Exercise # 7.1

Q:1 Establish the formulae by mathematical induction.

① $1+5+9+\cdots\cdots+(4n-3)=n(2n-1)$

Sol for $n=1$

Step #01

$$4n-3=n(2n-1)$$
$$\Rightarrow 4(1)-3=1[2(1)-1]$$
$$\Rightarrow 4-3=1(2-1)$$
$$1=1 \quad \text{True for } n=1$$

Step #02: Suppose true for $n=K$, i.e

$$1+5+9+\cdots\cdots+(4K-3)=K(2K-1) \longrightarrow (i)$$

Step #03:

To prove for $n=K+1$, add $(4K+1)$ to b.s of eqn(i), we get

$$1+5+9+\cdots\cdots+(4K-3)+(4K+1)=K(2K-1)+(4K+1)$$
$$=2K^2-K+4K+1$$
$$=2K^2+3K+1$$
$$=2K^2+2K+K+1$$
$$=2K(K+1)+1(K+1)$$
$$=(K+1)(2K+1)$$

L.H.S
$4n-3$
put $K+1$
$4(K+1)-3$
$=4K+4-3$
$=4K+1$

R.H.S
$n(2n-1)$
put $K+1$
$(K+1)[2(K+1)-1]$
$=(K+1)[2K+1]$

Hence true for $n=K+1$

Hence by M.I, the formula is true for all integral values of n.

② $3+6+9+\cdots\cdots+3n=\dfrac{3n(n+1)}{2}$

Sol **Step #01:** For $n=1$

L.H.S $\qquad$ R.H.S

$$3n=\frac{3n(n+1)}{2}$$
$$3(1)=\frac{3(1)(1+1)}{2}$$
$$3=3 \quad \text{True for } n=1$$

Step #02:

Suppose true for $n=K$, i.e

$$3+6+9+\cdots\cdots+3K=\frac{3K(K+1)}{2} \longrightarrow (i)$$

Step #03:

To prove for $n=K+1$, add $(3K+3)$ to b.s of eqn(i), we get

$$3+6+9+\cdots\cdots+3K+(3K+3)=\frac{3K(K+1)}{2}+(3K+3)$$
$$=\frac{3K(K+1)}{2}+3(K+1)$$

take $3(K+1)$ as common

$$=3(K+1)\left\{\frac{K}{2}+1\right\}$$
$$=3(K+1)\left(\frac{K+2}{2}\right)$$
$$=\frac{3}{2}(K+1)(K+2)$$

Hence true for $n=K+1$.

L.H.S
$3n$
put $K+1$
$3(K+1)$
$3K+3$

R.H.S
$\frac{3K(K+1)}{2}$
put $K+1$
$\frac{3(K+1)(K+1+1)}{2}$
$\frac{3(K+1)(K+2)}{2}$

So by M.I, the formula is true for all integral values of n.

③ $5+10+15+\cdots\cdots+5n = \frac{5n(n+1)}{2}$

Step #01: for $n=1$

$$5n = \frac{5n(n+1)}{2}$$
$$5(1) = \frac{5(1)(1+1)}{2}$$
$$5=5 \text{ true for } n=1$$

Step #02:

Suppose true for $n=k$, i.e

$$5+10+15+\cdots+5k = \frac{5k(k+1)}{2} \longrightarrow (i)$$

Step #03:

To prove for $n=k+1$, add $5(k+1)$ to b.s of eqn (i), we get

$$5+10+15+\cdots+5k+5(k+1) = \frac{5k(k+1)}{2}+5(k+1)$$

take $5(k+1)$ as common

$$=5(k+1)\left\{\frac{k}{2}+1\right\}$$
$$=5(k+1)\left(\frac{k+2}{2}\right)$$
$$=\frac{5}{2}(k+1)(k+2)$$

Hence true for $n=k+1$

R.H.S $\frac{5n(n+1)}{2}$ put $k+1$ $\frac{5(k+1)(k+2)}{2}$

L.H.S $5n$ put $k+1$ $5(k+1)$

So by M.I, the formula is true for all values

④ $a+(a+d)+(a+2d)+\cdots\cdots+a+(n-1)d = \frac{n}{2}\{2a+(n-1)d\}$

Sol Step #01

for $n=1$

$$a+(n-1)d = \frac{n}{2}\{2a+(n-1)d\}$$
$$a+(1-1)d = \frac{1}{2}\{2a+(1-1)d\}$$
$$a+0d = \frac{1}{2}(2a+0d)$$
$$a = \frac{1}{2}(2a)$$
$$a=a$$

true for $n=1$

Step #02: Suppose true for $n=k$ i.e

$$a+(a+d)+(a+2d)+\cdots\cdots+a+(k-1)d = \frac{k}{2}\{2a+(k-1)d\} \longrightarrow (i)$$

