Exercise # 9.1 CH-9

Q:1 Solve the following inequilities and graph the solution set in each case.

(i) $x + 3 < 7$

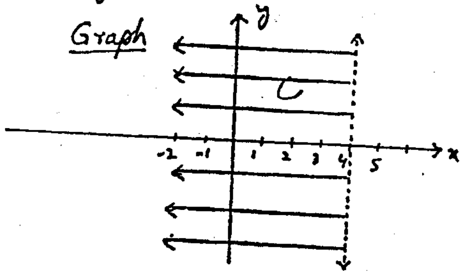
Sol

$x+3 < 7$ Subtract 3 from b.s

$x+3-3 < 7-3$

$\Rightarrow \boxed{x < 4}$ Ans

Graph



(ii) $-3x-2 \leq 4$

Sol

$-3x \leq 4+2$

$-3x \leq 6$

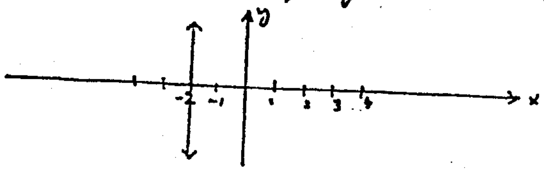
Multiply by -1, we get

$(-1)(-3x) \geq (-1)6$ (when we multiply an inequility by a -ve #, the inequility is reversed)

$\Rightarrow 3x \geq -6$

$\Rightarrow x \geq -\frac{6}{3}$

$\Rightarrow \boxed{x \geq -2}$

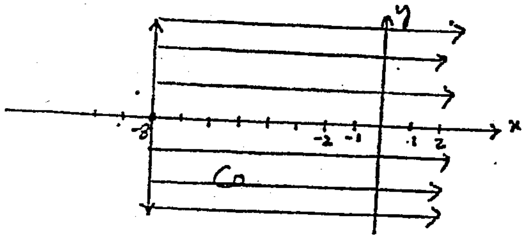


(iii) $2x+5 \geq x-3$

Sol

$2x - x \geq -3-5$

$\Rightarrow \boxed{x \geq -8}$



Q:2 Graph the following linear inequilities.

(i) $x-2y \geq 4$

Sol * The associated eqn is

$x-2y=4$

* For boundary points

Put $x=0$, we get & put $y=0$, we get

$0-2y=4$; $x-2(0)=4$

$y=-2$; $x=4$

$\Rightarrow A=(0,-2)$; $\Rightarrow B=(4,0)$

* Now draw the boundary line passing through points A and B. The boundary line will be continuous because equility is involved.

* Putting origin as a test point in $x-2y \geq 4$

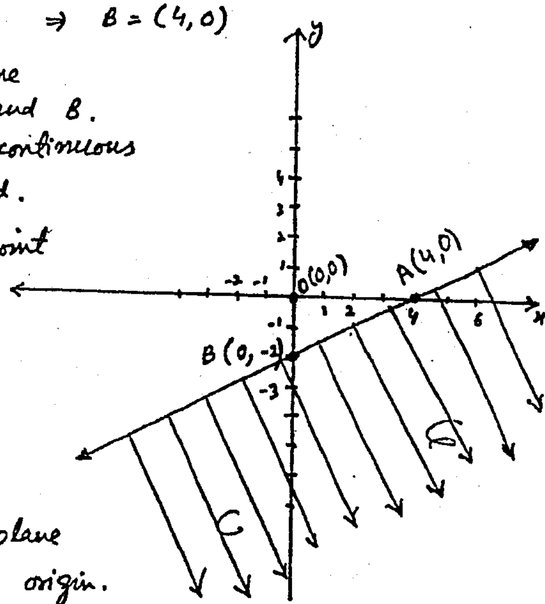
Put origin $(0,0)$

$\Rightarrow 0-2(0) \geq 4$

$\Rightarrow 0 \geq 4$

which is not true.

* Hence shade the half plane that does not contain origin. This shaded portion is the solution region.



Engr. Majid Amin

(ii) $x+y \leq 2$

Sol * The associated equation is

$$x+y=2$$

* For boundary points

(i) Put $x=0$, we get & (ii) Put $y=0$, we get

$0+y=2$ $\quad$ $x+0=2$

$\Rightarrow y=2$ $\quad$ $\Rightarrow x=2$

So $A=(0,2)$ & $B=(2,0)$ are boundary points.

* Draw the boundary line passing through A and B points. The boundary line will be continuous because equility is involved.

* Put origin (0,0) as a test point in the original inequility

$$x+y \leq 2$$

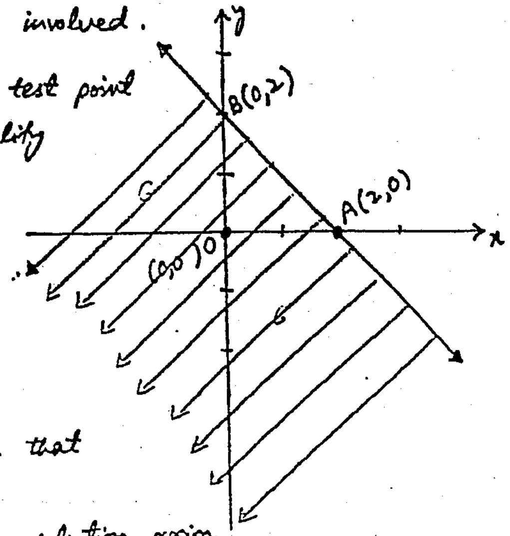
Put (0,0)

$\Rightarrow 0+0 \leq 2$

$\Rightarrow 0 \leq 2$

which is true.

Hence shade the half plane that contains the origin.
The shaded portion is the solution region.



(iii) $2x-3y>6$

Sol * The associated eqn will be

$$2x-3y=6$$

* For boundary points

(i) Put $x=0$, we get & (ii) put $y=0$, we get

$2(0)-3y=6$ $\quad$ $2x-3(0)=6$

$-3y=6$ $\quad$ $2x=6$

$\Rightarrow y=-2$ $\quad$ $\Rightarrow x=3$

Hence $A=(0,-2)$ $\quad$ Hence $B=(3,0)$

So $A(0,-2)$ and $B(3,0)$ are the boundary points

* Now draw a line passing through points A and B which is called boundary line. Here the boundary line will be dashed (-----) because equility is not present.

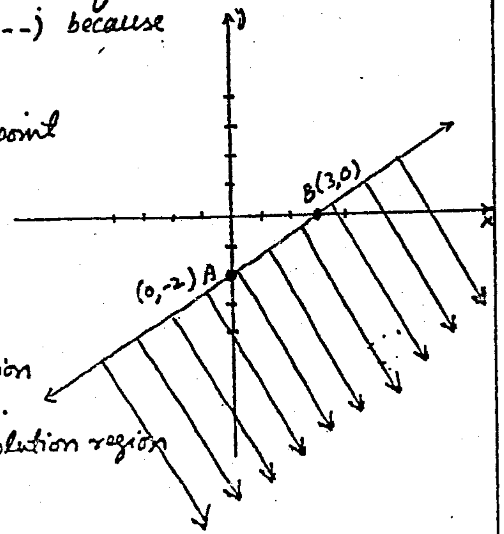
* Put origin (0,0) as a test point in $2x-3y>6$

$\Rightarrow 2(0)-3(0)>6$

$\Rightarrow 0>6$

which is not true.

Hence shade that half portion which does not contain origin.
That shaded portion is the solution region



Q:3 Graph the following systems of linear inequilities.

(i) $2x-3y \le 12$
$3x+2y \le 6$

Sol
$2x-3y \le 12$ & $3x+2y \le 6$

* Associated equs are
$2x-3y = 12$ & $3x+2y = 6$

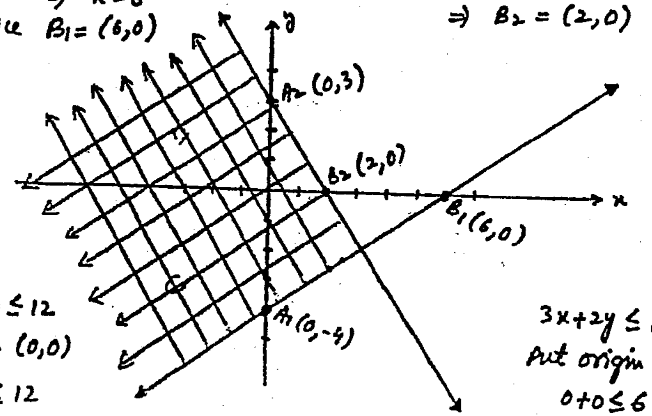
* Boundary points

Put $x=0 \Rightarrow 2(0)-3y=12$
$-3y=12$
$\Rightarrow y=-4$
Hence $A_1 = (0,-4)$

Put $y=0 \Rightarrow 2x-3(0)=12$
$2x=12$
$\Rightarrow x=6$
Hence $B_1 = (6,0)$

Put $x=0 \Rightarrow 3(0)+2y=6$
$\Rightarrow y=3$
$\Rightarrow A_2 = (0,3)$

Put $y=0 \Rightarrow 3x+2(0)=6$
$3x=6$
$\Rightarrow x=2$
$\Rightarrow B_2 = (2,0)$



* $2x-3y \le 12$
Put origin (0,0)
$\Rightarrow 0-0 \le 12$
$0 \le 12$
which is true
$\Rightarrow$ The half plane containing origin is solution for $2x-3y \le 12$

$3x+2y \le 6$
Put origin (0,0)
$0+0 \le 6$
$0 \le 6$
which is true.
$\Rightarrow$ The half plane containing origin is solution for $2x-3y \le 12$

Finally leave that portion which is common in the two regions.

