Exercise # 8.1

Q.1 Given that $f(x) = x^2 + x - 1$

(i) Find image of $-2, 0, 2, 5$

Sol

Image of $-2 = f(-2) = (-2)^2 + (-2) - 1 = 4 - 2 - 1 = 1$

" " $0 = f(0) = 0^2 + 0 - 1 = -1$

" " $2 = f(2) = 2^2 + 0 - 2 = 2$

" " $5 = f(5) = 5^2 + 5 - 1 = 29$

(ii) If $f(x) = 5$, find $x = ?$

Sol

$f(x) = x^2 + x - 1$

$\Rightarrow 5 = x^2 + x - 1$

$\Rightarrow x^2 + x - 6 = 0$

$\Rightarrow x^2 + 3x - 2x - 6 = 0$

$\Rightarrow x(x+3) - 2(x+3) = 0$

$\Rightarrow (x+3)(x-2) = 0$

$\Rightarrow x + 3 = 0$ or $x - 2 = 0$

$x = -3$ $\qquad x = 2$

Hence $x = 2, -3$ Ans

(iii) Find $f(x+1)$

Sol

$f(x) = x^2 + x - 1$

$f(x+1) = (x+1)^2 + (x+1) - 1$

$f(x+1) = (x^2 + 1 + 2x) + (x+1) - 1$

$\boxed{f(x+1) = x^2 + 3x + 1}$ Ans

Quote: Human history becomes more and more a race between education and catastrophe.

H. G. Wells (1866 – 1946).

(iv) find $\dfrac{f(x+h) - f(x)}{h}$

Sol

$f(x) = x^2 + x - 1$

$f(x+h) = (x+h)^2 + (x+h) - 1$

$\Rightarrow f(x+h) = x^2 + h^2 + 2xh + x + h - 1$

$\Rightarrow f(x+h) = x^2 + h^2 + 2xh + x + h - 1$

Then $\dfrac{f(x+h) - f(x)}{h} = \dfrac{x^2 + h^2 + 2xh + x + h - 1) - (x^2 + x - 1)}{h}$

$= \dfrac{x^2 + h^2 + 2xh + x + h - 1 - x^2 - x + 1}{h}$

$= \dfrac{h^2 + 2xh + h}{h}$

$= \dfrac{h(h + 2x + 1)}{h}$

$= h + 2x + 1$ Ans

Q.2 If $f(x) = 7x + 3$, $g(x) = \dfrac{2x}{x^2+9}$, $h(x) = 20\sqrt{25 - x^2}$, $k(x) = x^2$

Determine

(i) $f(6)$, $g(-1)$, $h(4) = 20\sqrt{25 - 4^2}$, $k(\frac{1}{2}) = (\frac{1}{2})^2$

Sol $f(6) = 7(6) + 3$

$= 45$

$g(-1) = \dfrac{2(-1)}{(-1)^2 + 9}$

$= \dfrac{-2}{10}$

$= -\dfrac{1}{5}$

$h(4) = 20\sqrt{25 - 16}$

$= 20\sqrt{9}$

$= 20(3)$

$= 60$

$k(\frac{1}{2}) = \frac{1}{4}$

223

(ii) $\dfrac{f(x)-f(2)}{x-2}$

Sol $f(x) = 7x+3$

$f(2) = 7(2)+3$

$= 14+3 = 17$

Then $\dfrac{f(x)-f(2)}{x-2} = \dfrac{(7x+3)-(17)}{x-2}$

$= \dfrac{7x-14}{x-2} = \dfrac{7(x-2)}{x-2} = 7$ Ans

225

Q:3 Find the domain and range of $f(x)$.

(i) $f(x) = 2x+1$

Let $y = 2x+1$

$\Rightarrow$ Domain $= R$

Get x $2x = y-1$

$x = \dfrac{y-1}{2}$

$\Rightarrow$ Range $= R$

(ii) $f(x) = \sqrt{x^2-9}$ $\qquad x^2-9 \geq 0$, $x^2 \geq 9$

Domain $\left.\begin{array}{l} x \geq 3 \\ \& \ x \leq -3 \end{array}\right\}$ Ans

OR Dom $= R - (-3 \ \ 3)$

Get x $y = f(x)$

$y = \sqrt{x^2-9}$

$\Rightarrow y^2 = x^2-9$

$\Rightarrow x^2 = y^2+9$ $\qquad y^2+9 \geq 0$

$x = \sqrt{y^2+9}$ $\qquad y^2 \geq -9$

Range $= R$

(iii) $f(x) = \dfrac{x-3}{x+5}$

Let $y = f(x)$

$y = \dfrac{x-3}{x+5}$

$x \neq -5$

Hence Dom $= R - \{-5\}$

Now get x

$y = \dfrac{x-3}{x+5}$

$\Rightarrow xy+5y = x-3$

$\Rightarrow xy - x = -5y-3$

$\Rightarrow x(y-1) = -5y-3$

$\Rightarrow x = \dfrac{-5y-3}{y-1} \Rightarrow y \neq 1$

Range $= R - \{1\}$ Ans

(iv) $f(x) = \dfrac{x}{x^2-16}$

Let $y = \dfrac{x}{x^2-16}$ $\quad \because y = f(x)$

$x \neq \pm 4$

Hence Dom $= R - \{4, -4\}$ Ans

Now get x

$y = \dfrac{x}{x^2-16}$

$\Rightarrow (x^2-16)y = x$

$\Rightarrow yx^2 - 16y - x = 0$

$\Rightarrow yx^2 - 1x - 16y = 0$

By quadratic formula

$x = \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4y(-16y)}}{2y}$

$\Rightarrow \quad x = \dfrac{1 \pm \sqrt{1 + 64y^2}}{2y}$

Hence $y \neq 0$

$\Rightarrow$ Range $= R - \{0\}$ Ans

Q:4 $f(x) = 2x^3 + ax^2 + 4x - 5$. If $f(2) = 3$ find a.

Sol $f(2) = 2(2)^3 + a(2)^2 + 4(2) - 5$

$\Rightarrow 3 = 16 + 4a + 8 - 5$

$\Rightarrow 3 = 4a + 19 \Rightarrow 3 - 19 = 4a \Rightarrow -16 = 4a$

$\Rightarrow \boxed{a = -4}$ Ans

Q.5 $f(x) = 2x^3 + ax^2 + bx + 1$

$f(2) = -3$ and $f(-1) = 0$. Find the values of a & b.

Sol $f(x) = 2x^3 + ax^2 + bx + 1$

$\Rightarrow f(2) = 2(2)^3 + a(2)^2 + b(2) + 1$ & $f(-1) = 2(-1)^3 + a(-1)^2 + b(-1) + 1$

$\Rightarrow -3 = 16 + 4a + 2b + 1$

$\Rightarrow -3 = 17 + 4a + 2b$

$\Rightarrow -20 = 4a + 2b$

$\div$ by 2

$\Rightarrow -10 = 2a + b \longrightarrow (i)$

$0 = 2(-1) + a(1) - b + 1$

$0 = -2 + a - b + 1$

$0 = -1 + a - b$

$1 = a - b \longrightarrow (ii)$

Eqn (i) + Eqn (ii)

$-10 = 2a + b$

$1 = a - b$

$-9 = 3a$

$\Rightarrow \boxed{-3 = a}$ Ans

Now $a - b = 1$

$\Rightarrow -3 - b = 1$

$\Rightarrow \boxed{-4 = b}$ Ans

Q:6 Determine whether the given function is Even, odd or neither.

(i) $f(x) = x^2 + 1$

Sol put $-x$

$f(-x) = (-x)^2 + 1$

$f(-x) = x^2 + 1$

$\Rightarrow \boxed{f(-x) = f(x)}$

Hence the given ftn is Even ftn.

(ii) $f(x) = (x-2)^2$

put $-x$

$\Rightarrow f(-x) = (-x-2)^2$

$\Rightarrow f(-x) = (-(x+2))^2$

226

$\Rightarrow f(-x) = +(x+2)^2$

Since $f(-x) \neq f(x) \Rightarrow$ Not Even

Now $f(-x) = (x+2)^2$

But $-f(x) = -(x-2)^2$

$\Rightarrow f(-x) \neq -f(x) \Rightarrow$ Not odd

So the given fn is neither even nor odd.

(iii) $f(x) = x\sqrt{x^2+3}$

Sol put $-x$

$f(-x) = (-x)\sqrt{(-x)^2+3}$

$f(-x) = -x\sqrt{x^2+3}$

Since $f(-x) \neq f(x)$

$\Rightarrow$ Not even

Now $-f(x) = -x\sqrt{x^2+3}$

So $f(-x) = -f(x)$

$\Rightarrow$ odd fn.

So the given fn is odd fn.

(iv) $f(x) = \dfrac{x-1}{x+1}$

Sol put $-x$

$f(-x) = \dfrac{-x-1}{-x+1}$

$f(-x) = \dfrac{-(x+1)}{-(x-1)}$

$f(-x) = \dfrac{x+1}{x-1}$

$f(-x) \neq f(x)$

$\Rightarrow$ Not even

Now $-f(x) = -\left(\dfrac{x-1}{x+1}\right)$

$-f(x) = \dfrac{-x+1}{x+1}$

$-f(x) = \dfrac{1-x}{x+1}$

$f(-x) \neq -f(x)$

$\Rightarrow$ Not odd

Hence neither Even nor odd.

(v) $f(x) = |x|$

Sol put $-x$

$f(-x) = |-x|$

Since $|-x| = |x|$

$\Rightarrow f(-x) = f(x)$

Hence the given fn is Even fn.

(vi) $f(x) = \dfrac{x^3+x+3}{x^2-2}$

Sol

Put $-x$

$\Rightarrow f(-x) = \dfrac{(-x)^3+(-x)+3}{(-x)^2-2}$

$\Rightarrow f(-x) = \dfrac{-x^3-x+3}{x^2-2}$

$f(-x) \neq f(x) \Rightarrow$ Not Even

Now $-f(x) = -\left(\dfrac{x^3+x+3}{x^2-2}\right)$

$-f(x) = \dfrac{-x^3-x-3}{x^2-2}$

Since $f(-x) \neq -f(x) \Rightarrow$ Not odd

Hence the given fn is neither Even nor odd

Q.7 Find the inverse of the following fn.

