DEFINITIONS

9th class Math (English med)

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UNIT 1

Matrix and Determinant of Matrices

Matrix: They rectangular array of Real numbers in the form of rows and columns is called matrix. $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Order of a Matrix: If a Matrix "A" has 'm' rows and 'n' columns then order of matrix A will be m-by-n.

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ has order 2-by-2.

Equal Matrix: Two Matrices are said to be equal if they have same order and their corresponding entries are equal.

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 1 & 1+1 \\ 3 & 4 \end{bmatrix}$

Row matrix: A matrix is called a row matrix, if it has only one row.

$A = \begin{bmatrix} 1 & 2 \end{bmatrix}$

Column Matrix: A matrix is called column matrix, if it has only one column.

$A = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

Rectangular Matrix: A matrix is called rectangular. If the number of rows not equal to number of columns.

$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

Square Matrix: A matrix is called square matrix, if the number of rows are equal to number of columns.

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Null or Zero Matrix: A matrix is called a Null or Zero matrix, if each of its entries is Zero (0).

$O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Symmetric Matrix: A matrix is called symmetric matrix if $A^t = A$

$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

Skew Symmetric Matrix: A square matrix is called skew symmetric $A^t = -A$

$A = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$

Transpose of a matrix: A matrix obtained by changing the rows into columns or to columns in to rows of a matrix is called transpose of matrix.

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, A^t = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

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Negative Matrix: Let A be a matrix then its negative –A is obtained by changing the signs of all the

entries. $\qquad A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, -A = \begin{bmatrix} -1 & -2 \\ -3 & -4 \end{bmatrix}$

Diagonal Matrix: A square matrix is called diagonal matrix if at least any one of the entries of its diagonal is not zero and non-diagonal entries must all be zero.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Scalar Matrix: A diagonal matrix is called scalar matrix if all the diagonal entries are same.

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Identity Matrix: A diagonal matrix is called identity (unit) matrix if all diagonal entries are 1.

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Singular Matrix: A square matrix is called singular matrix if its determinant is equal to zero.

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$$

Non Singular Matrix: A square matrix is called non-singular matrix, if its determinant equal to zero.

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Adjoint of a Matrix: Adjoint of a matrix is obtained by interchanging the diagonal entries and changing sign of other entries.

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, adjA = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Additive Identity: If 'A' is a matrix then a matrix 'O' of same order as 'A' is called Additive Identity of A if

A+O=A=O+A

$$O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, O = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Additive Inverse: 'A' and 'B' Matrices of same order are additive inverse of each other if

A+B=O=B+A

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} -1 & -2 \\ -3 & -4 \end{bmatrix}$$

Inverse of a Matrix:

If 'A' is a matrix then its inverse given as $\qquad A^{-1} = \dfrac{adjA}{|A|}$

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| **Determinant of Matrix (2-2)** | Determinant of a matrix is denoted by $\lvert A \rvert$ $$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \; \lvert A \rvert = ad - bc$$ |

Unit 2

Real and Complex numbers

Natural numbers:	The numbers which we use for counting certain objects are called natural numbers. $N = \{1, 2, 3........\}$
Whole numbers:	If we include 0 in the set of natural numbers then the resulting set is the set of whole numbers. $W = \{0, 1, 2, 3........\}$
Integers:	The set of integers consist of positive integers, 0 and negative integers. $Z = \{0, \pm 1, \pm 2, \pm 3........\}$
Rational numbers:	The numbers that can be written in the form of $\dfrac{p}{q}$ where $p, q \in Z$ and $q \neq 0$
Irrational numbers:	The numbers that cannot be written in the form of $\dfrac{p}{q}$ where $p, q \in Z$ and $q \neq 0$
Real Numbers:	The union of the set of rational numbers and irrational numbers is called set of real numbers. It is denoted by **R**. $\quad R = Q \cup Q^{`}$
Terminating Decimal Fractions:	The fraction in which there are finite number of digits in its decimal part is called a terminating decimal fraction. for example $\dfrac{1}{2} = 0.5$ and $\dfrac{2}{5} = 0.4$
Recurring and non-terminating decimal fractions:	It is a decimal fraction in which some digits are repeated again and again in the same order in its decimal part. for example $\dfrac{2}{9} = 0.2222....$ and $\dfrac{4}{11} = 0.363636....$
Radical and Radicand:	In $\sqrt[n]{x}$ real number x is called radicand, n is called index and $\sqrt{}$ is the sign of radical.
Complex or Imaginary numbers:	The number of the form $a + bi$ is called Complex number. Where a and b are real numbers and $i = \sqrt{-1}$.
Conjugate of Complex number:	$a + bi$ and $a - bi$ are the conjugate of each other.
Additive Identity:	In the set of real numbers R '0' is called additive identity. a+0=a=0+a $\quad \forall, a \in R$
Additive Inverse:	In Set of real numbers each number 'a' has additive inverse '-a' a+(-a)=0=(-a)+a $\quad \forall, a \in R$

