Multiple Choice Questions
For PPSC (Mathematics)

GROUP THEORY Compiled by: Akhtar Abbas

1. Which of the following are multiplicative tables for groups with four elements?

I.

	a	b	c	d
a	a	b	c	d
b	b	c	d	a
c	c	d	a	b
d	d	a	b	c

II.

	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	a	a
d	d	c	a	b

III.

	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	c	d
d	d	c	d	c

   A. I only

   B. I and II only

   C. II and III only

   D. None of these

2. If b and c are elements in a group G and if $b^5 = c^3 = e$, where e is the identity of G, then the inverse of $b^2cb^4c^2$ must be:

   A. $cb^2c^2b^4$

   B. $c^2b^4cb^2$

   C. cbc^2b^3

   D. $b^4c^2b^2c$

3. Let G_n be a cyclic group of order n. Which of the following direct product is not cyclic?

   A. $G_{22} \times G_{31}$

   B. $G_{222} \times G_{333}$

   C. $G_{17} \times G_{11}$

   D. $G_{17} \times G_{11} \times G_5$

4. Let p and q be distinct primes. There is a proper subgroup J of the additive group of integers which contain s exactly three elements of the set $\{p, p+q, pq, p^q, q^p\}$. Which three elements are in J?

   A. pq, p^q, q^p

   B. p, p^q, q^p

   C. p, pq, p^q

   D. $p, p+q, pq$

5. Two subgroups H and K of a groups have orders 12 and 30 respectively. Which of the following could not be the order of the subgroup G generated by H and K?

   A. 30

   B. 60

   C. 120

   D. Could not be determined

6. Let $\mathbb{Z}$ be the group of integers under the operation of addition. Which of the following subsets of $\mathbb{Z}$ is not a subgroup of $\mathbb{Z}$?

   A. $\mathbb{Z}$

   B. $\{n \in \mathbb{Z} : n \geq 0\}$

   C. $\{n \in \mathbb{Z} : n \textit{ is even}\}$

   D. $\{n \in \mathbb{Z} : 6|n \textit{ and } 9|n\}$

7. A cyclic group of order 15 has an element x such that the set $\{x^3, x^5, x^9\}$ has exactly two elements. The number of elements in the set $\{x^{13n} : n \textit{ is a positive integer}\}$ is

   A. 3

   B. 5

   C. 8

   D. 15

8. Let $\star$ be the binary operation on the rational numbers given by $a \star b = a + b + 2ab$. Which of the following are true?

   I. $\star$ is commutative

   II. There is a rational number that is a $\star$-identity

   III. Every rational number has a $\star$-inverse

   A. I only

   B. II only

   C. I and II only

   D. I and III only

9. For which integers n such that $3 \leq n \leq 11$ is there only one group of order n (upto isomorphism)?

   A. For no such integer n

   B. For 3, 5, 7 and 11 only

   C. For 4, 6, 8, 9 and 10 only

   D. For 3, 5, 7, 9 and 11 only

10. If a finite group G contains a subgroup of order seven but no element (other than identity) is its own inverse, then the order of G could be

    A. 27

    B. 28

    C. 35

    D. 37

11. A group G in which $(ab)^2 = a^2b^2$ for all $a,\ b$ in G, is necessarily

    A. Abelian

    B. Finite

    C. Cyclic

    D. Of order 2

12. The map $x \mapsto axa^2$ of a group G into itself is a homomorphism if and only if

    A. $a^3 = e$

    B. $a^2 = e$

    C. $a = e$

    D. G is abelian

13. Which of the following is not a group?

    A. The integers under addition

    B. The complex numbers under addition

    C. The nonzero integers under multiplication

    D. The nonzero real numbers under multiplication

14. What is the largest order of an element in the group of permutations of 5 objects?

    A. 5

    B. 6

    C. 12

    D. 60

15. Let $\mathbb{Z}_{17}^{\times}$ be the group of units of $\mathbb{Z}_{17}$ under multiplication. Which of the following are generators of $\mathbb{Z}_{17}^{\times}$?