(i) $f(x) = 2x-3$

Sol Method # 01

let $f(x) = y \Rightarrow x = f^{-1}(y)$

$\Rightarrow y = 2x-3$

Step # 01: Find x

$2x = y+3$

$\Rightarrow x = \frac{y+3}{2}$

Step # 02: Put $x = f^{-1}(y)$

$\Rightarrow f^{-1}(y) = \frac{y+3}{2}$

Step # 03: Replace y by x

$\boxed{f^{-1}(x) = \frac{x+3}{2}}$ Ans

Method # 02

$y = 2x-3$

interchange x with y

$x = 2y-3$

$\Rightarrow 2y = x+3$

$\boxed{y = \frac{x+3}{2}}$

is inverse of $y = 2x-3$

(ii) $f(x) = \frac{x}{3} - 5$

let $f(x) = y \Rightarrow x = f^{-1}(y)$

$\Rightarrow y = \frac{x}{3} - 5$

Step # 01 $y = \frac{x-15}{3}$

$3y = x-15$

$\Rightarrow x = 3y+15$

Step # 02: $f^{-1}(y) = 3y+15$

Step # 03: $f^{-1}(x) = 3x+15$ Ans

Method # 02

$y = \frac{x}{3} - 5$

$x \longleftrightarrow y$

$x = \frac{y}{3} - 5$

$x+5 = \frac{y}{3}$

$\Rightarrow \boxed{y = 3x+15}$ Ans

(iii) $f(x) = \frac{2x+1}{x-1}$

Sol let $f(x) = y \Rightarrow x = f^{-1}(y)$

$\Rightarrow y = \frac{2x+1}{x-1}$

Step # 01: Find x

$(x-1)y = 2x+1$

$\Rightarrow xy - y = 2x+1$

$\Rightarrow xy - 2x = y+1$

$\Rightarrow x(y-2) = y+1$

$\Rightarrow x = \frac{y+1}{y-2}$

Step # 02:

Put $x = f^{-1}(y)$

$f^{-1}(y) = \frac{y+1}{y-2}$

Step # 03

$f^{-1}(x) = \frac{x+1}{x-2}$ Ans

(iv) $f(x) = 4 + \sqrt{2x}$

let $y = f(x)$

$\Rightarrow y = 4 + \sqrt{2x}$

Step # 01: Find x

$\sqrt{2x} = y-4$

square b.s

$2x = (y-4)^2$

$\Rightarrow x = \frac{(y-4)^2}{2}$

Step # 02 Put $x = f^{-1}(y)$

$f^{-1}(y) = \frac{(y-4)^2}{2}$

Step # 03: Replace y by x

$f^{-1}(x) = \frac{(x-4)^2}{2}$ Ans

228

(8) $f(x) = x^3 - 2$. Find (i) $f^{-1}(x)$ (ii) $f^{-1}(3)$

Sol Let $y = f(x) \Rightarrow x = f^{-1}(y)$

$\Rightarrow y = x^3 - 2$

Step # 01: Find the value of x

$x^3 = y + 2$

$\Rightarrow x = (y+2)^{\frac{1}{3}}$

Step # 02:

Put $x = f^{-1}(y)$

$\Rightarrow f^{-1}(y) = (y+2)^{1/3}$

Step # 03: Replace y by x

$f^{-1}(x) = (x+2)^{1/3}$ Ans

Put $x = 3$

$f^{-1}(3) = (3+2)^{1/3} = (5)^{1/3} = \sqrt[3]{5}$ Ans

Q.9 Without find the inverse, determine the domain and range of f^{-1}.

(i) $f(x) = \frac{1}{x+2}$, $x \neq 2$

Sol Let $y = f(x)$

$\Rightarrow y = \frac{1}{x+2}$

Dom of $f = R - \{-2\}$

Now get x

$y(x+2) = 1$

$\Rightarrow xy + 2y = 1$

$\Rightarrow xy = 1 - 2y$

$\Rightarrow x = \frac{1-2y}{y}$

clearly Range of $f = R - \{0\}$

Now as we know that Dom of f = Range of f^{-1}

& Range of f = Dom of f^{-1}

So Domain of $f^{-1} = R - \{0\}$

and Range of $f^{-1} = R - \{-2\}$

(ii) $f(x) = \sqrt{x+3}$

$\Rightarrow y = \sqrt{x+3}$

$\Rightarrow$ Dom of $f = R$ $\Rightarrow$ $\boxed{\text{Range of } f^{-1} = R}$ Ans

and get x

$(y)^2 = (\sqrt{x+3})^2$

$y^2 = x + 3$

$\Rightarrow x = y^2 - 3$

$\Rightarrow$ Range of $f = R$ $\Rightarrow$ $\boxed{\text{Dom of } f^{-1} = R}$ Ans

(iii) $f(x) = \frac{x-1}{x-2}$

Sol $f(x) = y$

$\Rightarrow y = \frac{x-1}{x-2}$

As $x \neq 2$

$\Rightarrow$ Dom of $f = R - \{2\}$ $\Rightarrow$ $\boxed{\text{Range of } f^{-1} = R - \{2\}}$ Ans

Get x

$y = \frac{x-1}{x+2}$

$\Rightarrow y(x+2) = x-1$

$\Rightarrow xy + 2y = x-1$

$\Rightarrow xy - x = -2y-1$

$\Rightarrow x(y-1) = -2y-1$

$\Rightarrow x = \frac{-2y-1}{y-1}$

As $y \neq 1$

$\Rightarrow$ Range of $f = R - \{1\}$ $\Rightarrow$ $\boxed{\text{Domain of } f^{-1} = R - \{1\}}$ Ans

(iv) $f(x) = (x-7)^2$

Sol Dom of $f = R$ $\Rightarrow$ $\boxed{\text{Range of } f^{-1} = R}$ Ans

$y = (x-7)^2$

$\Rightarrow \sqrt{y} = x - 7$

$\Rightarrow x = \sqrt{y} + 7$

$\Rightarrow y \geq 0 \Rightarrow$ Range of $f \geq 0 \Rightarrow$ Dom of $f^{-1} \geq 0$ Ans

Exercise # 8.2

Q:1 Sketch the graphs of the functions.