(ii) $x+2y \ge 2$
$4x-y \ge 4$

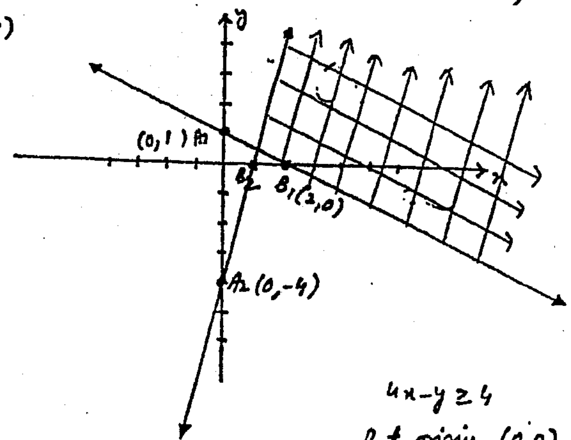
Sol
$x+2y \ge 2$ & $4x-y \ge 4$.

* Associated equs will be
$x+2y = 2$ & $4x-y = 4$

* Boundary points

Put $x=0 \Rightarrow 0+2y=2$
$\Rightarrow y=1$
$\Rightarrow A_1 = (0,1)$
& Put $y=0 \Rightarrow x+0=2$
$\Rightarrow x=2$
So $B_1 = (2,0)$

$x=0 \Rightarrow 0-y=4 \Rightarrow y=-4$
Hence $A_2 = (0,-4)$
$y=0 \Rightarrow 4x-0=4 \Rightarrow x=1$
Hence $B_2 = (1,0)$



$x+2y \ge 2$
Put origin (0,0)
$\Rightarrow 0+0 \ge 2$
$\Rightarrow 0 \ge 2$
False
shade the half plane that does not contain origin

$4x-y \ge 4$
Put origin (0,0)
$\Rightarrow 0-0 \ge 4$
$\Rightarrow 0 \ge 4$
$\Rightarrow$ False
shade the half plane that does not contain origin

$\Rightarrow$ Finally take the common region.

(iii) $x - y \le 1$
$x + y \ge 4$

Sol Associated equs are

$x - y = 1$ & $x + y = 4$

$x = 0 \Rightarrow 0 - y = 1$ & $x = 0 \Rightarrow 0 + y = 4$

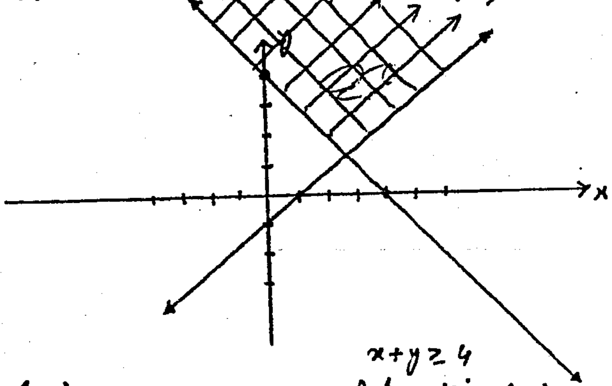
$\Rightarrow y = -1$ $\qquad$ $\Rightarrow y = 4$

$\Rightarrow A_1 = (0,-1)$ $\qquad$ $\Rightarrow A_2 = (0,4)$

$y = 0 \Rightarrow x - 0 = 1$ $\qquad$ $y = 0 \Rightarrow x + 0 = 4$

$\Rightarrow x = 1$ $\qquad$ $\Rightarrow x = 4$

$\Rightarrow B_1 = (1,0)$ $\qquad$ $\Rightarrow B_2 = (4,0)$



$x - y \le 1$
Put origin $(0,0)$
$\Rightarrow 0 - 0 \le 1$
$\Rightarrow 0 \le 1$
$\Rightarrow$ True

$x + y \ge 4$
Put origin $(0,0)$
$\Rightarrow 0 + 0 \ge 4$
$\Rightarrow 0 \ge 4$
$\Rightarrow$ False

First draw the solution region separately and then take the common region.

Q:4 Graph the following systems of linear inequalities.

(i) $2x + y \ge 4$
$x + y \ge 3$
$x \ge 0$

Sol $2x + y \ge 4$, $x + y \ge 3$

* Associated Equs are

$2x + y = 4$, $x + y = 3$, $x \ge 0$

Boundary points

$x = 0 \Rightarrow 0 + y = 4$ $\qquad$ $x = 0 \Rightarrow 0 + y = 3$

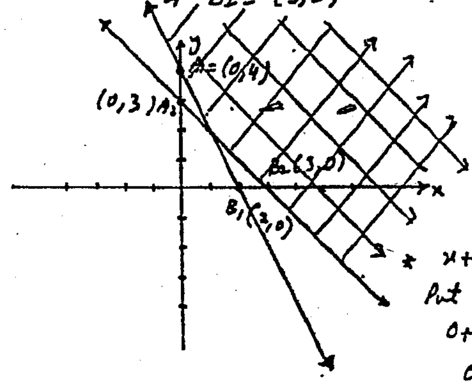
$\Rightarrow y = 4$ $\qquad$ $\Rightarrow y = 3$

$A_1 = (0,4)$ $\qquad$ $\Rightarrow A_2 = (0,3)$

$y = 0 \Rightarrow 2x + 0 = 4$ $\qquad$ $y = 0 \Rightarrow x + 0 = 3$

$x = 2$ $\qquad$ $\Rightarrow x = 3$

$\Rightarrow B_1 = (2,0)$ $\qquad$ $\Rightarrow B_2 = (3,0)$



* $2x + y \ge 4$
Put origin $(0,0)$
$\Rightarrow 0 + 0 \ge 4$
$0 \ge 4$
False

* $x + y \ge 3$
Put origin $(0,0)$
$0 + 0 \ge 3$
$0 \ge 3$
False.

Also $x \ge 0$. Shade the region which is solution for the given system.

(ii) $2x+y \le 8$
$x+y \le 6$
$y \ge 0$

Sol $2x+y \le 8$, $x+y \le 6$

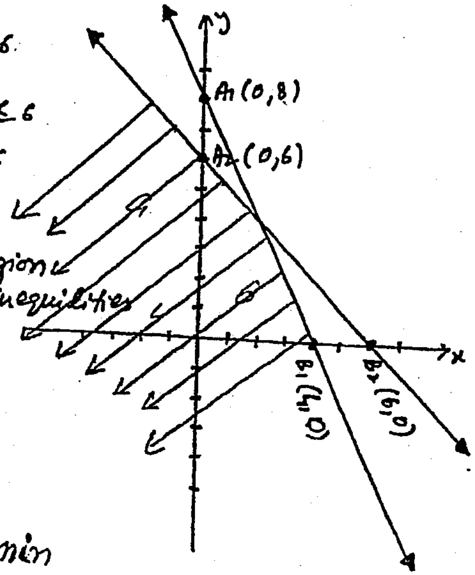
* Associated equs are

$2x+y=8$
$x=0 \Rightarrow 0+y=8$
$\Rightarrow y=8$
$\Rightarrow A_1=(0,8)$
$y=0 \Rightarrow 2x+0=8$
$\Rightarrow x=4$
$\Rightarrow B_1=(4,0)$

$x+y=6$
$x=0 \Rightarrow 0+y=6$
$\Rightarrow y=6$
$\Rightarrow A_2=(0,6)$
$y=0 \Rightarrow x+0=6$
$\Rightarrow x=6$
$\Rightarrow B_2=(6,0)$

Now $2x+y \le 8$ & $x+y \le 6$
Put origin $(0,0)$
$\Rightarrow 0+0 \le 8$ $\Rightarrow 0+0 \le 6$
$\Rightarrow 0 \le 8$ $\Rightarrow 0 \le 6$
$\Rightarrow$ True $\Rightarrow$ True

Now take that common region which satisfies the two inequilities and also make sure $y \ge 0$



Engr. Majid Amin
0333 9470690

(iii) $2x+y \ge 2$
$x+2y \le 10$
$x \ge 0$

Sol $2x+y \ge 2$, $x+2y \le 10$

* Associated equs are

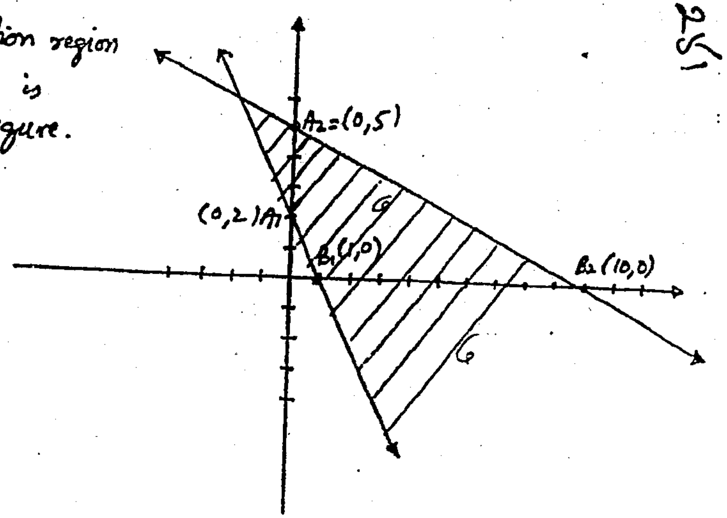
$2x+y=2$
$x=0 \Rightarrow 0+y=2$
$\Rightarrow y=2$
So $A_1=(0,2)$
$y=0 \Rightarrow 2x+0=2$
$\Rightarrow x=1$
So $A_1=(1,0)$

$x+2y=10$
$x=0 \Rightarrow 0+2y=10$
$\Rightarrow y=5$
So $A_2=(0,5)$
$y=0 \Rightarrow x+0=10$
$\Rightarrow x=10$
So $B_2=(10,0)$

Now $2x+y \ge 2$
Put origin $(0,0)$
$\Rightarrow 2(0)+0 \ge 2$
$0 \ge 2$
$\Rightarrow$ False

Now $x+2y \le 10$
Put origin $(0,0)$
$\Rightarrow 0+2(0) \le 10$
$0 \le 10 \Rightarrow$ True

Also $x \ge 0$
Then the solution region for the system is shown in the figure.