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Multiplicative Identity:	In set of real numbers R '1' is called multiplicative Identity.
	a.1=a=1.a $\forall, a \in R$
Multiplicative Inverse:	In set of Real numbers each number 'a' has multiplicative inverse '-a'
	$aa^{-1} = 1 = a^{-1}a, \forall a \in R \qquad a^{-1} = \dfrac{1}{a}$

Unit 3

Logarithms

Logarithm:	If $a^x = y$, then x is called the logarithm of y to the base 'a' and is written as $\log_a y = x$ where $a > 0, y > 0$ and $a, x, y \in R$
Scientific Notation:	Number written in the form a x 10n where 1≤a <10 and n is an integer called scientific notation.
Common or Brigs Logarithm:	If the base of logarithm is taken as 10 then logarithm is called common logarithm.
Natural Logarithm:	Logarithm having base e is called Napier logarithm or Natural logarithm.
Characteristic:	The integral part of the logarithm of any number is called the characteristic.
Mantissa:	The decimal part of the logarithm of a number is called the mantissa and is always positive.
Anti-Logarithm:	The number whose logarithm value is given is called anti-logarithm.
Laws of logarithm:	$\log_a (mn) = \log_a m + \log_a n$
	$\log_a \left(\dfrac{m}{n}\right) = \log_a m - \log_a n$
	$\log_a m^n = n \log_a m$
	$\log_b n \log_a b = \log_a n \quad \text{or} \quad \log_b n \log_a b = \dfrac{\log_b n}{\log_b a}$

Unit 4

Algebraic Expressions and Algebraic Formulas

Algebra:	Algebra is a generalization of arithmetic.
Algebraic Expression:	An algebraic expression is that in which constants and variables or both are combined by addition and subtraction. e.g. $5x^2$-4x
Polynomial:	A polynomial in the variable x is an algebraic expression of the from $P(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + ...a_1 x^1 + a_0$ where $a_n \neq 0$ and 'n' is a positive integer.
Degree Of Polynomial:	The degree of the polynomial in variable x is the highest power of the variable x.

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Rational Expression:	The expression of the form $\dfrac{p(x)}{q(x)}$ where $q(x) \neq 0$ is called rational expression if p(x) and q(x) are polynomials. $\dfrac{2x+1}{3x+1}$
Irrational Expression:	The expression of the form $\dfrac{p(x)}{q(x)}$ where $q(x) \neq 0$ is called irrational expression if p(x) and q(x) are not polynomials. $\dfrac{\sqrt{2x}+1}{\sqrt{3x}+1}$
Surd:	An irrational radical with radicand is called a surd. e.g $\sqrt{2}, \sqrt{3}, \sqrt{2}+\sqrt{3}$
Surd index (order) and Surd radicand:	In $\sqrt[n]{x}$ 'n' is called surd order and 'x' is called radicand.
Monomial Surd:	A surd which contain a single term is called monomial surd. e.g. $\sqrt{2}, \sqrt{3}$
Binomial Surd:	A surd which contain sum of two monomial surds is called binomial surd. e.g. $\sqrt{2}+\sqrt{3}$
Conjugate of Surd:	Conjugate of $\sqrt{x}+\sqrt{y}$ is $\sqrt{x}-\sqrt{y}$.

Unit 5

Factorization

Factorization:	The process of expressing an algebraic expression in term of its factor is called factorization.
Remainder theorem:	If a polynomial f(x) is a divided by a linear divisor (x-a), then the remainder is f(a).
Factor theorem:	The polynomial (x-a) is a factor of the polynomial f(x) if and only if f(a)=0.
Zero of polynomial:	If a specific number x=a is substituted for the variable x in a polynomial p(x) so that the value of P(a) is zero then x=a is called zero of polynomial p(x).

Unit 6

ALGEBRAIC MANIPULATION

HCF (Highest common factor):	If two or more algebraic expression are given then their common factor of highest power is called HCF of the expression.
LCM (Least common multiple):	The product of common factor together with non-common factors of the given expression is called LCM.
Relation between HCF and LCM:	LCM. HCF =p(x)+q(x) Where p(x) and q(x) are given expression.