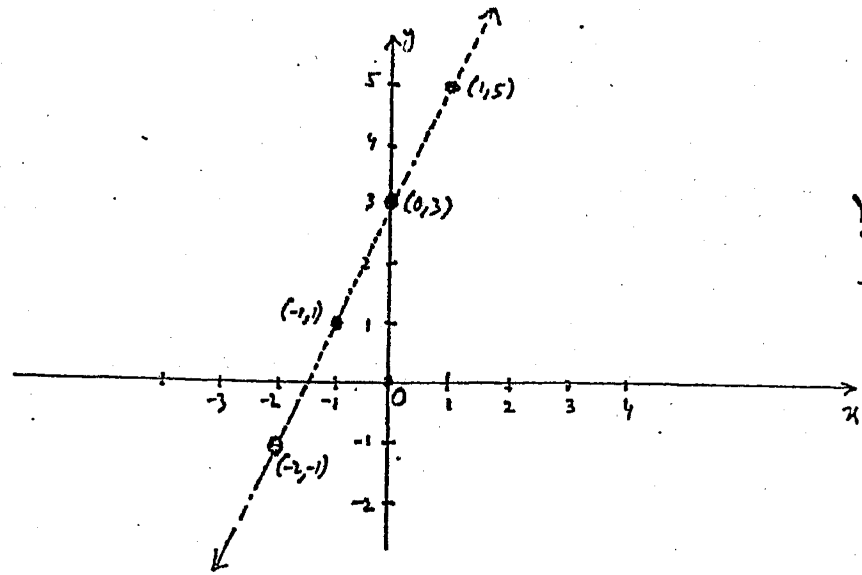
(a) $f(x) = 2x+3$

Sol Let $f(x) = y$

$\Rightarrow y = 2x+3$

Table

x (Inputs)	-2	-1	0	1	2
y (outputs)	-1	1	3	5	7

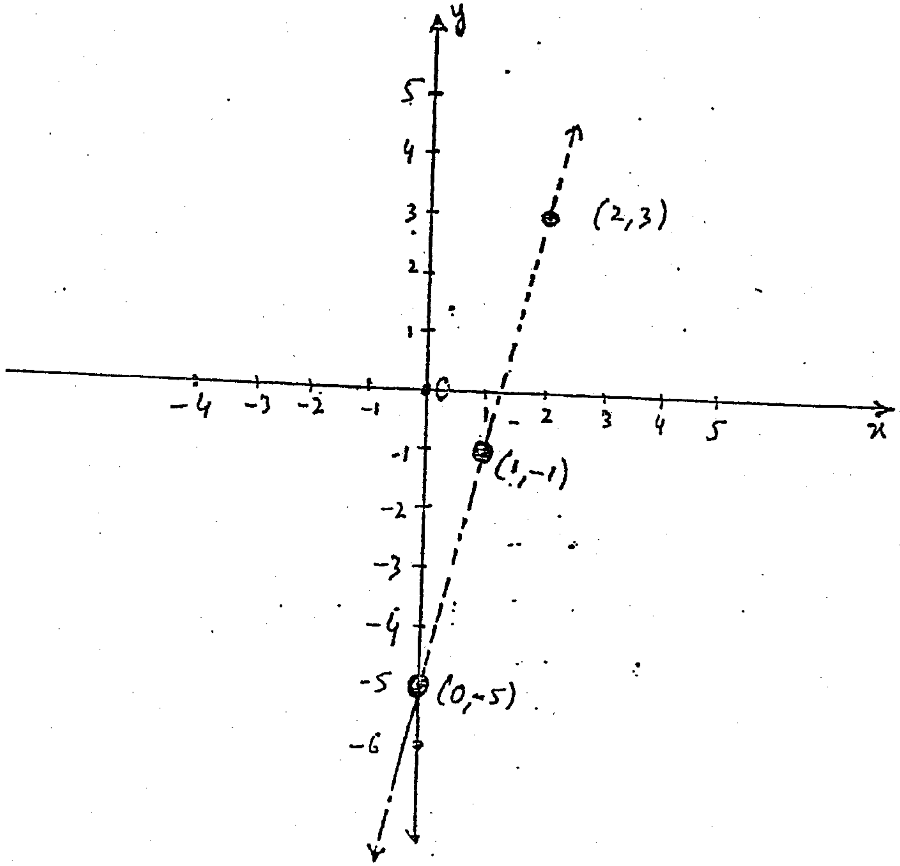


(b) $f(x) = 4x - 5$

Sol $\Rightarrow$ $y = 4x - 5$

Table

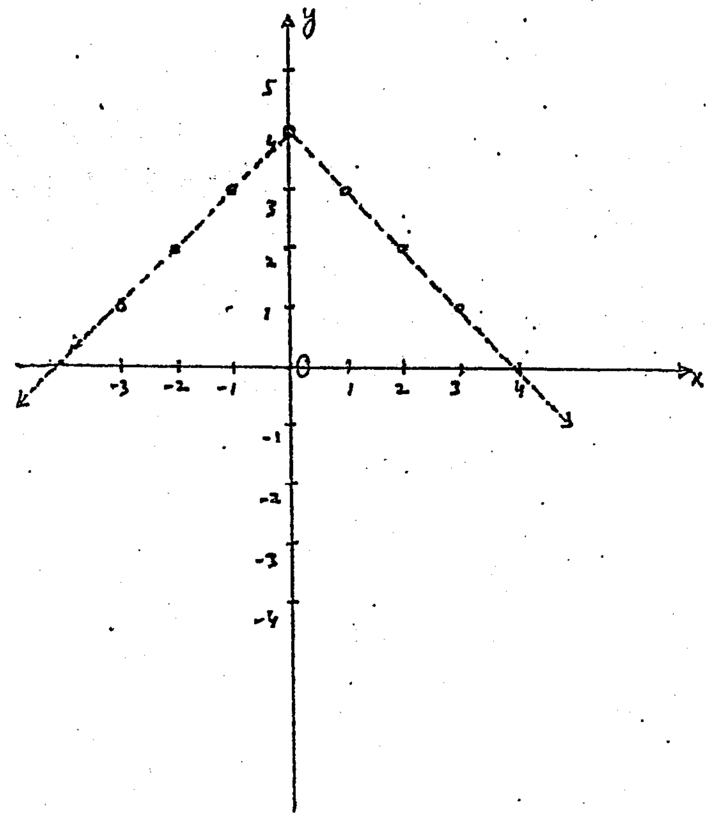
x	-2	-1	0	1	2	3	4
y	-13	-9	-5	-1	3	7	11



(c) $f(x) = 4 - |x|$

Sol $y = 4 - |x|$

x	-3	-2	-1	0	1	2	3
y	1	2	3	4	3	2	1

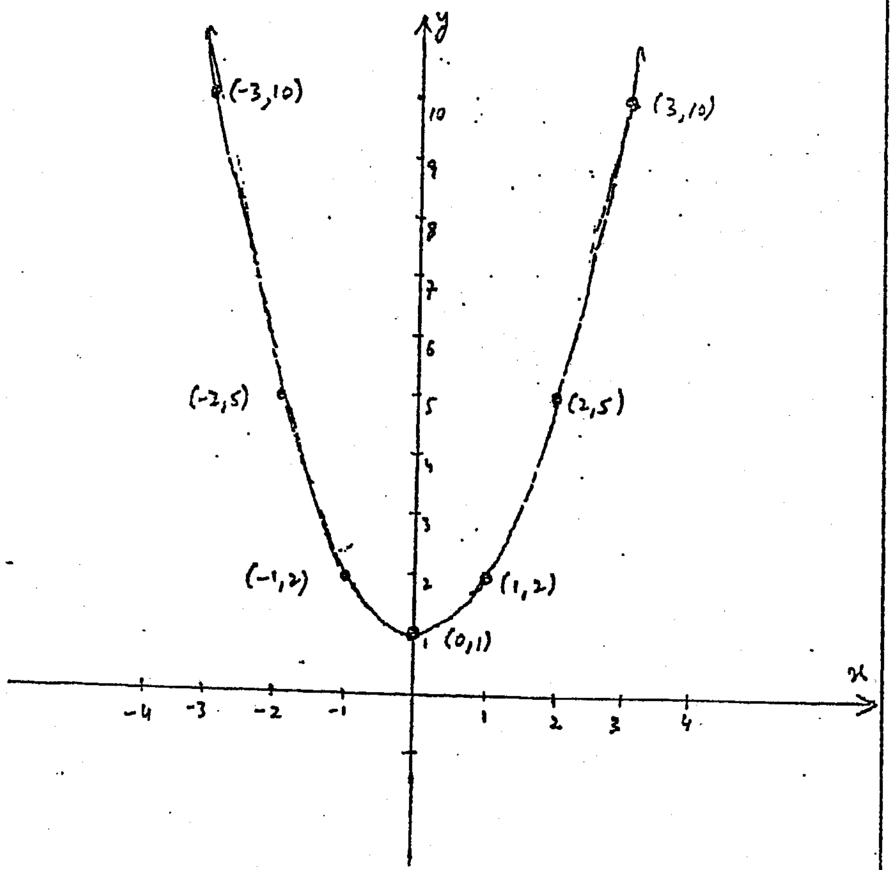


Q:2 Sketch the graphs of the following functions.

(a) $f(x) = x^2+1$ or $y = x^2+1$

Sol Table

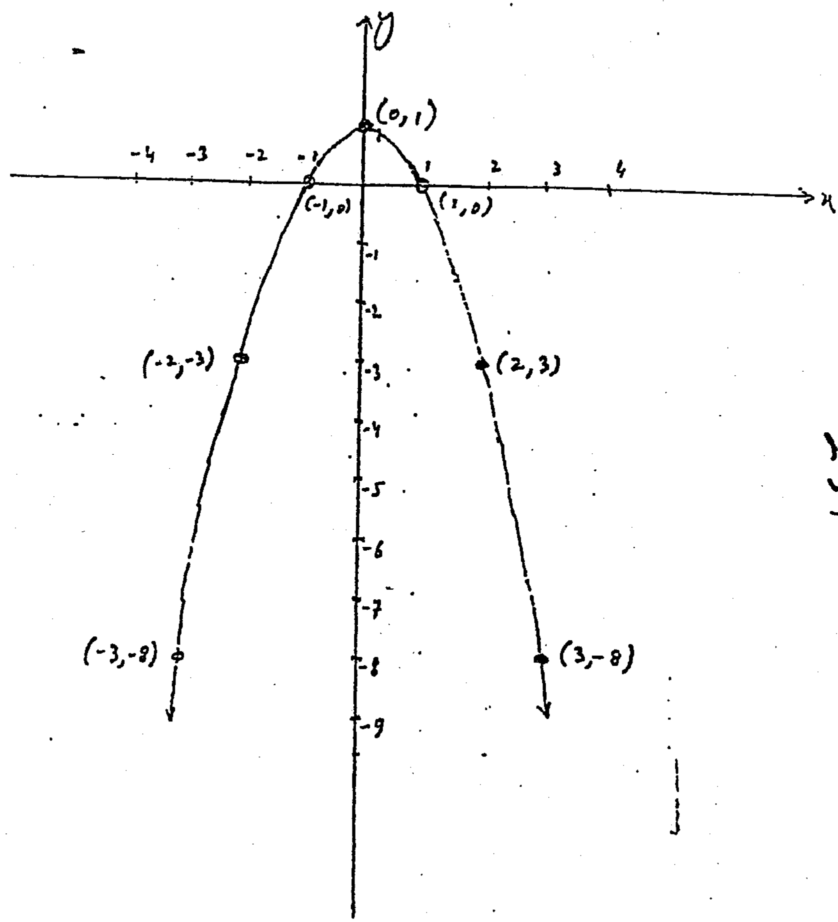
x	-3	-2	-1	0	1	2	3
y	10	5	2	1	2	5	10



(b) $f(x) = -x^2+1$ or $y = -x^2+1$

Sol Table

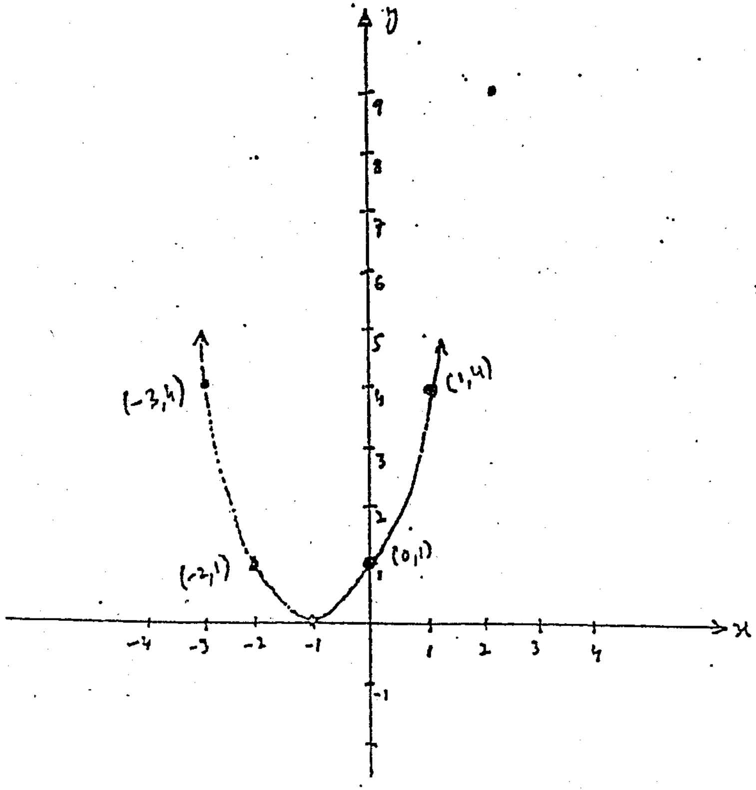
x	-3	-2	-1	0	1	2	3
y	-8	-3	0	1	0	-3	-8



232

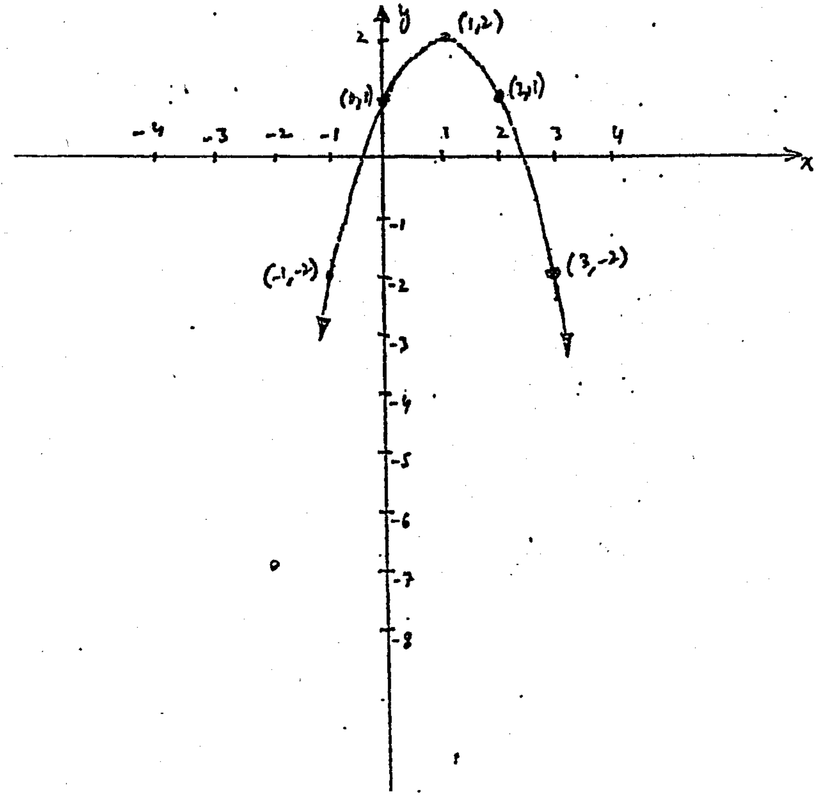
(c) $f(x) = x^2 + 2x + 1$
or
$y = x^2 + 2x + 1$

x	-4	-3	-2	-1	0	1	2	3	
y	9	4	1	0	1	4	9	16	



(d) $f(x) = -x^2 + 2x + 1$
or
$y = -x^2 + 2x + 1$

x	-4	-3	-2	-1	0	1	2	3	4
y	-23	-4	-7	-2	1	2	1	-2	-11



Q.3 Without graphing, find the vertex, all intercepts (if any) and axis of the graph of the following function. Also determine whether the graph opens upward or downward.

