - 64z^3$$

$$= (1)^3 - (4z)^3$$

$$= (1-4z)[(1)^2 + (1)(4z) + (4z)^2]$$

$$= (1-4z)(1+4z+16z^2)$$

v. $\mathbf{8x^3 - \frac{1}{27y^3}}$

Solution:

$$8x^3 - \frac{1}{27y^3}$$

$$= (2x)^3 - \left(\frac{1}{3y}\right)^3$$

$$= \left(2x - \frac{1}{3y}\right)\left[(2x)^2 + (2x)\left(\frac{1}{3y}\right) + \left(\frac{1}{3y}\right)^2\right]$$

$$= \left(2x - \frac{1}{3y}\right)\left(2x^2 + \frac{2x}{3y} + \frac{1}{9y^2}\right)$$

vi. $\mathbf{2y^2 + 5y - 3}$

Solution:

$$2y^2 + 5y - 3$$

$$= 2y^2 + 6y - y - 3$$

$$= 2y(y+3) - 1(y+3)$$

$$= (y+3)(2y-1)$$

vii. $\mathbf{x^3 + x^2 - 4x - 4}$

Solution:

$$x^3 + x^2 - 4x - 4$$

Rearrange the terms

$= x^3 - 4x + x^2 - 4$

$= x(x^2 - 4) + 1(x^2 - 4)$

$= (x + 1)(x^2 - 4)$

$= (x + 1)[(x)^2 - (2)^2]$

$= (x + 1)(x + 2)(x - 2)$

viii. $\mathbf{25m^2n^2 + 10mn + 1}$

Solution:

$25m^2n^2 + 10mn + 1$

$= (5mn)^2 + 2(5mn)(1) + (1)^2$

$= (3mn + 1)^2$

ix. $\mathbf{1 - 12pq + 36p^2q^2}$

Solution:

$1 - 12pq + 36p^2q^2$

$= (1)^2 - 2(1)(6pq) + (6pq)^2$

$= (1 - 6pq)^2$

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☺ Best of Luck ☺

֍ Happy Learning ֍