Q:5 Graph the solution region of the following system of linear inequilities and find the corner points in each case. Also tell whether the graph is bounded or unbounded?

(i) $2x + y \le 6$
$x + 2y \le 6$
$x \ge 0$

Sol $2x + y \le 6$, $x + 2y \le 6$, $x \ge 0$

Associated equs are

$2x + y = 6$
$x = 0 \Rightarrow 2(0) + y = 6$
$\Rightarrow y = 6$
$\Rightarrow A_1 = (0,6)$
$y = 0 \Rightarrow 2x + 0 = 6$
$\Rightarrow x = 3$
$\Rightarrow B_1 = (3,0)$

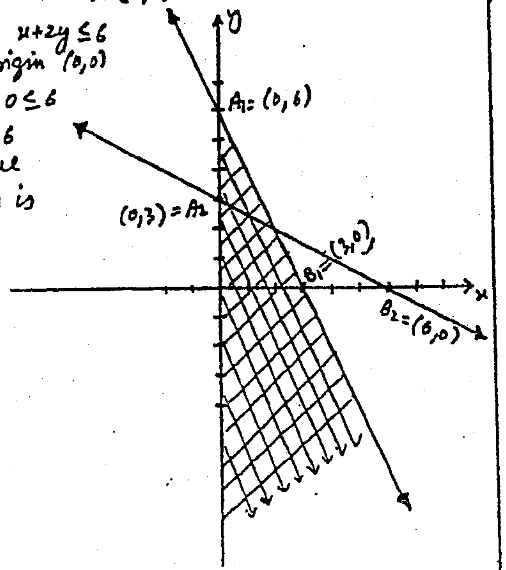
$x + 2y = 6$
$x = 0 \Rightarrow 0 + 2y = 6$
$\Rightarrow y = 3$
$\Rightarrow A_2 = (0,3)$
$y = 0 \Rightarrow x + 2(0) = 6$
$\Rightarrow x = 6$
$\Rightarrow B_2 = (6,0)$

$x = 0$

Now $2x + y \le 6$
put origin $(0,0)$
$\Rightarrow 0 \le 6$
True

Now $x + 2y \le 6$
put origin $(0,0)$
$0 + 0 \le 6$
$0 \le 6$
True

Also $x \ge 0$
The required solution region is shown.



To find corner points

solve $2x + y = 6 \longrightarrow (1)$
$x + 2y = 6 \longrightarrow (2)$

Equ (1) multiply by 2
$4x + 2y = 12 \longrightarrow (3)$

Equ (3) − Equ (2)

$$\begin{array}{r} 4x + 2y = 12 \\ -\ x \pm 2y = 6 \\ \hline 3x \quad = 6 \end{array}$$

$\Rightarrow \boxed{x = 2}$

Now $x + 2y = 6$
$\Rightarrow 2 + 2y = 6$
$\Rightarrow 2y = 4$
$\Rightarrow y = 2$ Hence one corner point is $(2,2)$.

Now solve
$2x + y = 6$
& $x = 0$
$\Rightarrow 2(0) + y = 6$
$\Rightarrow y = 6$ $\Rightarrow (0,6)$ is another corner point

Now solve

$$\begin{array}{r} x + 2y = 6 \\ \&\ \ x \quad = 0 \\ \hline 2y = 6 \end{array}$$

$\Rightarrow y = 3$
$\Rightarrow (0,3)$ is another corner point.

Hence the corner points are $(2,2), (0,6), (0,3)$.

Also the solution region is unbounded because it can't be enclosed in a circle of known radius.

Available at
www.mathcity.org

(ii) $2x + 3y \geq 6$

$x + y \geq 4$

$y \geq 0$

Sol The associated eqn are

$2x + 3y = 6$ ⓘ , $x + y = 4$ (ii) , $y = 0$ (iii)

$x=0 \Rightarrow 0+3y=6$

$\Rightarrow y=2$

$\Rightarrow A_1=(0,2)$

$y=0 \Rightarrow 2x+0=6$

$\Rightarrow x=3$

$\Rightarrow B_1=(3,0)$

$x=0 \Rightarrow 0+y=4$

$\Rightarrow y=4$

$\Rightarrow A_2=(0,4)$

$y=0 \Rightarrow x+0=4$

$\Rightarrow x=4$

$\Rightarrow B_2=(4,0)$

Now $2x+3y \geq 6$

put origin $(0,0)$

$\Rightarrow 0+0 \geq 6$

$\Rightarrow 0 \geq 6$

False

$x+y \geq 4$

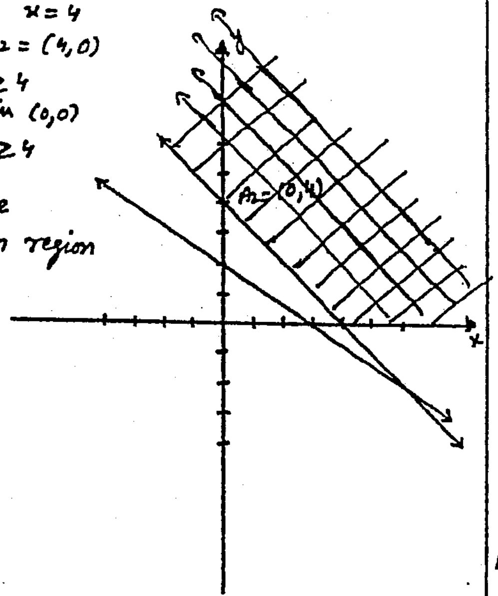
Put origin $(0,0)$

$\Rightarrow 0+0 \geq 4$

$\Rightarrow 0 \geq 4$

$\Rightarrow$ False

Also $y \geq 0$. So the solution region is shown in the figure



CORNER POINTS

Solve $2x+3y=6 \longrightarrow$ ⓘ

$x+y=4 \longrightarrow$ (ii)

Eqn (ii) multiply by 2

$\Rightarrow 2x+2y=8 \longrightarrow$ ⓐ

Eqn ⓐ − Eqn ⓘ

$2x+2y=8$

$\underline{-2x \underset{-}{+}3y = \underset{-}{6}}$

$-y=2$

$\Rightarrow y=-2$

Eqn (ii) $x+y=4$

$\Rightarrow x-2=4 \Rightarrow x=6$ Hence $(6,-2)$

$\Rightarrow (6,-2)$ is one boundary point

Now solve eqn ⓘ & (iii)

$2x+3y=6$

$y=0$

$\Rightarrow 2x \quad =6$

$\Rightarrow x=3 \Rightarrow (3,0)$ is a corner point

Now solve eqn (ii) & (iii)

$x+y=4$

$y=0$

$x=4 \Rightarrow (4,0)$ is another corner point.

Hence the corner points are $(6,-2)$, $(3,0)$, $(4,0)$.

The solution region is unbounded because it can't be enclosed in a circle of known radius.

Q:6 Graph the solution region of the following system of linear inequalities and find the corner points in each case. Also tell whether the graph is bounded or unbounded.

ⓘ $2x+3y \leq 12$

$3x+y \leq 12$

$x+y \geq 2$

Sol * The associated eqns are

$2x+3y=12 \to$ ⓘ, $3x+y=12 \to$ (ii), $x+y=2 \to$ (iii)

Put $x=0 \Rightarrow 0+3y=12$, $0+y=12$, $0+y=2$

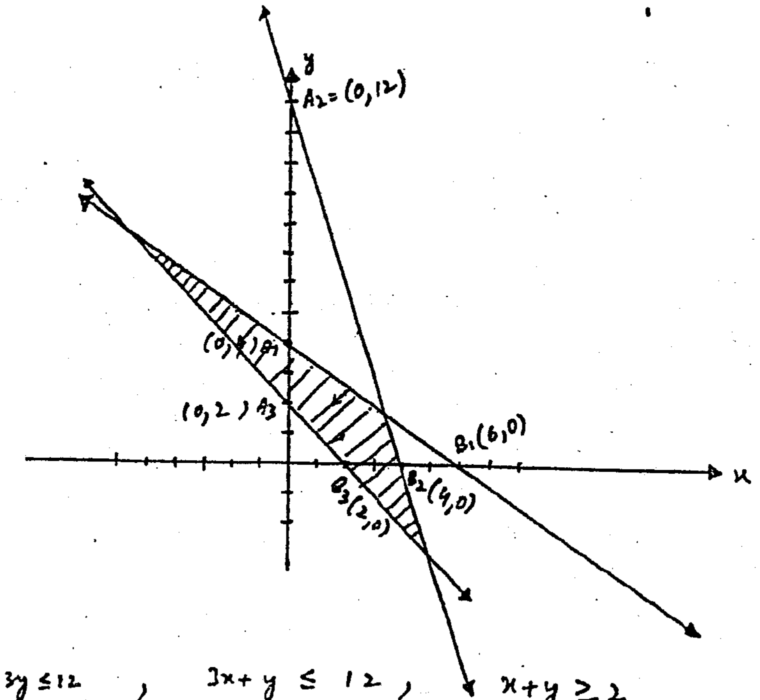
$\Rightarrow y=4$, $\Rightarrow y=12$, $\Rightarrow y=2$