(a) $f(x) = \frac{3}{4}x^2 \Rightarrow y = \frac{3x^2}{4}$

Sol $f(x) = \frac{3}{4}x^2 + 0x + 0$

compare with $f(x) = ax^2 + bx + c = 0$, we get

$a = \frac{3}{4}, \quad b = 0, \quad c = 0$

Vertex

$V = (h, k)$ where $h = \frac{-b}{2a}$ & $k = c - \frac{b^2}{4a}$

$V = \left(-\frac{b}{2a}, c - \frac{b^2}{4a}\right)$

$\Rightarrow V = \left(\frac{-0}{2a}, 0 - \frac{0^2}{4a}\right) \Rightarrow \boxed{V = (0,0)}$ Ans

Intercepts

For X intercept put $y = 0$

$\Rightarrow \frac{3x^2}{4} = 0 \Rightarrow x^2 = 0 \Rightarrow \boxed{x = 0}$ Ans

For Y intercept put $x = 0$

$y = \frac{3(0)^2}{4} \Rightarrow \boxed{y = 0}$ Ans

Axis

$x = h$

$\Rightarrow x = -\frac{b}{2a}$

$x = -\frac{0}{2a} \Rightarrow \boxed{x = 0}$ Ans

As $a = \frac{3}{4} > 0 \Rightarrow$ The graph opens upward.

(b) $f(x) = x^2 + 1$

$\Rightarrow f(x) = 1x^2 + 0x + 1$ or $y = 1x^2 + 0x + 1$

Compare with $y = ax^2 + bx + c$, we get

$a = 1, \quad b = 0 \quad \& \quad c = 1$

Vertex

$V = (h, k)$

$= \left(-\frac{b}{2a}, c - \frac{b^2}{4a}\right)$

$= \left(-\frac{0}{2(1)}, 1 - \frac{0^2}{4(1)}\right) = (0, 1)$ Ans

X-intercept

$y = 0$

$\Rightarrow x^2 + 0x + 1 = 0$

$x^2 = -1$

$x = \sqrt{-1}$

$x = \infty$

$\Rightarrow$ The fn has no x-intercept

Y-intercept

$x = 0$

$y = 0^2 + 1$

$y = 1$

$\boxed{y = 1}$ Ans

Axis

$x = h \Rightarrow x = \frac{-b}{2a}$

$x = -\frac{0}{2(1)} \Rightarrow \boxed{x = -0}$ Ans

As $a = 1 > 0 \Rightarrow$ The graph opens upward.

(c) $f(x) = -2x^2 + 8$

Sol $f(x) = -2x^2 + 0x + 8 \Rightarrow y = -2x^2 + 0x + 8$

$a = -2, \quad b = 0, \quad c = 8$

vertex $V = (h, k)$

$V = \left(-\frac{b}{2a}, \; c - \frac{b^2}{4a}\right)$

$V = \left(\frac{-0}{2(-2)}, \; 8 - \frac{(0)^2}{4(-2)}\right)$

$V = \left(0, \; 8 - \frac{0}{-8}\right) \Rightarrow V = (0, 8)$

x-intercept

$y = 0$

$-2x^2 + 0x + 8 = 0$

$2x^2 = 8$

$x^2 = 4$

$\boxed{x = \pm 2}$

y-intercept

$x = 0$

$y = -2(0)^2 + 0 + 8$

$\boxed{y = 8}$

Axis:

$x = h$

$x = -\frac{b}{2a} \Rightarrow x = -\frac{0}{2(-2)} \Rightarrow \boxed{x = 0}$

As $a = -2 < 0$

$\Rightarrow$ The graph opens downward.

(d) $f(x) = -x^2 + 6x - 5$

Sol $y = -1x^2 + 6x - 5$

$a = -1, \quad b = 6, \quad c = -5$

vertex $V = (h, k)$

$V = \left(-\frac{b}{2a}, \; c - \frac{b^2}{4a}\right)$

$\Rightarrow V = \left(\frac{-6}{2(-1)}, \; -5 - \frac{(6)^2}{4(-1)}\right)$

$V = \left(3, \; -5 - \frac{36}{-4}\right) \Rightarrow V = (3, -5 + 9) \Rightarrow V = (3, 4)$ Ans

X-intercepts

$y = 0$

$-1x^2 + 6x - 5 = 0$

×ing by -1 $\Rightarrow$ $x^2 - 6x + 5 = 0$

$x^2 - 5x - 1x + 5 = 0$

$x(x-5) - 1(x-5) = 0$

$\Rightarrow (x-5)(x-1) = 0$

$x - 5 = 0$ or $x - 1 = 0$

$\boxed{x = 5 \;\&\; x = 1}$ Ans

Y-intercepts

$x = 0$

$y = -1(0)^2 + 6(0) - 5$

$\boxed{y = -5}$

Axis

$x = h$

$x = -\frac{b}{2a} \Rightarrow x = -\frac{6}{2(-1)} \Rightarrow \boxed{x = 3}$ Ans

As $a = -1 < 0$

$\Rightarrow$ The graph opens downward.

(e) $f(x) = 1x^2 + 2x - 3$

$\Rightarrow y = 1x^2 + 2x - 3 \Rightarrow a = 1,\ b = 2,\ c = -3$

vertex $= v = (h, K)$

$v = \left(\frac{-b}{2a},\ c - \frac{b^2}{4a}\right)$

$v = \left(\frac{-2}{2(1)},\ -3 - \frac{2^2}{4(1)}\right)$

$v = \left(\frac{-2}{2},\ -3 - \frac{4}{4}\right)$

$\Rightarrow v = (-1, -3-1)$

$\Rightarrow v = (-1, -4)$ Ans

X-intercept

Put $y = 0$

$\Rightarrow 0 = x^2 + 2x - 3$

$\Rightarrow x^2 + 3x - x - 3 = 0$

$\Rightarrow x(x+3) - 1(x+3) = 0$

$\Rightarrow (x+3)(x-1) = 0$

$x + 3 = 0$ or $x - 1 = 0$

$\boxed{x = -3}$ & $\boxed{x = 1}$

Y-intercept

Put $x = 0$

$y = 0^2 + 2(0) - 3$

$\boxed{y = -3}$ Ans

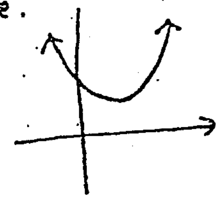
Axis

$x = h$

$x = \frac{-b}{2a} \Rightarrow x = -\frac{2}{2(1)} \Rightarrow \boxed{x = -1}$ Ans

As $a = 1 > 0 \Rightarrow$ The graph opens upward.

Q:4 Guess the quadratic function for the curve given in the figure.



(a) $f(x) = x^2 + 2x + 3$

(b) $f(x) = -x^2 - 2x + 3$

(c) $f(x) = x^2 - 2x + 3$

(d) $f(x) = -x^2 + 2x + 3$

Sol. • options (b) and (d) are not correct because $a = -1 < 0$

$\Rightarrow$ Graph opens downward but the given graph opens upward.

• Now either option (a) or (c) is correct

option (a)

$f(x) = y = x^2 + 2x + 3$

$a = 1,\ b = 2,\ c = 3$

vertex

$v = \left(\frac{-b}{2a},\ c - \frac{b^2}{4a}\right)$

$v = \left(\frac{-2}{2(1)},\ 3 - \frac{2^2}{4(1)}\right)$

$v = \left(\frac{-2}{2},\ 3 - \frac{4}{4}\right)$

$v = \left(\frac{-2}{2},\ 3 - 1\right)$

$v = (-1, 2)$

= IInd quadrant

But vertex of the given graph is in 1st quadrant

$\Rightarrow$ option (a) is not correct

option (c)

$f(x) = y = x^2 - 2x + 3$

$a = 1,\ b = -2,\ c = 3$

vertex

$v = \left(\frac{-b}{2a},\ c - \frac{b^2}{4a}\right)$

$v = \left(-\frac{-2}{2(1)},\ 3 - \frac{(-2)^2}{4(1)}\right)$

$v = \left(+\frac{2}{2},\ 3 - \frac{4}{4}\right)$

$v = (1, 3 - 1)$

$v = (1, 2) \Rightarrow$ 1st quadrant

$\Rightarrow$ option (c) is correct.

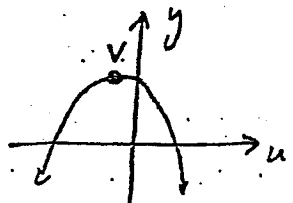
226

(5) (a) $g(x) = x^2 - 2x - 5$

(b) $g(x) = x^2 + 2x + 5$

(c) $g(x) = -x^2 - 2x + 5$

(d) $g(x) = -1x^2 + 2x + 5$



Sol: option (a) and (b) are not correct because $a = 1 > 0$ implies graph opens upward.

Now either option (c) or (d) is correct.

Option (c)

$y = 1x^2 - 2x + 5$

$a = 1,\ b = -2,\ c = 5$

$V = \left(\frac{-b}{2a},\ c - \frac{b^2}{4a}\right)$

$V = \left\{-\frac{(-2)}{2(1)},\ 5 - \frac{(-2)^2}{4(1)}\right\}$

$V = \left\{\frac{2}{2},\ 5 - \frac{4}{4}\right\}$

$V = (1, 4)$

$\Rightarrow$ 2nd quadrant

option (d)

$y = -1x^2 + 2x + 5$

$a = -1,\ b = 2,\ c = 5$

$V = \left(\frac{-b}{2a},\ c - \frac{b^2}{4a}\right)$

$V = \left(\frac{-2}{2(-1)},\ 5 - \frac{2^2}{4(-1)}\right)$

$V = \left(1,\ 5 - \frac{4}{-4}\right)$

$V = (1, 6)$

1st quadrant

Since the vertex of the given graph is in 2nd quadrant

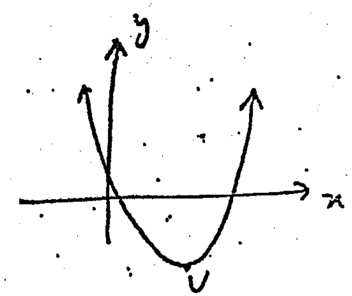
$\Rightarrow$ option (c) is correct.