    A. 5

    B. 8

    C. 5 and 8

    D. 5, 8 and 16

16. The subgroup H of a group G is called *characteristic* if for every automorphism $\phi : G \to G$, $\phi(H) \subseteq H$. Which of the following statements is true?
    A. Every characteristic subgroup is normal.
    B. Every normal subgroup is characteristic.
    C. If N is a normal subgroup of G and M a characteristic subgroup of N, then M is a normal subgroup of G
    D. Both A and C are true

17. Which of the following statements is true?
    A. If G is a non-abelian group with non-trivial center C, then the center of G/C is non-trivial.
    B. If G is a group of order 2, then the number of subgroups of $G \times G \times G$ is 6.
    C. A subgroup H of G is normal if and only if every cyclic subgroup of G is normal.
    D. $\mathbb{Z}_m/n\mathbb{Z}_m \cong \mathbb{Z}_n$

18. Let G be a group and H a subgroup of G such that $[G : H] = 2$. Which of the following statements is true?
    A. If $a \in H$ and $b \notin H$, then $ab \in H$
    B. If $a \notin H$ and $b \notin H$, then $ab^{-1} \in H$
    C. If $a \notin H$ and $b \notin H$, then $ab \in H$
    D. Both B and C are true

19. Which of the following statements is not true? (p is an odd prime number)
    A. If $|G| = p^n$, then G is cyclic
    B. If $|G| = p^n$, then $|Z(G)| > 1$
    C. If $|G| = p^n$ and G is non-abelian then G contains a subgroup which is not normal
    D. Both A and C are true

20. The order of the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 5 & 4 & 7 & 1 & 8 & 3 & 6 \end{pmatrix}$ is
    A. 4
    B. 5
    C. 6
    D. 8

21. The set of all generators of a cyclic group $G =< a >$ of order 8 is

    A. $\{a^2, a^4, a^6\}$

    B. $\{a^1, a^3, a^5, a^7\}$

    C. $\{a^4, a^8\}$

    D. $\{a^3, a^5, a^7\}$

22. The inverse of an element a in the group $G = \{a \in \mathbb{R} : a > 0, a \neq 0\}$ under the operation $\star$ defined by $a \star b = a^{\log b}$ is

    A. $e^{\frac{1}{\log a}}$

    B. $\frac{1}{\log a}$

    C. $\frac{1}{e^{\log a}}$

    D. 1

23. Which of the following statement is not correct?

    A. The Klein four group is abelian

    B. The Klein four group is not cyclic

    C. S_3 is abelian

    D. $\mathbb{Z}_4$ and $\mathbb{Z}_2 \times \mathbb{Z}_2$ are nonisomorphic groups

24. In the group $(\mathbb{Q} - \{-1\}, \star)$, where $\star$ is defined by $a \star b = a + b + ab$, for all $a, b \in \mathbb{Q} - \{-1\}$, the inverse of 15 is

    A. -15

    B. $\frac{15}{16}$

    C. $-\frac{15}{16}$

    D. $\frac{1}{15}$

25. Let H be a subgroup of G and

$$N_G(H) = \{g \in G : g^{-1}Hg = H\}.$$

    Then which of the following statements is not true?

    A. $N_G(H)$ is not subgroup of G

    B. $N_G(H)$ is a subgroup of G

    C. H is normal in $N_G(H)$

    D. H is normal in G if and only if $N_G(H) = G$

26. The kernel of the homomorphism $\phi : (\mathbb{Z}, +) \to (\mathbb{C}, \cdot)$ defined by $\phi(x) = e^{\pi i x}$ is
    A. $\{0\}$
    B. $4\mathbb{Z}$
    C. $2\mathbb{Z}$
    D. $\mathbb{Z}$

27. Consider the following statements
    I. Every cyclic group is abelian
    II. Every abelian group is cyclic
    III. Every group of order less than 4 is cyclic
    A. Only I is correct
    B. I and II are correct
    C. I and III are correct
    D. II and III are correct

28. Let G be a group of order np^k and $gcd(n, p) = 1$. Then G contains a subgroup H of order p^r only if
    A. G is abelian and $r = k$
    B. $r = k$
    C. r less than or equal to k
    D. G is abelian and r is less than or equal to k

29. G has an element of order 7 only if
    A. $|G| = 7^n$ for some positive integer n
    B. $gcd(7, |G|) = 1$
    C. $|G| = 7n$ for some positive integer n
    D. $|G| = 7$

30. How many generators does the group $(\mathbb{Z}_{24}, +)$ have?
    A. 2
    B. 10
    C. 12
    D. 24

31. How many subgroups does the group $\mathbb{Z}_3 \times \mathbb{Z}_{16}$ have?

    A. 6

    B. 10

    C. 12

    D. 20

32. Let p and q be distinct primes. How many (mutually nonisomorphic) groups are there of order p^2q^4?

    A. 6

    B. 8

    C. 10

    D. 12

33. Let G be the group generated by the elements x and y and subject to the following relations: $x^2 = y^3, y^6 = 1, x^{-1}yx = y^{-1}$. Express in simplest form the inverse of the element $z = x^{-2}yx^3y^3$?