Hence $A_1=(0,4)$, $A_2=(0,12)$, $A_3=(0,2)$

Put $y=0 \Rightarrow 2x+0=12$, $3x+0=12$, $x+0=2$

$x=6$, $x=4$, $\Rightarrow x=2$

Hence $B_1=(6,0)$, $B_2=(4,0)$, $B_3=(2,0)$



Now $2x+3y \le 12$, $3x+y \le 12$, $x+y \ge 2$

At origin $(0,0)$

$\Rightarrow 0+0 \le 12$	$0+0 \le 12$	$0+0 \ge 2$
$\Rightarrow 0 \le 12$	$\Rightarrow 0 \le 12$	$0 \ge 2$
True	$\Rightarrow$ True	False

<u>To find CORNER POINTS</u>

Solve eqn (i) & (ii)

$2x+3y=12 \longrightarrow$ (i)

$3x+y=12 \longrightarrow$ (ii)

Eqn (ii) multiply by 3

$\Rightarrow 9x+3y=36 \longrightarrow$ (a)

Eqn (a) $-$ Eqn (i)

$$\begin{array}{r} 9x+3y=36 \\ -2x \mp 3y = \mp 12 \\ \hline 7x \quad = 24 \end{array} \Rightarrow x = 24/7$$

Eqn (ii) $\Rightarrow 3x+y=12$

$\Rightarrow 3\left(\frac{24}{7}\right)+y=12$

$\Rightarrow \frac{72}{7}+y=12 \Rightarrow y = 12-\frac{72}{7}$

$y = \frac{84-72}{7} = \frac{12}{7}$

Hence one corner point is $\left(\frac{24}{7}, \frac{12}{7}\right)$

Now solve eqn (i) & (iii)

$2x+3y=12 \longrightarrow$ (i)

$x+y=2 \longrightarrow$ (iii)

Eqn (iii) multiplied by 2

$\Rightarrow 2x+2y=4 \longrightarrow$ (b)

Eqn (i) $-$ Eqn (b)

$$\begin{array}{r} 2x+3y=12 \\ -2x \mp 2y = \mp 4 \\ \hline y = 8 \end{array}$$

Eqn (iii) $x+y=2 \Rightarrow x+8=2 \Rightarrow x=-6$

Hence $(-6,8)$ is another corner point

Now solve eqn (ii) & (iii)

$$\begin{array}{r} 3x+y=12 \longrightarrow \text{(ii)} \\ -x \mp y = -2 \longrightarrow \text{(iii)} \\ \hline \end{array}$$

$\Rightarrow 2x = 10 \Rightarrow x=5$

Eqn (iii) $x+y=2$

$\Rightarrow 5+y=2 \Rightarrow y=-3$

Hence $(5,-3)$ is another corner point

Hence the three corner points are $\left(\frac{24}{7}, \frac{12}{7}\right)$, $(-6,8)$, $(5,-3)$ Ans

The solution region is bounded because it can be enclosed in a circle of known radius.

Engr. Majid Amin

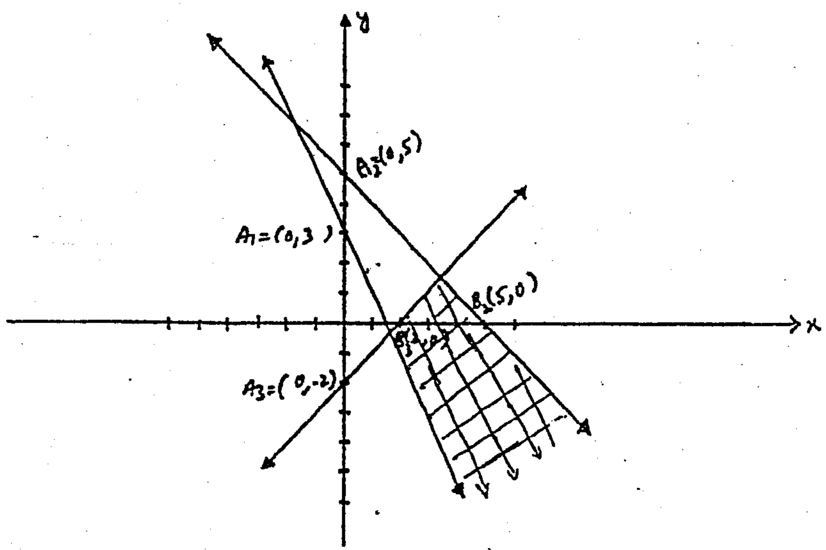
(ii) $2x+y \ge 3$
$x+y \le 5$
$x-y \ge 2$

Sol The associated eqns are

$2x+y=3 \to$ (i) , $x+y=5 \to$ (ii) , $x-y=2 \to$ (iii)

$x=0 \Rightarrow 0+y=3 \Rightarrow y=3$ So $A_1=(0,3)$ | $\Rightarrow 0+y=5 \Rightarrow y=5$ So $A_2=(0,5)$ | $\Rightarrow 0-y=2 \Rightarrow y=-2$ So $A_3=(0,-2)$

& $y=0 \Rightarrow 2x+0=3 \Rightarrow x=3/2$ So $B_1=(3/2,0)$ | $x+0=5 \Rightarrow x=5$ $B_2=(5,0)$ | $x-0=2 \Rightarrow x=2$ $B_3=(2,0)$



Now $2x+y \ge 3$, $x+y \le 5$, $x-y \ge 2$

Put origin (0,0)

$\Rightarrow 0+0 \ge 3 \Rightarrow 0 \ge 3 \Rightarrow$ False

$0+0 \le 5 \Rightarrow 0 \le 5 \Rightarrow$ True

$0-0 \ge 2 \Rightarrow 0 \ge 2 \Rightarrow$ False

To find CORNER POINTS

Solve Eqns (i) & (ii)

$$\begin{array}{r} 2x+y=3 \\ -x+y=5 \\ \hline x \quad =-2 \end{array}$$

Put in $x+y=5 \Rightarrow -2+y=5 \Rightarrow y=7$

one corner point is $(-2,7)$

Solve eqn (i) & (iii)

$$\begin{array}{r} 2x+y=3 \\ x-y=2 \\ \hline 3x \quad =5 \end{array} \Rightarrow x=5/3$$

Put in $x-y=2$

$\Rightarrow \frac{5}{3}-y=2$

$\Rightarrow \frac{5}{3}-2=y$

$\Rightarrow \frac{5-6}{3}=y \Rightarrow -\frac{1}{3}=y$

So $(5/3, -1/3)$ is another corner point.

Solve Eqn (ii) & (iii)

$$\begin{array}{r} x+y=5 \\ x-y=2 \\ \hline 2x \quad =7 \end{array} \Rightarrow x=7/2$$

& $x-y=2 \Rightarrow \frac{7}{2}-y=2 \Rightarrow \frac{7}{2}-2=y \Rightarrow \frac{7-4}{2}=y \Rightarrow 3/2=y$

So $(7/2, 3/2)$ is another corner point.

Hence the corner points are $(-2,7)$, $(5/3,-1/3)$ & $(7/2, 3/2)$ Ans

The solution region is unbounded because it can't be enclosed in a circle of sufficient radius.

Exercise # 9.2

Q.1 Graph the feasible region of the following system of linear inequilities and also find the corner points.

(i)
$2x + y \le 6$
$4x + y \le 8$
$x \ge 0$
$y \ge 0$

Sol First find the solution region for

$2x + y \le 6$ & $4x + y \le 8$

* Associated equs are

$2x + y = 6$ & $4x + y = 8$

$x = 0 \Rightarrow 0 + y = 6$; $0 + y = 8$

$\Rightarrow y = 6$; $\Rightarrow y = 8$

So $A_1 = (0,6)$; So $A_2 = (0,8)$

$y = 0 \Rightarrow 2x + 0 = 6$; $4x + 0 = 8$

$\Rightarrow x = 3$; $\Rightarrow x = 2$

So $B_1 = (3,0)$; So $B_2 = (2,0)$

* Draw the continuous boundary lines.