(6) (a) $h(x) = -1x^2 - 6x + 5$

(b) $h(x) = x^2 - 6x + 5$

(c) $h(x) = x^2 + 6x + 5$

(d) $h(x) = -x^2 - 6x - 5$



Sol: options (a) and (d) are not correct because $a = -1 < 0$ means graph opens downward.

Either option (b) or (c) is correct.

Option (b)

$y = 1x^2 - 6x + 5$

$a = 1,\ b = -6\ \ c = 5$

vertex

$V = \left(\frac{-b}{2a},\ c - \frac{b^2}{4a}\right)$

$V = \left(-\frac{-6}{2(1)},\ 5 - \frac{(-6)^2}{4(1)}\right)$

$V = \left(\frac{6}{2},\ 5 - \frac{36}{4}\right)$

$V = (3, 5 - 9)$

$V = (3, -4)$.

$\Rightarrow$ 4th quadrant

option (c)

$y = 1x^2 + 6x + 5$

$a = 1\ \ b = 6\ \ c = 5$

$V = \left(\frac{-b}{2a},\ c - \frac{b^2}{4a}\right)$

$V = \left(-\frac{6}{2(1)},\ 5 - \frac{6^2}{4(1)}\right)$

$= \left(-\frac{6}{2},\ 5 - \frac{36}{4}\right)$

$= (-3, 5 - 9)$

$V = (-3, -4)$.

IIIrd quadrant.

Since the vertex of the given graph is in 4th quadrant $\Rightarrow$ option (b) is correct option.

Exercise # 8.3

Q.1 Sketch the graphs of the following functions.

(a) $f(x) = (x-1)(x-3)$

$\Rightarrow \quad y = (x-1)(x-3)$

To find x-intercepts, put
$y = 0$

$\Rightarrow (x-1)(x-3) = 0$

$\Rightarrow x-1=0 \qquad x-3=0$

$x=1 \qquad x=3$

Now put in y or $f(x)$

$\Rightarrow f(1)=0 \quad \& \quad f(3)=0$

To find y-intercept (i.e point where graph touches y-axis)
put $x=0$

$f(0) = y = (0-1)(0-3)$

$f(0) = (-1)(-3)$

$f(0) = 3$

Now some other points are

x	-2	-1	2	4	5
y	15	8	-1	3	8

x	1	0	3
y	0	3	0

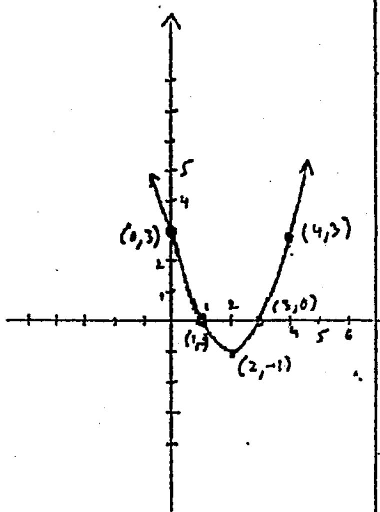
OR

$y = (x-1)(x-3)$

$y = x^2 - 4x + 3$

x	-2	-1	0	1	2
y	15	8	3	0	-1

Then draw the graph as given below



(b) $f(x) = (x+4)(x+1)$
or
$y = (x+4)(x+1)$

Sol: To find x-intercepts,
put $y=0$

$0 = (x+4)(x+1)$

$x+4=0 \quad$ or $\quad x+1=0$

$x=-4 \qquad x=-1$

i.e $f(-4)=0 \ \& \ f(-1)=0$

For y-intercept
$x=0$

$y=(0+4)(0+1)$

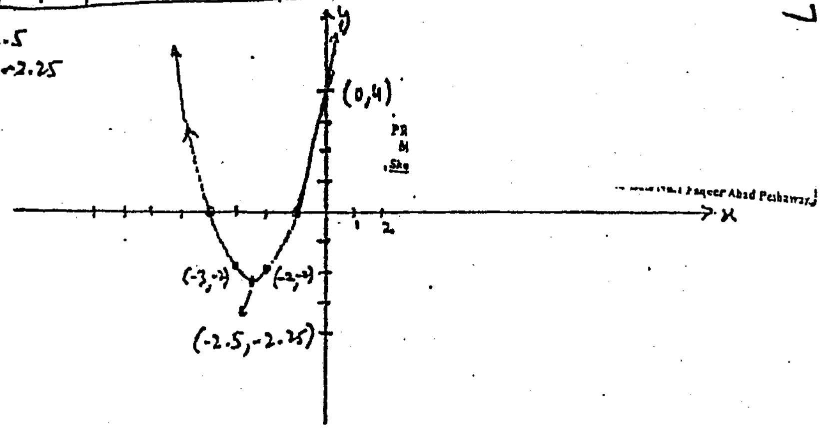
$y=(4)(1)$

$y=4$

complete table is

x	-3	-2	-1	0	1	2	3
y	-2	-2	0	4	10	18	

$x=-2.5$
$y=-2.25$



OR

$y = (x+4)(x+1)$

$y = x^2 + x + 4x + 4$

$y = x^2 + 5x + 4$

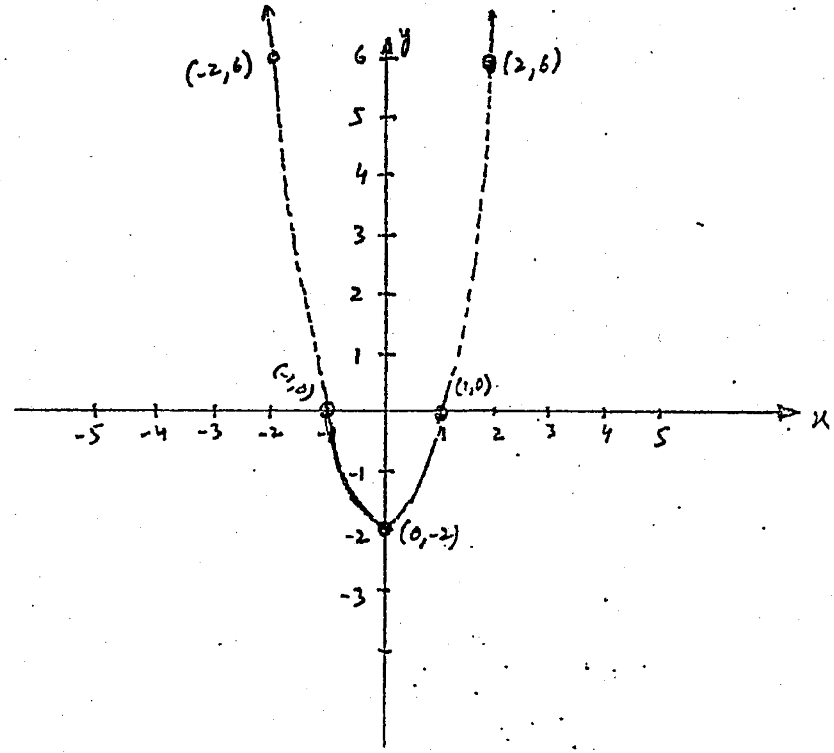
Then draw the graph

x	-2	-1	0	1	2
y	-2	0	4	10	18

etc
as shown

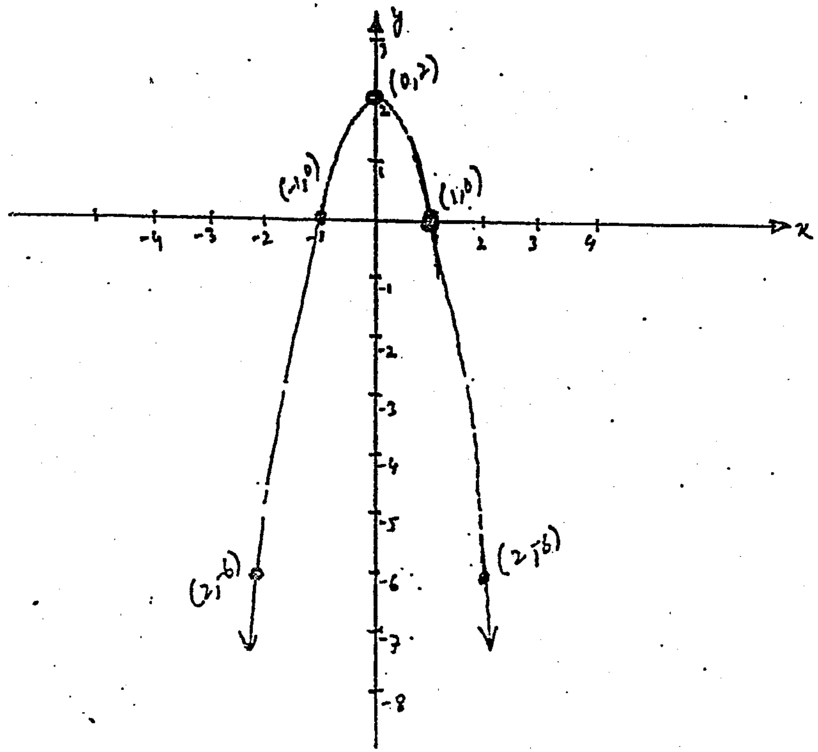
(c) $f(x) = 2(x+1)(x-1)$
$= 2(x^2-1)$
$\Rightarrow \quad y = 2x^2 - 2$