    A. xy

    B. yx

    C. xy^2

    D. $y^{-2}x^{-1}$

34. Let H be the set of all group homomorphisms from $\mathbb{Z}_3$ to $\mathbb{Z}_6$. How many functions does H contain?

    A. 1

    B. 2

    C. 3

    D. 6

35. Let G be a group of order 9 and let e denote the identity of G. Which one of the following statements about G cannot be true?

    A. G is cyclic

    B. There exists an element $x \in G$ such that $x \neq e$ and $x^{-1} = x$

    C. There exists an element $x \in G$ such that $x \neq e$ and $x^2 = x^5$

    D. There exists an element $x \in G$ such that $< x >$ has order 3

36. Let p divides the order of a finite group G and let G have k distinct p-sylow subgroups of G. Which one is not a correct statement?

    A. k is a multiple of p

    B. k is not a power of p

    C. k is a divisor of $|G|$

    D. k is relatively prime to p

37. Let G be the symmetric groups on 5 objects. Then the number of distinct conjugacy classes in G is:-

    A. 5

    B. 7

    C. 25

    D. 120

38. Let$H$ be a finite subset of a group G and has 4 elements. Then H is not a subgroup of G if

    A. G is an infinite group

    B. $|G| = 26$

    C. $|G| = 4$

    D. G is isomorphic to a permutation group S_n, $n \geq 4$

39. Let G be a group of order np^r where p does not divide n. Then the number of subgroup of order p^r is of the form

    A. $1 + kp$ where p does not divide k

    B. $1 + kp$ where p divides k

    C. kp where p does not divide k

    D. kp where p divides k

40. The binary operation $\star$ is defined on a set of ordered pairs of real numbers as

$$(a, b) \star (c, d) = (ad + bc, bd)$$

and $\star$ is associative. Then $(1, 2) \star (3, 5) \star (3, 4)$ is

    A. $(32, 40)$

    B. $(23, 11)$

    C. $(74, 30)$

    D. $(7, 11)$

41. If a finite group G has two elements a, b having orders 6 and 15, then
    - A. 90 divides $|G|$
    - B. 30 divides $|G|$ but 90 need not divide $|G|$
    - C. 3 divides $|G|$ but 30 need not divide $|G|$
    - D. 3 does not divide $|G|$

42. Which of the following statements is true?
    - A. In an infinite group every element is of infinite order
    - B. If in a group every element is of finite order, then the group must be a finite group
    - C. In a finite group every element is of finite order
    - D. If every proper subgroup of a group is cyclic, then the group must be cyclic

43. If K is kernel of a group homomorphism $f : G \to H$, then which statement is not true?
    - A. K is an abelian subgroup of G
    - B. K is a normal subgroup of G
    - C. $K = \{e\}$ for some homomorphisms
    - D. $K = G$ for some homomorphisms

44. The number of group homomorphisms from S_3 to $\mathbb{Z}_6$ are
    - A. 1
    - B. 2
    - C. 3
    - D. 6

45. Let G be a group of order 77, then the center of G is iomorphic to
    - A. $\mathbb{Z}_1$
    - B. $\mathbb{Z}_7$
    - C. $\mathbb{Z}_{11}$
    - D. $\mathbb{Z}_{77}$

46. The total number of non-isomorphic groups of order 122 is
    - A. 1
    - B. 2
    - C. 4
    - D. 61

47. The number of cyclic subgroups of K_4 is

    A. 1

    B. 2

    C. 3

    D. 4

48. Let G be a non-abelian group of order 343. then $|Z(G)|=$

    A. 1

    B. 7

    C. 49

    D. 343

49. Suppose G is a finite group and H is the only subgroup of G of order $|H|$, then

    A. H is abelian

    B. H is cyclic

    C. H is normal

    D. H is of prime order

50. If for a prime p, p^n divides, but p^n+1 does not divide the order of a finite group G, then

    A. For every $d \leq p^n$, G has a subgroup of order d

    B. For every divisor d of $|G|$, G has a subgroup of order d

    C. For every positive integer $r \leq n$, G has a subgroup of order p^r

    D. G has a subgroup of order p^r if $r = n$, but G need not have subgroups of order p^r if $r < n$

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