Now $2x + y \le 6$ & $4x + y \le 8$

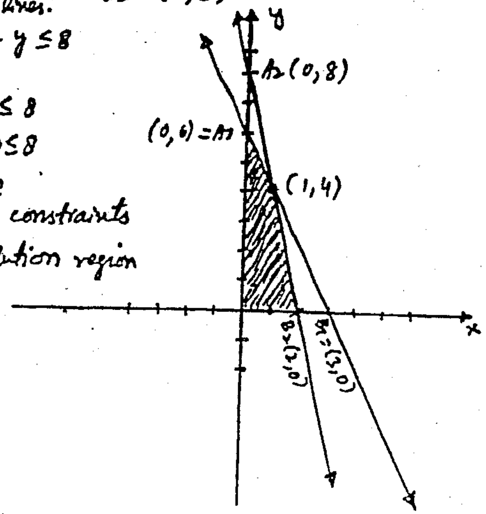
Put origin $(0,0)$

$\Rightarrow 0 + 0 \le 6$; $0 + 0 \le 8$

$\Rightarrow 0 \le 6$; $\Rightarrow 0 \le 8$

True ; True

Finally applying the non negative constraints $x \ge 0$ & $y \ge 0$ & the solution region is drawn which is shown



The corner points of the feasible region are

$(0,0)$, $(2,0)$, $(0,6)$, $(1,4)$

(ii)
$3x - y \ge -4$
$x + y \le 5$
$x \ge 0$
$y \ge 0$

Sol First find the solution region for

$3x - y \ge -4$ & $x + y \le 5$

* Associated equs are

$3x - y = -4$ & $x + y = 5$

$x = 0 \Rightarrow 0 - y = -4$; $\Rightarrow 0 + y = 5$

$\Rightarrow y = 4$; $\Rightarrow y = 5$

So $A_1 = (0, 4)$; So $A_2 = (0,5)$

$y = 0 \Rightarrow 3x - 0 = -4$ & $x + 0 = 5$

$x = -4/3$; $x = 5$

So $B_1 = (-4/3, 0)$; So $B_2 = (5,0)$

* Draw the continuous boundary lines.

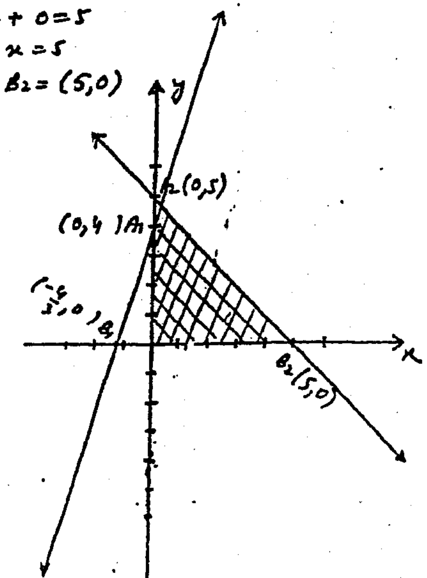
Now $3x - y \ge -4$ & $x + y \le 5$

Put origin $(0,0)$

$\Rightarrow 3(0) - 0 \ge -4$; $0 + 0 \le 5$

$\Rightarrow 0 \ge -4 \Rightarrow$ True ; $\Rightarrow 0 \le 5 \Rightarrow$ True

Finally apply the non-negative constraints $x \ge 0$ & $y \ge 0$, draw the feasible solution region which is shown.



From the figure three corner points are $(0,0)$, $(0,4)$, $(5,0)$.

To find the 4th corner points solve

$3x - y = -4 \longrightarrow$ (i)
$x + y = 5 \longrightarrow$ (ii)
$4x = 1 \Rightarrow x = 1/4$

equ (ii) $\Rightarrow \frac{1}{4} + y = 5 \Rightarrow y = 5 - \frac{1}{4} = \frac{20-1}{4} = 19/4$

So 4th corner point is $(\frac{1}{4}, \frac{19}{4})$

(iii)

$$x + 2y \le 6$$
$$2x + y \le 6$$
$$x \ge 0$$
$$y \ge 0$$

Sol: First solve for

$x + 2y \le 6$ & $2x + y \le 6$

* Associated equs are

$x + 2y = 6 \longrightarrow$ (i) $2x + y = 6 \longrightarrow$ (ii)

$x = 0 \Rightarrow 0 + 2y = 6$ $0 + y = 6$

$\Rightarrow y = 3$ $y = 6$

So $A_1 = (0,3)$ So $A_2 = (0,6)$

$y = 0 \Rightarrow x + 2(0) = 6$ $\Rightarrow 2x + 0 = 6$

$\Rightarrow x = 6$ $\Rightarrow x = 3$

So $B_1 = (6,0)$ So $B_2 = (3,0)$

* Draw the continuous boundary lines.

Now $x + 2y \le 6$ & $2x + y \le 6$

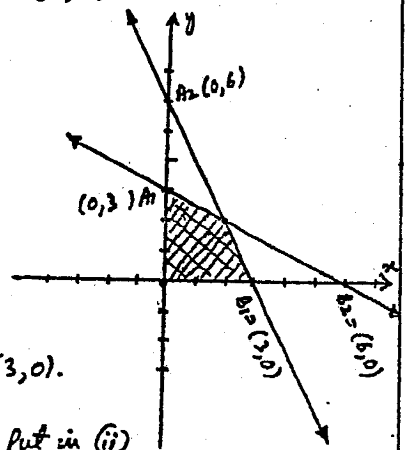
Put origin (0,0)

$\Rightarrow 0 + 2(0) \le 6$ $2(0) + 0 \le 6$

$\Rightarrow 0 \le 6$ $\Rightarrow 0 \le 6$

True True

* Also apply the non negative constraints $x \ge 0$ & $y \ge 0$, the final solution region is shown.



Three corner points are (0,0), (0,3), (3,0).

To find the 4th corner point solve

$x + 2y = 6 \longrightarrow$ (i) $\Rightarrow x = 6 - 2y$ Put in (ii)

$2x + y = 6 \longrightarrow$ (ii) $\Rightarrow 2(6 - 2y) + y = 6$

$\Rightarrow 12 - 4y + y = 6$

$\Rightarrow -3y = -6 \Rightarrow y = 2$

Now $x = 6 - 2y$

$= 6 - 2(2) = 2$

Hence 4th corner point is (2,2) Ans

Q:2 Graph the feasible region subject to the constraints and also find the corner points.

(i)

$$x + 2y \le 8$$
$$x + y \le 5$$
$$2x + y \le 8$$
$$x \ge 0$$
$$y \ge 0$$

Sol: First find the solution region of

$x + 2y \le 8$, $x + y \le 5$, $2x + y \le 8$

* Associated equs are

$x + 2y = 8 \rightarrow$ (i) , $x + y = 5 \rightarrow$ (ii) , $2x + y = 8 \longrightarrow$ (iii)

$x = 0 \Rightarrow 0 + 2y = 8$ $0 + y = 5$ $2(0) + y = 8$

$\Rightarrow y = 4$ $\Rightarrow y = 5$ $\Rightarrow y = 8$

So $A_1 = (0,4)$ So $A_2 = (0,5)$ So $A_3 = (0,8)$

$y = 0 \Rightarrow x + 2(0) = 8$ $\Rightarrow x + 0 = 5$ $\Rightarrow 2x + 0 = 8$

$\Rightarrow x = 8$ $\Rightarrow x = 5$ $\Rightarrow x = 4$

So $B_1 = (8,0)$ So $B_2 = (5,0)$ So $B_3 = (4,0)$

Now considre

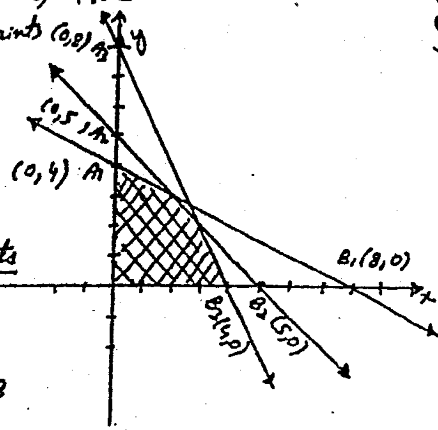
$x + 2y \le 8$, $x + y \le 5$, $2x + y \le 8$

Put origin (0,0)

$\Rightarrow 0 \le 8$ $\Rightarrow 0 \le 5$ $0 \le 8$

True $\Rightarrow$ True $\Rightarrow$ True

* Finally apply the non-negative constraints $x \ge 0$ & $y \ge 0$, the final feasible solution region is shown



* Three corner points are

(0,0) , (0,4) , (4,0)

To find the other two corner points

Sol: Equ (i) & (iii)

$x + 2y = 8 \Rightarrow x = 8 - 2y$

$2x + y = 8 \Rightarrow 2(8 - 2y) + y = 8$

$\Rightarrow 16 - 4y + y = 8$

$\Rightarrow -3y = -8 \Rightarrow y = 8/3$

Now $x = 8 - 2y$

$\Rightarrow x = 8 - 2\left(\frac{8}{3}\right)$

$\Rightarrow x = \frac{24-16}{3} = 8/3$

So the 4th corner point is $(8/3, 8/3)$

Now solve (ii) & (iii)

$$\begin{array}{r} x+y=5 \\ -2x \mp y = -8 \\ \hline -x \quad = -3 \end{array} \Rightarrow x = 3$$

Now $x+y=5$

$\Rightarrow 3+y=5$

$\Rightarrow y=2$

So the 5th corner point is $(3,2)$.