x	-2	-1	0	1	2	3
y	6	0	-2	0	6	16



(d) $f(x) = -2(x+1)(x-1)$
$\Rightarrow \quad y = -2(x^2-1)$
$y = -2x^2 + 2$

x	-3	-2	-1	0	1	2	3
y	-16	-6	0	2	0	-6	-16



Q.2 Using factors to sketch the graphs of the following fns

(a) $f(x) = x^2 - 2x - 3$

Sol

$y = x^2 - 2x - 3$

$y = x^2 - 3x + x - 3$

$y = x(x-3) + 1(x-3)$

$y = (x-3)(x+1)$

$\Rightarrow y = 1(x-3)(x+1)$ $\quad y = a(x-p)(x-q)$

$\Rightarrow a = 1 > 0 \Rightarrow$ Graph will open upward

For x-intercepts put $y = 0$

$\Rightarrow (x-3)(x+1) = 0$

$x - 3 = 0$ or $x + 1 = 0$

$x = 3 \quad x = -1$

Hence $(3,0)$, $(-1,0)$

& for y-intercept $x = 0$

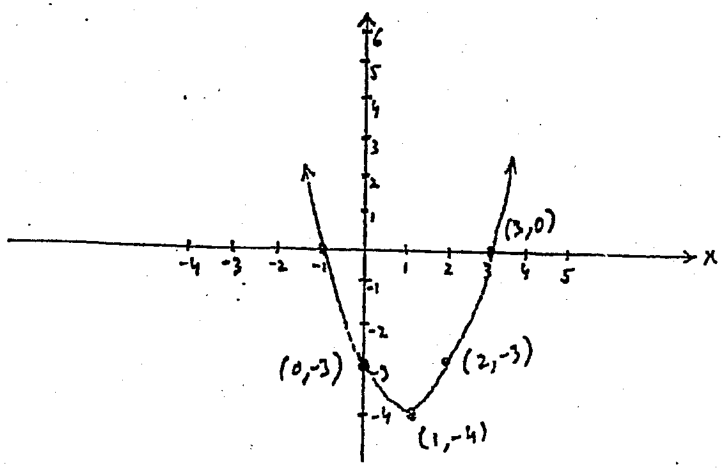
$y = 0^2 - 2(0) - 3$

$y = -3$

Hence $(0,-3)$

Some other points

x	-3	-2	1	2	4
y	12	5	-4	-3	5



(b) $f(x) = -(x^2 - x - 2)$

$\Rightarrow y = -(x^2 - 2x + x - 2)$

$\Rightarrow y = -\{x(x-2) + 1(x-2)\}$

$\Rightarrow y = -1(x-2)(x+1) \Rightarrow y = +a(x-p)(x-q)$ form

$\Rightarrow a = -1 < 0 \Rightarrow$ Graph will open downward

For x-intercept put $y = 0$

$-1(x-2)(x+1) = 0$

$x - 2 = 0$ or $x + 1 = 0$

$x = 2 \quad x = -1$

Hence $(2,0)$ & $(-1,0)$

and for y-intercept put $x = 0$

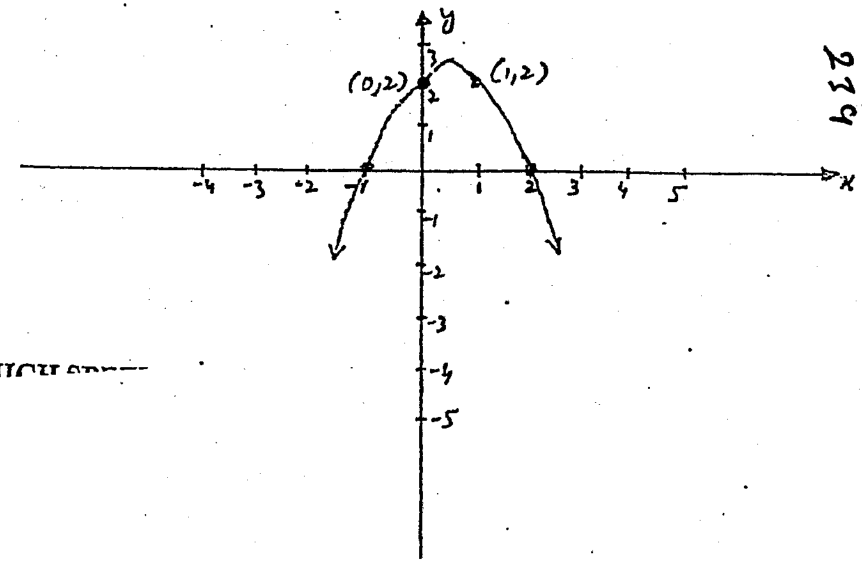
$y = -(0^2 - 0 - 2)$

$y = 2$

Hence $(0,2)$

Some other points

x	-3	-2	1	3	4
y	-10	-4	2	-4	-10



(c) $f(x) = -x^2 - 4x - 4$

$\Rightarrow y = -(x^2+2x+2)$

$y = -(x+2)^2$

$y = -1(x+2)(x+2)$ $\quad a<0 \Rightarrow$ opens downward

Put $y=0$ for x-intercepts & $x=0$ for y-intercept

$0 = -1(x+2)(x+2)$

$\Rightarrow x+2=0$

$x=-2$

$\Rightarrow (-2,0)$ is one point

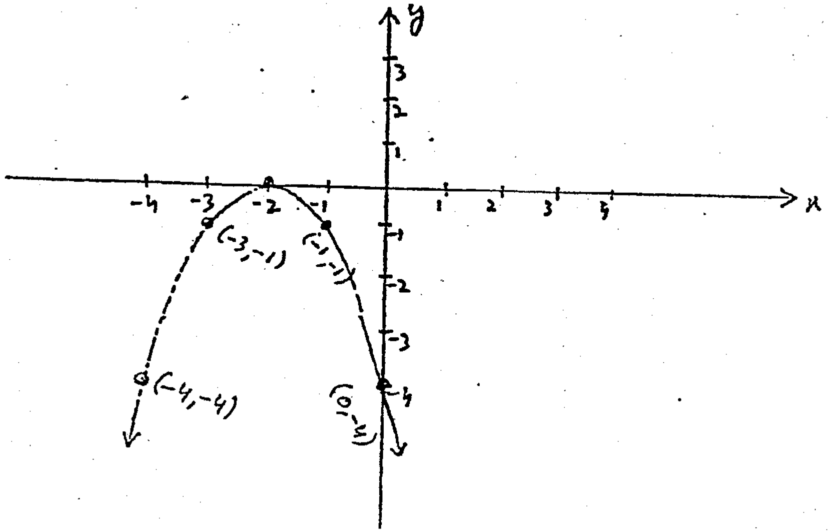
$y = -0-0-4$

$y=-4$

$(0,-4)$ is another point

Table

x	-4	-3	-1	1	2	
y	-4	-1	-1	-9	-16	



Q:3 Find the equation of the graph of the function of the type $y = x^2 + bx + c$ which cross the x-axis at the points $(3,0)$ & $(4,0)$

Sol

$y = x^2 + bx +$

Put $(3,0) \Rightarrow 0 = 3^2 + b(3) + c$

$0 = 9 + 3b + c$

$\Rightarrow 3b + c = -9 \longrightarrow$ (i)

Put $(4,0) \Rightarrow 0 = 4^2 + b(4) + c$

$0 = 16 + 4b + c$

$\Rightarrow 4b + c = -16 \longrightarrow$ (ii)

Eqn (i) − Eqn (ii)

$$\begin{array}{r} 3b + c = -9 \\ \underset{-}{4b} + \underset{-}{c} = \underset{+}{-16} \\ \hline -b \quad = 7 \end{array}$$

$\boxed{b=-7}$

Now $3b + c = -9$

$\Rightarrow 3(-7) + c = -9$

$\Rightarrow -21 + c = -9$

$c = 21 - 9$

$\boxed{c = 12}$

P.T.V in $y = x^2 + bx + c$

$\boxed{y = x^2 - 7x + 12}$ is the required eqn.

Q:4 Find the equ of the graph of the fn of the type $y = ax^2 + bx + c$ which

(a) Cross x-axis at point $(-5,0)$ and $(3,0)$ and also passes through $(-1,8)$.

Sol As we know that the equ of the graph which touches x-axis at $(p,0)$ and $(q,0)$ is

$y = a(x-p)(x-q)$ Here $(p,0) = (-5,0)$ $\Rightarrow p = -5$

Put the values & $(q,0) = (3,0)$ $\Rightarrow q = 3$

$\Rightarrow y = a(x-(-5))(x-3)$

$y = a(x+5)(x-3) \longrightarrow (i)$

Also the graph passes through $(-1,8)$ so put $x=-1$ & $y=8$

Then $8 = a(-1+5)(-1-3)$

$\Rightarrow 8 = a(4)(-4)$

$\Rightarrow 8 = -16a \Rightarrow a = \frac{8}{-16} \Rightarrow a = -\frac{1}{2}$

equ (i) $\Rightarrow y = \frac{-1}{2}(x+5)(x-3)$

$\Rightarrow y = -\frac{1}{2}(x^2 - 3x + 5x - 15)$

$\Rightarrow y = -\frac{1}{2}(x^2 + 2x - 15)$ Ans

(b) Cross x-axis at $(-7,0)$ and $(10,0)$ and also pass through $(4,11)$.

Sol Here $(p,0) = (-7,0) \Rightarrow p = -7$

$(q,0) = (10,0) \Rightarrow q = 10$

Equ is $y = a(x-p)(x-q)$

$y = a(x-(-7))(x-10)$

$\Rightarrow y = a(x+7)(x-10) \longrightarrow (i)$

Also the graph passes through $(4,11)$, so put $x = 4$ and $y = 11$ in equ (i)

$\Rightarrow 11 = a(4+7)(4-10)$

$\Rightarrow 11 = a(11)(-6) \Rightarrow 1 = -6a \Rightarrow \boxed{a = -1/6}$

equ (i) $\Rightarrow y = -\frac{1}{6}(x+7)(x-10)$

$\Rightarrow y = -\frac{1}{6}(x^2 - 10x + 7x - 70)$

$\Rightarrow y = -\frac{1}{6}(x^2 - 3x - 70)$ Ans

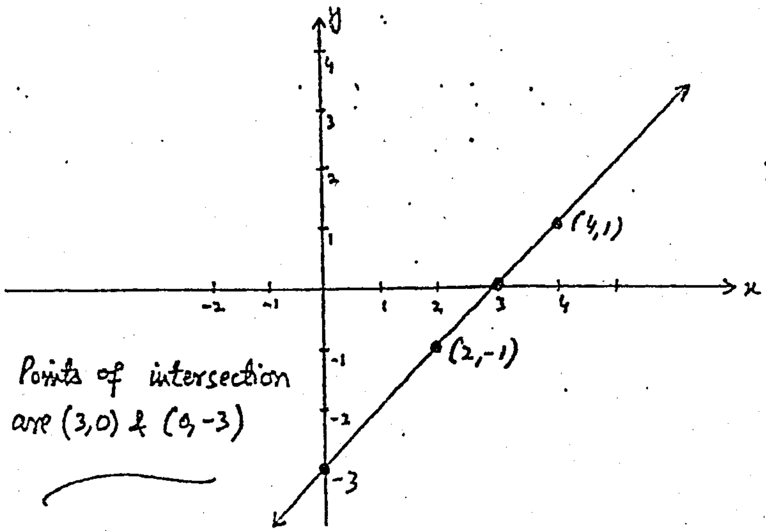
Q:5 Find the points of intersection graphically of the following linear functions with coordinate axes.