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Unit 7

Linear Equations and Inequalities

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Linear EQUATION:	A linear equation in one unknown variable x is an equation of the form ax+b=0. Where $a, b \in R$ and a $\neq 0$
Identity equation:	An identity is an equation that is satisfied by every number for which both sides are defined. e.g x+3=3+x
Conditional equation:	A conditional equation is an equation that is satisfied by at least one number but is not an identity. e.g 2x+1=9.
Inconsistent equating:	An inconsistent equation is an equation whose solution set is the empty set. eg. x =x+5 because no value of x satisfied it
Rational equation:	When the variable in an equation occurs under a radical sign the equation is called rational equation. e.g $2x - 3 - 7 = 0$
Extraneous solution:	A solution that does not satisfy the original equation is called extraneous solution.
Linear inequality:	A linear inequality is one variable x is an inequality in which the variable x occurs only to the first power and is of the form $ax + b < 0$ where a $\neq 0$
Absolute Value of Real numbers:	Absolute value of a is denoted by $\|a\|$ and $$\|a\| = \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } a < 0 \end{cases}$$
Properties of Absolute value:	$\left\|\dfrac{a}{b}\right\| = \dfrac{\|a\|}{\|b\|}$ $\qquad$ $\|ab\| = \|a\|.\|b\|$ $\qquad$ $\|-a\| = \|a\|$ $\qquad$ $\|a\| \geq 0$ If $\|x\| = a$ then $x = a$ or $x = -a$
Tracheotomy law:	If $a, b \in R$ then $a > b$ or $a = b$ or $a < b$
Transitive law:	$a > b$ and $b > c \Rightarrow a > c$

Unit 8

Linear Graphs & Their Applications

Cartesian plane:	The plane formed by two straight lines perpendicular to each other is called Cartesian plane.
Coordinate axes:	The line intersecting each other perpendicularly are called Co-Ordinate Axes.

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Order pair of elements: An order pair is a pair of elements in which elements are written in specific order.

Origin: The point of intersection of two co-ordinate axes is called origin. It is denoted by $O(0,0)$.

Abscissa and ordinate: The x coordinate of point is called Abscissa and y coordinate is called ordinate. e.g (3,6) have 3 is Abscissa and 5 is ordinate

Collinear point: The set of points which lie on the same line are called Collinear point.
Line segment: On line L the two points P and Q and set of all points between them form a line segment $\overline{PQ}$.

P   Q

$\longleftrightarrow$

L
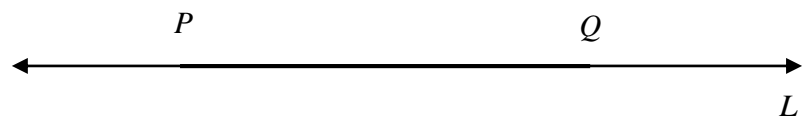
Unit 9

Introduction to Coordinate Geometry

Plane geometry and co-ordinate geometry: The study of geometrical shapes in a plane is called plane geometry. Co- ordinate geometry is the study of geometrical shapes in the Cartesian plane.

Distance formula: The distance between two points A(x1,y1) and B (x2,y2) is define as

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Collinear point: Two or more than two points which lie on the same straight line are called collinear point.

Non collinear point: Points are non collinear is they do not lie on the same straight line are called non collinear point.

Rectangle: A figure formed in the plane by four non collinear points is called a rectangle if
i. Its opposite sides are of equal measure
ii. Its opposite sides are parallel to each other

Parallelogram: A figure formed by four none collinear point in the plane is called parallelogram if
i. Its opposite sides are of equal measure
ii. Its opposite sides are parallel
iii. Measure of none of the angle is 90º

Triangle: A simple closed figure which consist of three sides and three angles is called triangle.

Equilateral triangle: If the length of all the three sides of a triangle are same then the triangle is called an equilateral triangle.

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Isosceles triangle: If length of two sides of the triangle is equal and length of the 3rd side is different than that triangle is called isosceles triangle.

Right triangle: A right triangle is that in which one of the angles has measure equal to 90°.

Square: A square is a closed figure in the plane formed by four non collinear points such that length of all sides equal and measure of each angle is 90°.

Scalene triangle: Its measure of three sides of the triangle are different then the triangle is called scalene triangle.

Unit 10

Congruent Triangles

Congruent Triangles: Two triangles as said to the congruent if there exists a Correspondence between them such that all the corresponding sides and angles are congruent.

A.S.A postulate: In any correspondence of two triangles if one side and any two angles of the triangle are congruent to the corresponding side and angles of the other triangle then the triangles are congruent. This statement is called ASA postulate

S.S.S postulate: In any correspondence of two triangles if three sides of a triangle are congruent to the corresponding three sides of the other triangle then the triangles are congruent. This statement is called SSS postulates.