Hence the five corner points are

$(0,0)$, $(0,4)$, $(4,0)$, $(3,2)$, $(8/3, 8/3)$ Ans

(ii)
$$\begin{aligned} 2x+y &\geq 6 \\ 2x+3y &\leq 12 \\ -x+y &\leq 2 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$$

Sol 1st solve

$2x+y \geq 6$, $2x+3y \leq 12$, $-x+y \leq 2$

* Associated equs are

$2x+y=6 \longrightarrow$ (i) $\qquad 2x+3y=12 \longrightarrow$ (ii) $\qquad -x+y=2 \longrightarrow$ (iii)

* $x=0$

$\Rightarrow 2(0)+y=6$ $\qquad \Rightarrow 2(0)+3y=12$ $\qquad \Rightarrow -0+y=2$

$\Rightarrow y=6$ $\qquad \Rightarrow y=4$ $\qquad \Rightarrow y=2$

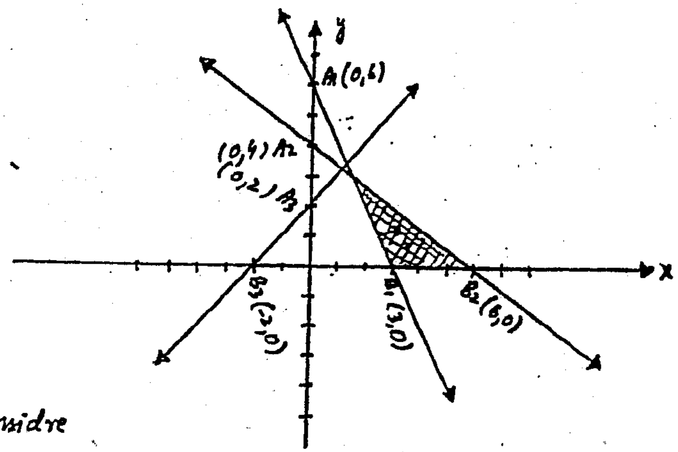
So $A_1=(0,6)$ $\qquad$ So $A_2=(0,4)$ $\qquad$ So $A_3=(0,2)$

* $y=0$

$\Rightarrow 2x+0=6$ $\qquad \Rightarrow 2x+3(0)=12$ $\qquad \Rightarrow -x+0=2$

$x=3$ $\qquad \Rightarrow x=6$ $\qquad \Rightarrow x=-2$

So $B_1=(3,0)$ $\qquad$ So $B_2=(6,0)$ $\qquad$ So $B_3=(-2,0)$



Now considre

$2x+y \geq 6$, $2x+3y \leq 12$, $-x+y \leq 2$

Put origin $(0,0)$

$\Rightarrow 2(0)+0 \geq 6$ $\qquad \Rightarrow 2(0)+3(0) \leq 12$ $\qquad \Rightarrow -0+0 \leq 2$

$\Rightarrow 0 \geq 6$ $\qquad \Rightarrow 0 \leq 12$ $\qquad \Rightarrow 0 \leq 2$

False $\qquad$ True $\qquad$ True

* Finally apply the non-negative constants $x \geq 0$ & $y \geq 0$ draw the feasible region which is shown.

Two corner points are $(3,0)$, $(6,0)$.

To find the third corner point solve

$$\begin{array}{r} 2x+y=6 \\ 2x+3y=12 \\ \hline -2y=-6 \end{array}$$

$\boxed{y=3}$

Now $2x+y=6$

$2x+3=6$

$2x=3 \Rightarrow x=3/2$

Hence the third corner point is $(3/2, 3)$

So the three corner points are

$(3,0)$, $(6,0)$, $(3/2, 3)$

(iii) $x+y\ge 3$
$2x+3y\le 12$
$x-y\le 12$
$x\ge 0$
$y\ge 0$

Sol* First solve the 1st three inequilities

$x+y\ge 3$, $2x+3y\le 12$, $x-y\le 12$

* The associated equs are

$x+y=3$ ... (i) , $2x+3y=12$ ... (ii) , $x-y=12$ ... (iii)

* Put $x=0$

$0+y=3$, $2(0)+3y=12$, $0-y=12$

$\Rightarrow y=3$, $\therefore y=4$, $y=-12$

So $A_1=(0,3)$, $A_2=(0,4)$, $A_3=(0,-12)$

* Put $y=0$

$x+0=3$, $2x+3(0)=12$, $x-0=12$

$\Rightarrow x=3$, $x=6$, $x=12$

So $B_1=(3,0)$, $B_2=(6,0)$, $B_3=(12,0)$.

* Now draw the boundary lines. All the boundary lines will be continuous because they have equility.

Now considre

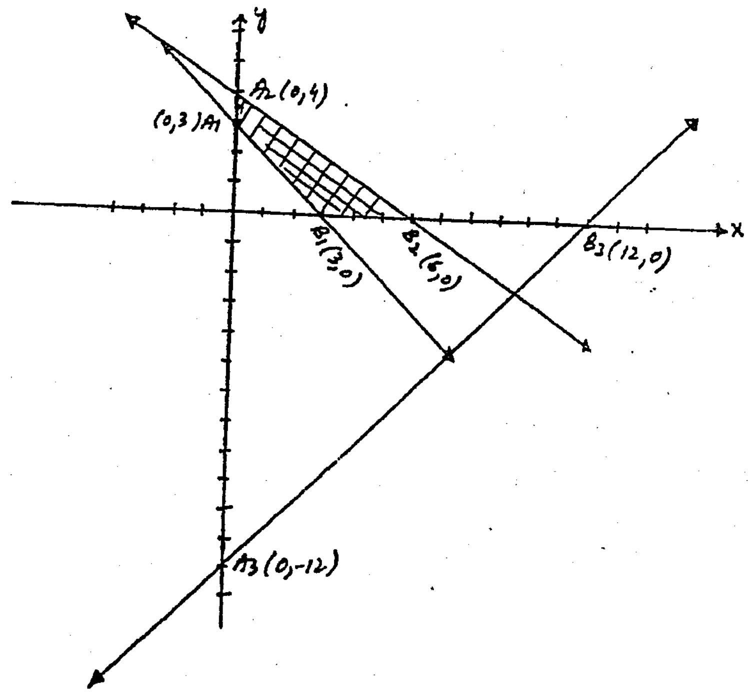
Put origin $(0,0)$

$x+y\ge 3$, $2x+3y\le 12$, $x-y\le 12$

$0+0\ge 3$, $2(0)+3(0)\le 12$, $0-0\le 12$

$0\ge 3$, $0\le 12$, $0\le 12$

False , True , True



* Now apply the non-negative constraints the final feasible region is drawn as shown in the figure.

* Clearly we see from the figure that the corner points are

$(3,0)$, $(6,0)$, $(0,3)$, $(0,4)$

Engr. Majid Amin

Exercise # 9.3

Q:1 Maximize $f(x,y) = 2x + 1y$ subject to the constraints

$$x + y \le 6$$
$$x + y \ge 1$$
$$x \ge 0$$
$$y \ge 0$$

Sol * The objective ftn is $f(x,y) = 2x + 1y$

* First draw the feasible region

$x + y \le 6$, $x + y \ge 1$

* Associated eqns are

$x + y = 6$, $x + y = 1$

$x = 0 \Rightarrow 0 + y = 6$, $0 + y = 1$

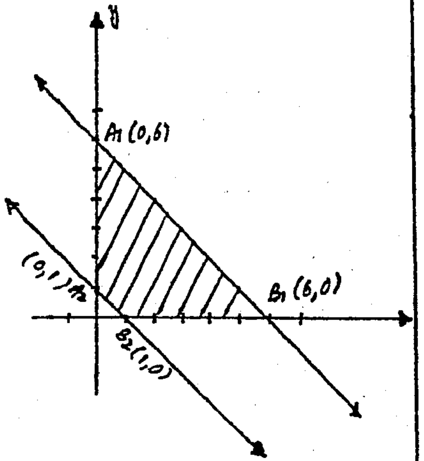
$\Rightarrow y = 6$, $\Rightarrow y = 1$

So $A_1 = (0,6)$, $A_2 = (0,1)$

$y = 0 \Rightarrow x + 0 = 6$, $\Rightarrow x + 0 = 1$

$\Rightarrow x = 6$, $\Rightarrow x = 1$

$\Rightarrow B_1 = (6,0)$, $\Rightarrow B_2 = (1,0)$



* Put origin in

$x + y \le 6$ & $x + y \ge 1$

$\Rightarrow 0 + 0 \le 6$, $\Rightarrow 0 + 0 \ge 1$

$\Rightarrow 0 \le 6$, $\Rightarrow 0 \ge 1$

True , $\Rightarrow$ False

* Applying the non negative constraints the feasible region is shown

* It is clear from the figure that the corner points are $(1,0)$, $(0,1)$, $(0,6)$, $(6,0)$.