(a) $f(x) = x - 3$

$\Rightarrow y = x - 3$

Table

x	-2	-1	0	1	2	3	4
y	-5	-4	-3	-2	-1	0	1



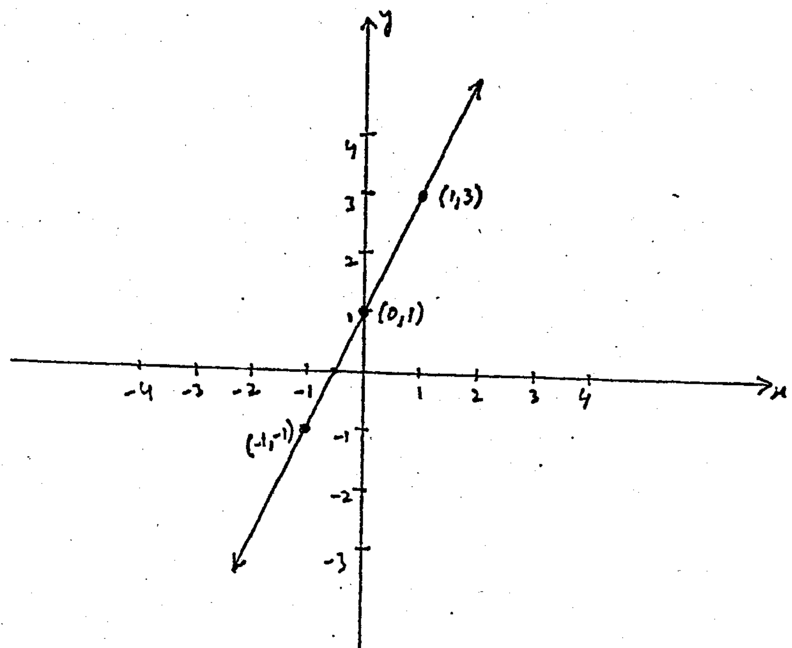
Points of intersection are $(3,0)$ & $(0,-3)$

(b) $f(x) = 2x+1$

$\Rightarrow \quad y = 2x+1$

Sol

x	-3	-2	-1	0	1	2	3
y	-5	-3	-1	1	3	5	7



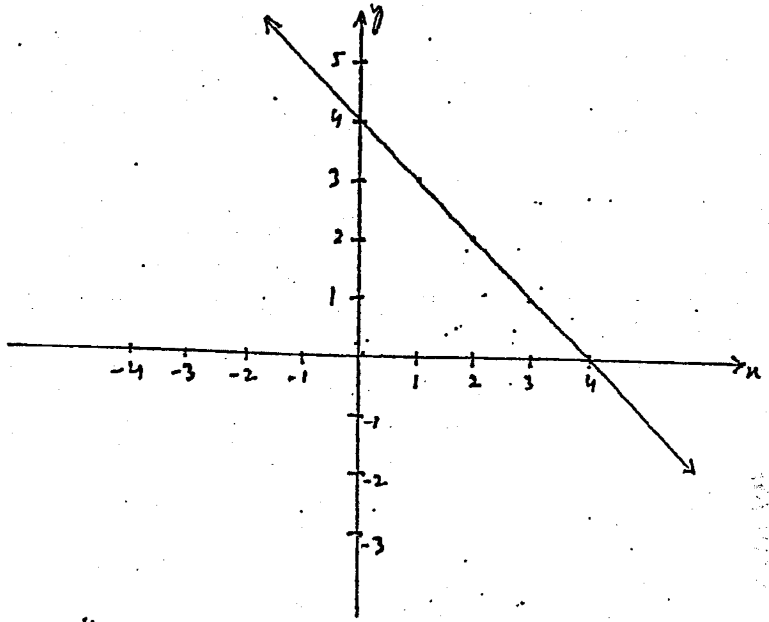
From the figure x-intercept $= (-0.5, 0)$

& y-intercept $= (0,1)$

(c) $f(x) = -x+4$

Sol

x	-2	-1	0	1	2	3
y	-6	-5	4	3	2	-1



Hence X-intercept $= (4,0)$

Y-intercept $= (0,4)$

Q:6 Find the point of intersection graphically of the following functions.

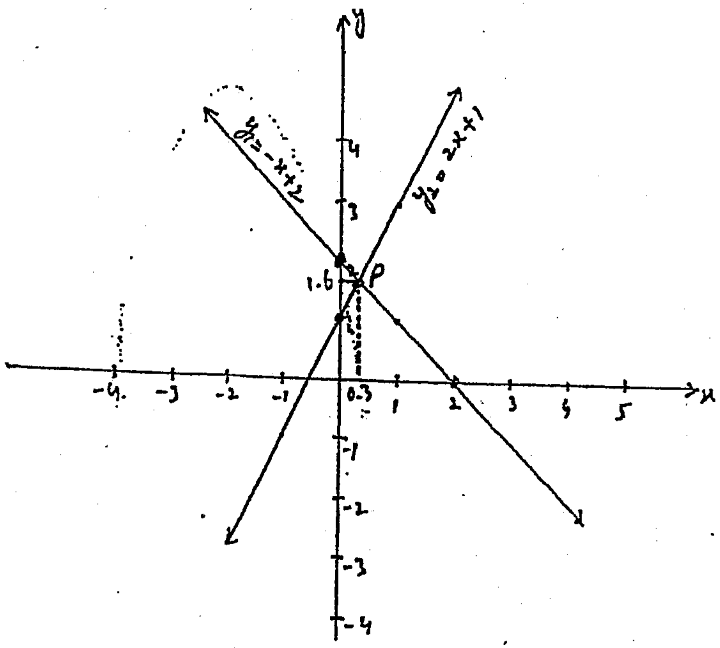
(a) $f(x) = -x+2$ & $g(x) = 2x+1$

let $f(x) = y_1$ & $g(x) = y_2$

$\Rightarrow y_1 = -x+2$ & $y_2 = 2x+1$

x	-1	0	1	2
y_1	3	2	1	0

x	-1	0	1	2
y_2	-1	1	3	5



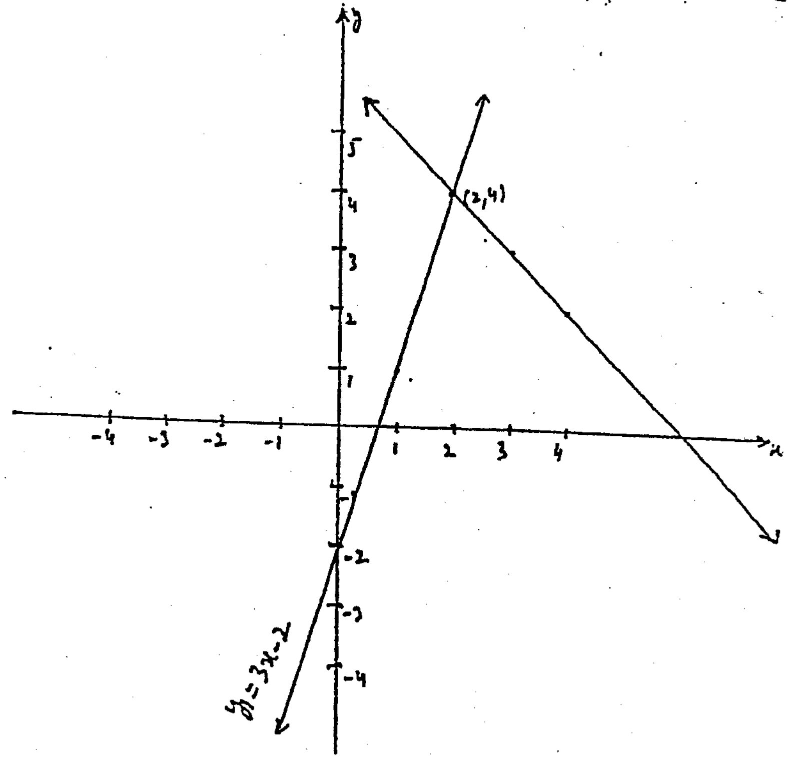
Point of intersection = (0.3, 1.6) Ans

(b) $f(x) = 3x-2$ & $g(x) = -x+6$

$\Rightarrow y_1 = 3x-2$ & $y_2 = -x+6$

x	-2	-1	0	1	2
y_1	-8	-5	-2	1	4

x	-2	-1	0	1	2	3
y_2	8	7	6	5	4	3



Point of intersection of the two graphs is (2, 4).

(c) $f(x) = x+4$

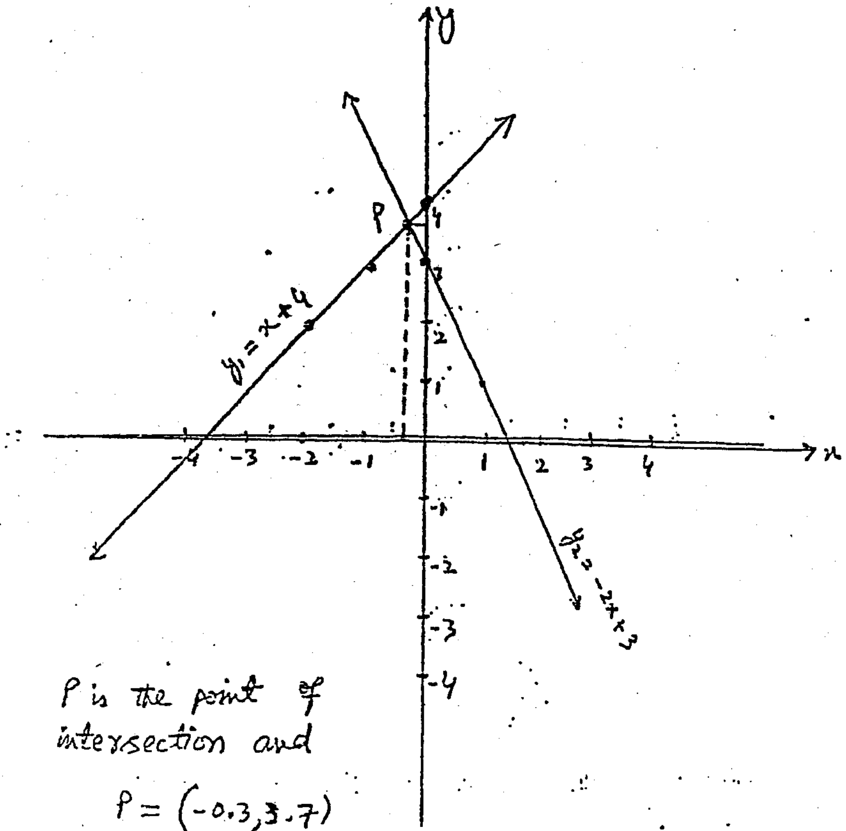
$\Rightarrow y_1 = x+4$

x	-2	-1	0	1	2
y_1	2	3	4	5	6

$g(x) = -2x+3$

$y_2 = -2x+3$

x	-2	-1	0	1	2
y_2	7	5	3	1	-1



P is the point of intersection and

$P = (-0.3, 3.7)$ approximately.