H.S postulate: In any correspondence of two right angles triangles if hypotenuse and one side of a triangle are congruent to the corresponding hypotenuse and side of the other then the triangle are congruent.

S.A.S Postulate: In any correspondence of two triangles if two sides and one angle of a triangle are congruent to the corresponding to two sides and one angle of other triangle then the triangles are congruent. This statement is called SSS postulates.

S.A.A Postulate: In any correspondence of two triangles if two angles and one side of a triangle are congruent to the corresponding to two angles and one side of other triangle then the triangles are congruent. This statement is called SSS postulates.

Unit 11

Parallelograms and Triangles

Trapezium: A trapezium is a quadrilateral with two parallel sides and two non-parallel sides.

Parallelogram: If two opposite sides of a quadrilateral and congruent and parallel. It is a parallelogram.

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Unit 12

Line Bisectors and Angle Bisectors

Right bisector of a line segment:	A line is called a right bisector of a line segment if it is perpendicular to the line segment and passes through its mid-point.
Angle bisector:	Angle bisector is the ray which divide and angle into two equal parts.

Unit 13

Sides & Angles of Triangles

Scalene triangle:	Its measure of three sides of the triangle are different then the triangle is called scalene triangle.

Unit 14

Ratio and Proportions

Ratio:	Ratio $a : b = \dfrac{a}{b}$ is the compassion of two alike quantities having same unit.
Proportion:	The equality of two ratios is called proportional four number a,b,c,d are in proportion if $a : b = c : d$
Similar triangles:	Triangles are called similar if they are equiangular and measure of their corresponding sides are proportional.
Practical application of similar triangle:	Photographer can develop prints of different sizes from the same negative. In spite of the difference in sizes. These pictures look like each other. One photograph is simply on enlargement of another.
Congruent Triangles:	Two triangles are said to be congruent if there exists a correspondence between them. Such that all the corresponding sides are angles are congruent.

Unit 15

Pythagoras' Theorem

Pythagoras' theorem:	In a right angle triangle the square of the length of hypotenuse is equal to the sum of the square of the length of the other two sides. $(hyp)^2 = (base)^2 + (perp)^2$
Converse to Pythagoras' theorem:	The square of one side of a triangle is equal to sum of the squares of the other two sides, then the triangle is a right angled triangle.
Right angle:	A right triangle is that in which one of the angles as measure equal to 90º.
Acute angle triangle:	If in a triangle sum of square of two sides is greater than the square of 3rd side then the triangle is called acute angle triangle
Obtuse angle triangle:	The triangle in which of sum of squares are two sides is less then the square of 3rd side is called obtuse angle triangle.

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Supplementary angles:	Two angles whose sum is equal to 180° are called Supplementary angles. e.g $120^o, 60^o$
Complementary angles:	Two angles whose sum is equal to 90° are called Complementary angles. e.g $30^0, 60^0$

Unit 16

Theorems Related with Area

Area of the figure:	The region enclosed by bounding lines of a closed figure is called the area of the figure.
Triangular region:	A triangular region is the union of a triangle and its interior i.e The three line segments forming the triangle and its interior.
Area of triangle:	A triangular region means the union of triangle and its interior and triangular region is also called area of triangle.
Rectangular region:	A Rectangular region is the union of a rectangle and its interior.
Parallel lines:	Two lines which on extending in both the directions infinitely never intersect at a point are called parallel Lines.
Parallelogram:	A parallelogram is a quadrilateral in which opposite sides are parallel opposite sides are of equal length and the measure of opposite angles are equal.
Altitude or height of a parallelogram:	If one side of parallelogram is taken as its base the perpendicular distance between that sides parallel to it is called altitude or height parallelogram.

Unit 17

Practical Geometry _ Triangles

Incentre of a triangle:	The point where the internal bisectors of the angles of a triangle meet is called incentre of triangle.
Circumscentre of a triangle:	Circumscentre of a triangle means the point of concurrency of the three perpendicular bisectors of the sides of a triangle.
Median of a triangle:	Median of a triangle means a line segment joining a vertex of a triangle to the midpoint of the opposite side.
Centroid of triangle:	The point where the three medians of a triangle meet is called the centroid of the triangle.
Orthocentre of triangle:	Orthocentre of a triangle means the point of concurrency of three altitudes of a triangle.
Concurrent lines:	The lines passing through the same point are called concurrent lines.
Altitude of a triangle:	If one side of a triangle is taken as base the perpendicular distance between the sides and the side parallel to it is called altitude or height of the triangle.

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