* Now put these corner points in the objective ftn

$$f(x,y) = 2x + 1y$$

$(1,0) \Rightarrow f(1,0) = 2(1) + 1(0) = 2$

$(0,1) \Rightarrow f(0,1) = 2(0) + 1(1) = 1$

$(0,6) \Rightarrow f(0,6) = 2(0) + 1(6) = 6$

$(6,0) \Rightarrow f(6,0) = 2(6) + 1(0) = 12$

So the maximum value is 12 at $(6,0)$

Q:2 Maximize $f(x,y) = 3x + 5y$ subject to the constraints

$$2x + 3y \le 12$$
$$3x + 2y \le 12$$
$$x + y \ge 2$$
$$x \ge 0$$
$$y \ge 0$$

Sol * The objective function is $f(x,y) = 3x + 5y$

* $2x + 3y \le 12$, $3x + 2y \le 12$, $x + y \ge 2$

* The associated eqns are

$2x + 3y = 12$, $3x + 2y = 12$, $x + y = 2$

$x = 0 \Rightarrow 2(0) + 3y = 12$, $3(0) + 2y = 12$, $0 + y = 2$

$3y = 12$, $y = 6$, $y = 2$

$\Rightarrow y = 4$

So $A_1 = (0,4)$, So $A_2 = (0,6)$, So $A_3 = (0,2)$

$y = 0 \Rightarrow 2x + 3(0) = 12$, $3x + 2(0) = 12$, $x + 0 = 2$

$\Rightarrow x = 6$, $\Rightarrow x = 4$, $\Rightarrow x = 2$

So $B_1 = (6,0)$, So $B_2 = (4,0)$, So $B_3 = (2,0)$

* Now put origin in the original inequalities

$2x + 3y \le 12$, $3x + 2y \le 12$, $x + y \ge 2$

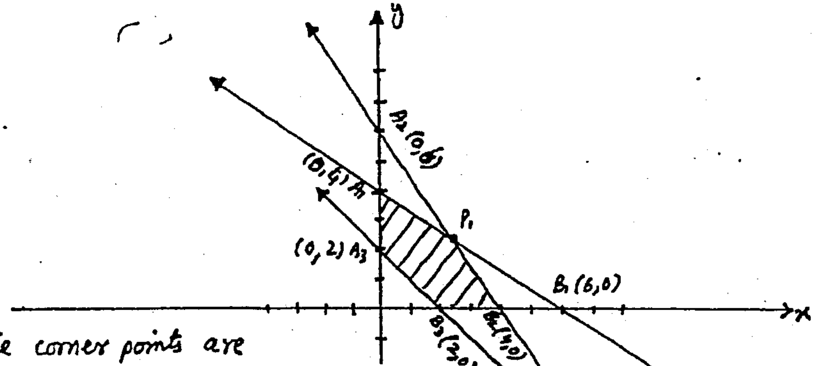
$\Rightarrow 2(0) + 3(0) \le 12$, $3(0) + 2(0) \le 12$, $0 + 0 \ge 2$

$\Rightarrow 0 \le 12$, $0 \le 12$, $0 \ge 2$

True , True , False

* Finally apply the non-negative constraints $x \ge 0$, $y \ge 0$ the feasible region is drawn as shown in the figure

260



The corner points are

$A_1=(4,0)$ $A_3=(0,2)$, $B_2=(4,0)$, $B_3(2,0)$ & P_1

To find P_1 (which is the intersection of two lines)

$2x+3y=12$ (i) $\quad 3x+2y=12$ (ii)

Multiply eqn (i) by 3 and eqn (ii) by 2 and then subtracting

$$\begin{array}{r} 6x+9y=36 \\ -\ 6x+4y=24 \\ \hline 5y=12 \end{array} \Rightarrow y=12/5$$

Eqn (i) $\Rightarrow 2x+3\left(\frac{12}{5}\right)=12$

$\Rightarrow 2x+\frac{36}{5}=12 \Rightarrow 2x=12-\frac{36}{5} = \frac{60-36}{5} = \frac{24}{5}$ So $2x=\frac{24}{5}$

$\Rightarrow x=12/5$

Hence $P_1=(12/5,\ 12/5)$

So the five corner points are

$A_1=(0,4)$, $A_3=(0,2)$, $B_2=(4,0)$, $B_3=(2,0)$ & $P_1=(12/5,\ 12/5)$

Put these corner points in the objective ftn $f(x,y)=3x+5y$

$B_2\ (4,0) \Rightarrow f(4,0)=3(4)+5(0)=12$

$A_3\ (0,2) \Rightarrow f(0,2)=3(0)+5(2)=10$

$A_1\ (0,4) \Rightarrow f(0,4)=3(0)+5(4)=20$

$B_3\ (2,0) \Rightarrow f(2,0)=3(2)+5(0)=6$

$P_1(12/5,12/5) \Rightarrow f(12/5,12/5)=3\left(\frac{12}{5}\right)+5\left(\frac{12}{5}\right)=\frac{36+60}{5}=96/5=19.2$

So the maximum value is 20 at $A_1=(0,4)$

Q.3 Minimize $f(x,y)=3x+4y$ subject to the constraints

CH-09
P-08

$$2x+3y\ge 6$$
$$x+y\le 8$$
$$x\ge 0$$
$$y\ge 0$$

Sol First considre

$2x+3y\ge 6$, $x+y\le 8$

* Associated eqns are

$2x+3y=6$, $x+y=8$

* $x=0 \Rightarrow 2(0)+3y=6$, $0+y=8$

$\Rightarrow y=2$ $\qquad \Rightarrow y=8$

$\Rightarrow A_1=(0,2)$ $\qquad \Rightarrow A_2=(0,8)$

$y=0 \Rightarrow 2x+3(0)=6$, $x+0=8$

$\Rightarrow x=3$ $\qquad \Rightarrow x=8$

So $B_1=(3,0)$ $\qquad$ So $B_2=(8,0)$

* Now

$2x+3y\ge 6$, $x+y\le 8$

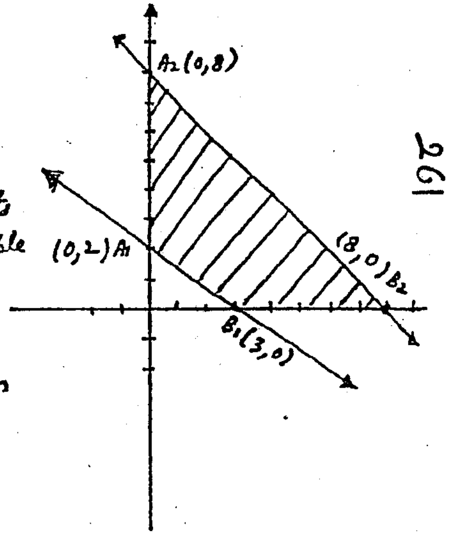
Put origin $(0,0)$, we get

$2(0)+3(0)\ge 6$ $\qquad 0+0\le 8$

$\Rightarrow 0\ge 6$ $\qquad 0\le 8$

False $\qquad$ True

* Now apply the non-negative constraints $x\ge 0$ and $y\ge 0$ and draw the feasible region which is shown



* clearly the corner points are

$(0,2)$, $(3,0)$, $(0,8)$, $(8,0)$

Now put in the objective function

$$f(x,y)=3x+4y$$

$A_1=(0,2) \Rightarrow f(0,2)=3(0)+4(2)=8$

$B_1=(3,0) \Rightarrow f(3,0) \Rightarrow 3(3)+4(0)=9$

$A_2=(0,8) \Rightarrow f(0,8)=3(0)+4(8)=32$

$B_2=(8,0) \Rightarrow f(8,0)=3(8)+4(0)=24$

So the minimum value is 8 at $A_1=(0,2)$

Q.4 Find the maximum and minimum value of the function $f(x,y) = 5x+2y$ subject to the constraints

$$2x+y \geq 2$$
$$x+2y \leq 10$$
$$x \geq 0$$
$$y \geq 0$$

Sol Considre

$2x+y \geq 2$, $x+2y \leq 10$

* The associated equs are

$2x+y=2$, $x+2y=10$

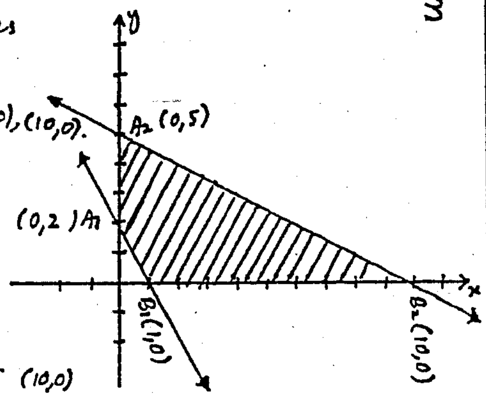
$x=0 \Rightarrow 2(0)+y=2 \Rightarrow y=2$; $0+2y=10 \Rightarrow y=5$

So $A_1=(0,2)$; So $A_2=(0,5)$

$y=0 \Rightarrow 2x+0=2 \Rightarrow x=1$; $x+2(0)=10 \Rightarrow x=10$

So $B_1=(1,0)$; $B_2=(10,0)$

* The feasible region is drawn as shown in the figure.



* The corner points are $(0,2),(0,5),(1,0),(10,0)$.
* Now put in the objective function

$$f(x,y) = 5x+2y$$

$(0,2) \Rightarrow f(0,2) = 5(0)+2(2) = 4$

$(0,5) \Rightarrow f(0,5) = 5(0)+2(5) = 10$

$(1,0) \Rightarrow f(1,0) \Rightarrow 5(1)+2(0) = 5$

$(10,0) \Rightarrow f(10,0) \Rightarrow 5(10)+2(0) = 50$

Hence the maximum value is 50 at $(10,0)$ and minimum value is 4 at $(0,2)$

Engr. Majid Amin

Q.5. Find the maximum and minimum values of the ftn $f(x,y) = 7x+21y$ subject to the constraints

$$2x+y \geq 2$$
$$2x+3y \leq 6$$
$$x+2y \leq 8$$
$$x \geq 0$$
$$y \geq 0$$

Sol * The associated equs are

$2x+y=2$, $2x+3y=6$, $x+2y=8$

* The boundary points are

$A_1=(0,2)$, $A_2=(0,2)$, $A_3=(0,4)$

$B_1=(1,0)$, $B_2=(3,0)$, $B_3=(8,0)$

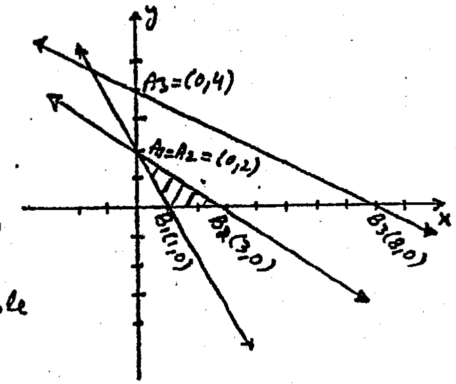
* Draw the boundary lines.
* Now put origin $(0,0)$ in

$2x+y \geq 2$, $2x+3y \leq 6$, $x+2y \leq 8$

$2(0)+0 \geq 2$, $2(0)+3(0) \leq 6$, $0+2(0) \leq 8$

$0 \geq 2$ False , $0 \leq 6$ True , $0 \leq 8$ True

* Then apply $x \geq 0$ & $y \geq 0$ and the final solution region is drawn as shown in the figure.