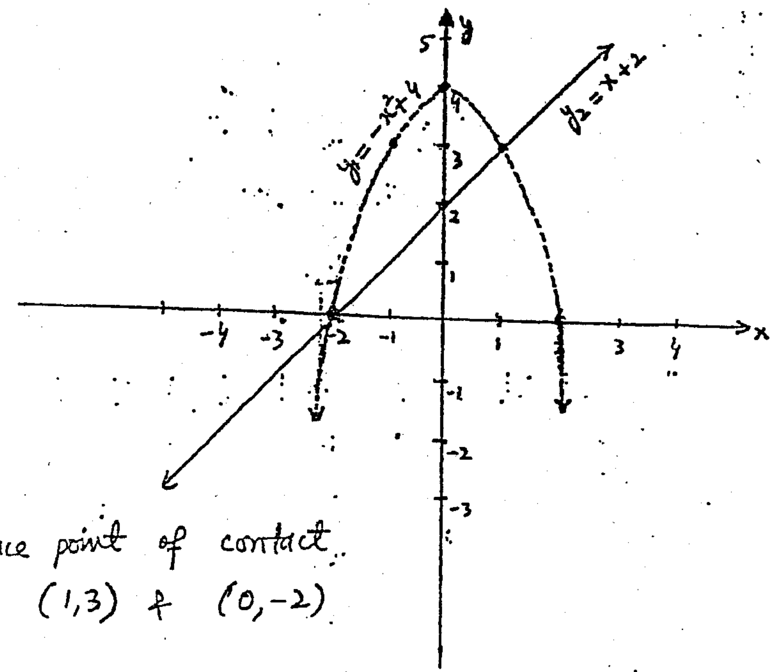
Q.7 Find the point of intersection graphically of the following functions.

(a) $f(x) = -x^2+4$ $\qquad g(x) = x+2$

Sol $y_1 = -x^2+4$ $\qquad y_2 = x+2$

x	-2	-1	0	1	2
y	0	3	4	3	0

x	-1	0	1	2
y	1	2	3	4



Hence point of contact are $(1,3)$ & $(0,-2)$

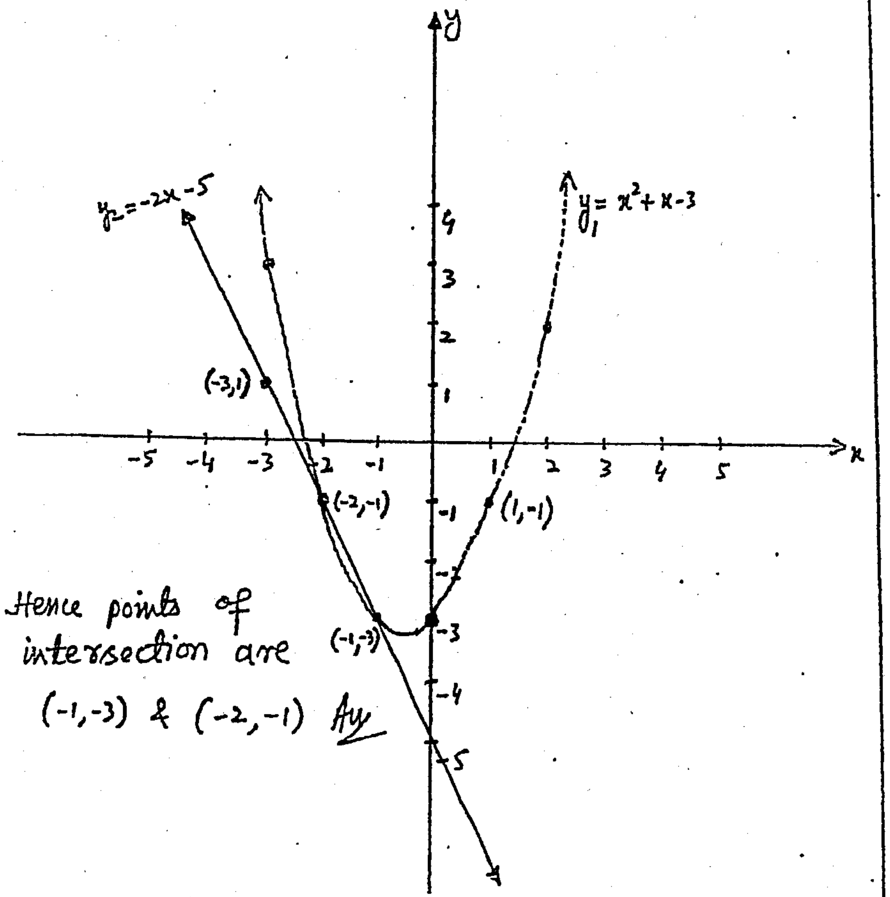
Available at www.mathcity.org

(b) $f(x) = x^2 + x - 3$, $g(x) = -2x - 5$

or $y_1 = x^2 + x - 3$ or $y_2 = -2x - 5$

x	-2	-1	0	1	2
y_1	-1	-3	-3	-1	3

x	-3	-2	-1	0	1
y	1	-1	-3	-5	-7



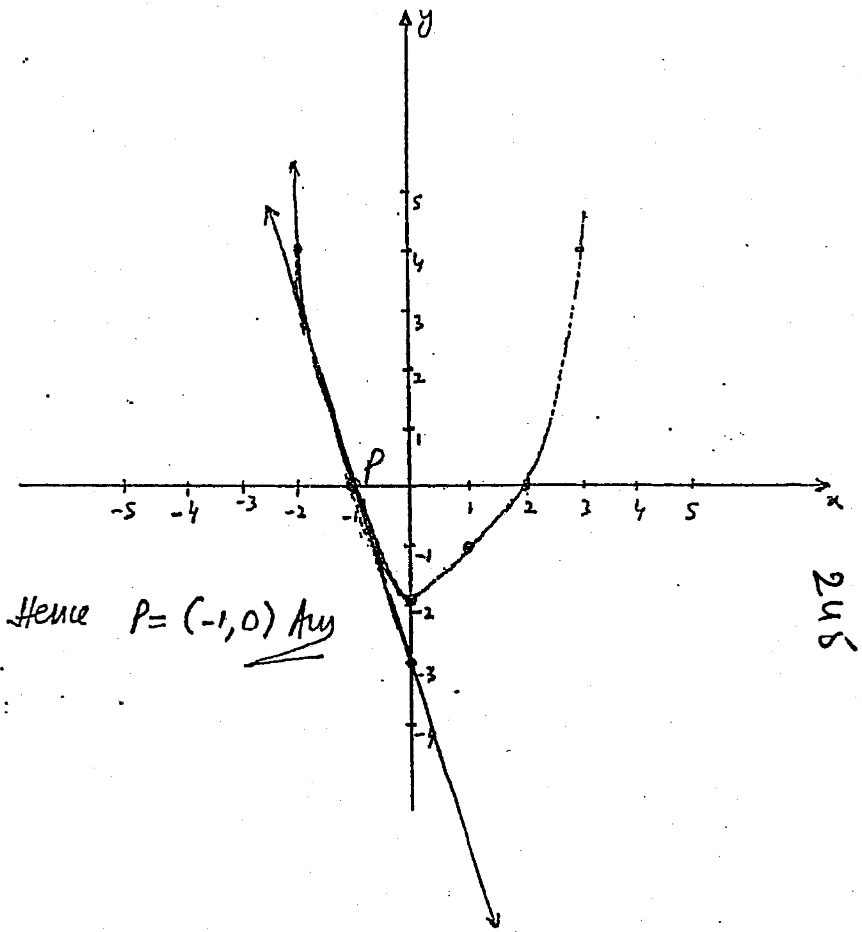
Hence points of intersection are $(-1,-3)$ & $(-2,-1)$ Ans

(c) $f(x) = x^2 - x - 2$ $g(x) = -3x - 3$

$\Rightarrow y_1 = x^2 - x - 2$ $y_2 = -3x - 3$

x	-3	-2	-1	0	1	2	3
y_1	10	4	0	-2	-1	0	4

x	-1	0	1	2
y_2	0	-3	-6	-9



Hence $P = (-1, 0)$ Ans

246

Q.8 The paths of two airplanes A and B in the plane are determined by the straight lines $2x-y=6$ and $3x+y=4$ respectively. Find the point where the two paths cross each other.

Sol Path of airplane A. = $2x-y=6 \longrightarrow$ (i)

" " " B = $3x+y=4 \longrightarrow$ (ii)

eqn (i) + eqn (ii)

$$\begin{aligned} 2x-y&=6\\ 3x+y&=4\\ \hline 5x&=10 \end{aligned} \Rightarrow x=\frac{10}{5} \Rightarrow \boxed{x=2}$$

Now $2x-y=6$

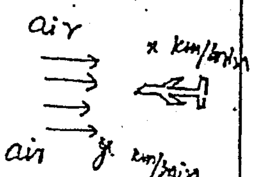
$$\Rightarrow 2(2)-y=6 \Rightarrow 4-y=6 \Rightarrow 4-6=y \Rightarrow \boxed{-2=y}$$

Hence point of intersection $=(2,-2)$

Q.9 A pilot makes a check flight in an air. Going directly into the wind, he covers a distance of 24 km in 6 minutes. Going with wind, he covers the distance in 4 minutes. Find his air speed and velocity of the wind in km/min.

Sol Let speed of airplane $= x$

& velocity of wind $= y$

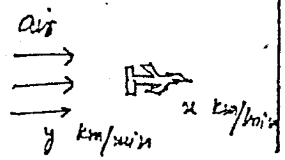


Then eqn for travelling against air

$$x-y=6 \longrightarrow (1)$$

Travelling with air

$$x+y=4 \longrightarrow (2)$$



Eqn (1) + Eqn (2)

$$\begin{aligned} x-y&=6\\ x+y&=4\\ \hline 2x&=10 \end{aligned}$$

$$\Rightarrow \boxed{x=5 \text{ km/min}} \text{ Ans}$$

i.e eqn (1) $\Rightarrow$ $x-y=6$

$5-y=6$

$5-6=y$

$\Rightarrow y=-1$ but speed can't be $-$ve

$$\Rightarrow \boxed{y=1 \text{ km/min}} \text{ Ans}$$

Hurrah! Thats the end of chapter # 08

Available at
www.mathcity.org