* The corner points of the feasible region are

$(1,0)$, $(3,0)$, $(0,2)$

* Now put in the objective ftn

$$f(x,y) = 7x+21y$$

$(1,0) \Rightarrow f(1,0) = 7(1)+21(0) = 7$

$(3,0) \Rightarrow f(3,0) = 7(3)+21(0) = 21$

$(0,2) \Rightarrow f(0,2) = 7(0)+21(2) = 44$

So Maximum value is 44 at $(0,2)$ & Minimum value is 7 at $(1,0)$

Q:6 A company manufactures two models of bicycle, model A and model B using two machines M_1 and M_2. Machine M_1 has at most 120 hours available and Machine M_2 has maximum of 144 hours available. Manufacturing a model A bicycle requires 5 hours in machine M_1 and 4 hours in machine M_2 and manufacturing of a model B bicycle requires 4 hours in machine M_1 and 8 hours in machine M_2. If the company gets profit of Rs 40 per model A bicycle and profit of Rs 50 per model B bicycle, how many of each model should be manufactured for maximum profit.

Sol Let x = units manufactured of model A

y = " " " model B

The condition that model A requires 5 hours on machine M_1 and 4 hours on machine M_2 gives the constraint

$5x+4y \le 120$

Similarly $4x+8y \le 144$

and equation for profit is $P(x,y) = 40x+50y$

* The associated eqns will be

$5x+4y=120$ and $4x+8y=144$

* The boundary points are

$(0,30), (24,0)$ and $(0,18), (36,0)$

* The boundary lines are shown

* Now $5x+4y \le 120$ & $4x+8y \le 144$

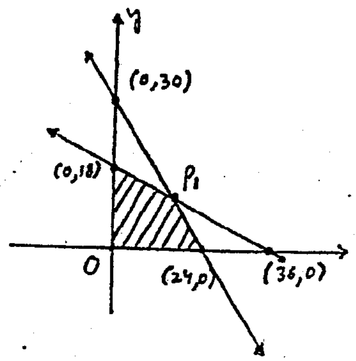
Put origin $(0,0)$

$\Rightarrow 5(0)+4(0) \le 120$ & $4(0)+8(0) \le 144$

$0 \le 120$. $0 \le 144$

True True

* finally apply $x \ge 0$ & $y \ge 0$ and draw the feasible region which is shown



* Three corner points are $(0,0), (0,18), (24,0)$.

To find P_1

Sol

$5x+4y=120$
$4x+8y=144$ $\Rightarrow$

$10x+8y = 240$ (eqn ① by 2)
$4x+8y = 144$
$6x = 96$

$\Rightarrow 6x = 96$
$\Rightarrow x = 16$

Then $5x+4y=120$
$\Rightarrow 5(16)+4y=120$
$\Rightarrow 80+4y=120$
$\Rightarrow 4y=40 \Rightarrow y=10$

So $P_1 = (16,10)$.

Now the objective fn is

$P(x,y) = 40x+50y$

$(0,0) \Rightarrow P(0,0) = 40(0)+50(0) = 0$

$(0,18) \Rightarrow P(0,18) = 40(0)+50(18) = 900$

$(24,0) \Rightarrow P(24,0) = 40(24)+50(0) = 960$

$(16,10) \Rightarrow P(16,10) = 40(16)+50(10) = 640+500 = 1140$

So the maximum profit is Rs 1140 at $(16,10)$.

i.e The company should manufacture 16 units of M_1
& 10 " " M_2.

Q:7 A company manufactures and sells two models of lamps L_1 & L_2. Use the following table to determine how many of each type of lamps should be produced to achieve a maximum profit?

	Model L_1	Model L_2	Max Time available
Manufacturing time per lamp	2 hours	1 hour	40 hours
Finishing time per lamp	1 hour	1 hour	32 hours
Profit per lamp	Rs 70	Rs 50	

Sol Let x = profit of model L_1 $\Rightarrow$ Eqn of profit

y = " " " L_2

$P(x,y) = 70x+50y \rightarrow$ ①

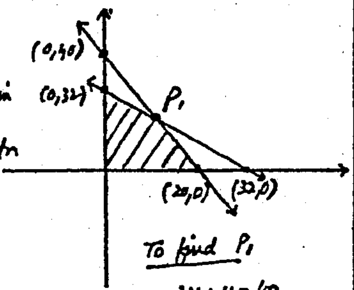
* The constraints are $2x+1y \le 40$ & $x+y \le 32$ (Also x & y can't be -ve)

* The associated eqns are $2x+y=40$ & $x+y=32$

* Boundary points are $(0,40), (20,0)$ & $(0,32), (32,0)$

* Draw continuous boundary lines as shown

* Now After putting origin (0,0) in the inequilities and apply $x \geq 0$ and $y \geq 0$, the feasible region is drawn as shown.



* The corner points are (0,32), (20,0) & P_1 & origin where $P_1 = (8,24)$
* Now put the corner points in the objective ftn

$$P(x,y) = 70x + 50y$$

$(0,32) \rightarrow P(0,32) = 70(0) + 50(32) = 1600$

$(0,0) \Rightarrow P(0,0) = 70(0) + 50(0) = 0$

$(20,0) \Rightarrow P(20,0) = 70(20) + 50(0) = 1400$

$(8,24) \Rightarrow P(8,24) = 70(8) + 50(24)$

$= 560 + 1200 = 1760$

So the maximum profit is Rs 1760 at (8,24).

To find P_1

$$\begin{array}{r} 2x + y = 40 \\ -x \mp y = -32 \\ \hline x = 8 \end{array}$$

Now $x + y = 32$

$8 + y = 32$

$\Rightarrow \boxed{y = 24}$

Q:8 A machine can produce product A by using 2 units of chemical and 1 unit of a compound or can produce product B by using 1 unit of chemical and 2 units of the compound. Only 800 units of chemical and 1000 units of the compound are available. The profit per unit of A and B are Rs 30 and Rs 20 respectively. Determine how many units of each product should be produced to achieve the maximum profit.

Sol: Let x = unit of product A

y = " " " B

The profit $P(x,y) = 30x + 20y \longrightarrow$ ①

But the constraints are

$2x + y \leq 800$

$x + 2y \leq 1000$ (Also x and y can't be -ve)

The associated equs are

$2x + y = 800 \longrightarrow$ (i) and $x + 2y = 1000 \longrightarrow$ (ii)

Equ (i) by 2 and then subtract equ (ii)

$$\begin{array}{r} 4x + 2y = 1600 \\ -x \mp 2y = -1000 \\ \hline 3x = 600 \end{array} \Rightarrow x = 200$$

Now $2x + y = 800$

$\Rightarrow 2(200) + y = 800 \Rightarrow 400 + y = 800 \Rightarrow y = 400$

So $P_1 = (200, 400)$

Now $2x + y = 800$, $x + 2y = 1000$

$x = 0 \Rightarrow 2(0) + y = 800$, $0 + 2y = 1000$

$\Rightarrow y = 800$, $\Rightarrow y = 500$

$\Rightarrow A_1 = (0, 800)$, $\Rightarrow A_2 = (0, 500)$

$y = 0 \Rightarrow 2x + 0 = 800$, $x + 2(0) = 1000$

$\Rightarrow x = 400$, $\Rightarrow x = 1000$

So $B_1 = (400, 0)$, So $B_2 = (1000, 0)$

* The corner points are after drawing the feasible region (0,0), (400,0), (0,500), (200,400).
* Now the objective ftn is

$$P(x,y) = 30x + 20y$$

Put the corner points are

$(0,0) \Rightarrow P(0,0) = 30(0) + 20(0) = 0$

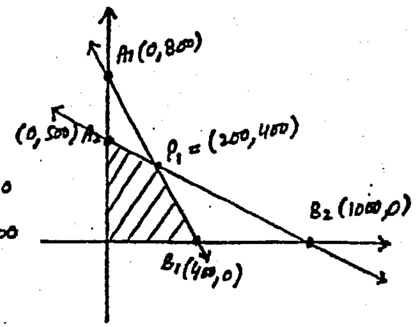
$(400,0) \Rightarrow P(400,0) = 30(400) + 20(0) = 12000$

$(0,500) \Rightarrow P(0,500) = 30(0) + 20(500) = 10000$

$(200,400) \Rightarrow P(200,400) = 30(200) + 20(400)$

$= 6000 + 8000$

$= 14000$



So the maximum profit is Rs 14000 at (200, 400)

End of chapter # 09

Written by Engr. Majid Amin
(MS. Mechanical Engineering)
(UET Peshawar.