Step #03: To prove for $n=k+1$, add $(a+kd)$ to b.s of equ(i)

$$a+(a+d)+\cdots\cdots+(a+(k-1)d)+(a+kd) = \frac{k}{2}\{2a+(k-1)d\}+(a+kd)$$
$$=\frac{k}{2}\{2a\}+\frac{k}{2}\{k-1\}d+a+kd$$
$$=ak+\frac{k^2d}{2}-\frac{kd}{2}+a+kd$$
$$=a+ak+\frac{k^2d}{2}+kd-\frac{kd}{2}$$
$$=a(1+k)+\frac{k^2d}{2}+\frac{2kd-kd}{2}$$
$$=a(k+1)+\frac{k^2d}{2}+\frac{kd}{2}$$
$$=a(k+1)+\frac{kd}{2}(k+1)$$

take $(k+1)$ as common

$$=(k+1)\left\{a+\frac{kd}{2}\right\}$$
$$=(k+1)\left\{\frac{2a+kd}{2}\right\}$$
$$=\frac{k+1}{2}\{2a+kd\}$$

Hence true for $n=k+1$

So by M.I, the formula is true for all values of n

⑤ $a + ar + ar^2 + \dots\dots + ar^{n-1} = \dfrac{a(r^n - 1)}{r-1}$

Step # 01: For $n = 1$

$$ar^{1-1} = \frac{a(r^1-1)}{r-1}$$

$$ar^0 = \frac{a(r-1)}{r-1}$$

$$a = a \quad (\text{true for } n=1)$$

Step # 02:

Suppose true for $n = k$, i.e

$$a + ar + ar^2 + \dots\dots + ar^{k-1} = \frac{a(r^k-1)}{r-1} \longrightarrow (i)$$

Step # 03:

To prove for $n = k+1$,

Add ar^k to b.s of eqn (i)

$$a + ar + ar^2 + \dots\dots + ar^{k-1} + ar^k = \frac{a(r^k-1)}{r-1} + ar^k$$

$$= \frac{a(r^k-1) + ar^k(r-1)}{r-1}$$

$$= \frac{ar^k - a + ar^{k+1} - ar^k}{r-1}$$

$$= \frac{-a + ar^{k+1}}{r-1}$$

$$= \frac{ar^{k+1} - a}{r-1} = a\left(\frac{r^{k+1}-1}{r-1}\right)$$

true for $n = k+1$.

Hence by M.I, the formula is true for all values of n.

⑥ $1^2 + 3^2 + 5^2 + \dots\dots + (2n-1)^2 = \dfrac{n(4n^2-1)}{3}$

Step # 01: For $n = 1$

$$(2(1)-1)^2 = \frac{1(4(1)^2-1)}{3}$$

$$1^2 = \frac{1(4-1)}{3}$$

$$1 = 1 \Rightarrow \text{True for } n = 1$$

Step # 02: suppose true for $n = k$

$$1^2 + 3^2 + 5^2 + \dots\dots + (2k-1)^2 = \frac{k(4k^2-1)}{3} \longrightarrow (i)$$

Step # 03: To prove for $n = k+1$,

add $(2k+1)^2$ to b.s of eqn (i)

$$1^2 + 3^2 + 5^2 + \dots\dots + (2k-1)^2 + (2k+1)^2 = \frac{k(4k^2-1)}{3} + (2k+1)^2$$

$$= \frac{k(2k+1)(2k-1) + 3(2k+1)^2}{3}$$

take $(2k+1)$ as common

$$= (2k+1)\frac{\{k(2k-1) + 3(2k+1)\}}{3}$$

$$= (2k+1)\frac{\{2k^2 - k + 6k + 3\}}{3}$$

$$= (2k+1)\frac{\{2k^2 + 5k + 3\}}{3}$$

$$= (2k+1)\frac{\{2k^2 + 2k + 3k + 3\}}{3}$$

$$= (2k+1)\frac{\{2k(k+1) + 3(k+1)\}}{3}$$

$$= \frac{(2k+1)\ (k+1)(2k+3)}{3}$$

R.H.S

$$\frac{n(4n^2-1)}{3}$$

put $(k+1)$

$$\frac{(k+1)\{4(k+1)^2-1\}}{3}$$

$$= \frac{(k+1)\{4(k^2+1+2k)-1\}}{3}$$

$$= \frac{(k+1)(4k^2+4+8k-1)}{3}$$

$$= \frac{(k+1)(4k^2+8k+3)}{3}$$

$$= \frac{(k+1)(2k+1)(2k+3)}{3}$$

True for $n = k+1$

Hence by M.I, the formula is true for all values

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⑦ $2^2+4^2+6^2+\cdots+(2n)^2 = \frac{2}{3}n(n+1)(2n+1)$

Step # 01: For $n=1$

$$(2(1))^2 = \frac{2}{3}(1)(1+1)(2(1)+1)$$
$$2^2 = \frac{2}{3}\cdot(2)(3)$$
$$2^2 = 2^2 \quad \text{True for } n=1$$

Step # 02: Suppose true for $n=k$

$$2^2+4^2+6^2+\cdots\cdots+(2k)^2 = \frac{2}{3}k(k+1)(2k+1) \longrightarrow (i)$$

Step # 03: To prove for $n=k+1$, add $(2(k+1))^2$ to b.s of eq (i)

$$\Rightarrow 2^2+4^2+6^2+\cdots\cdots+(2k)^2+(2(k+1))^2 = \frac{2}{3}k(k+1)(2k+1)+(2(k+1))^2$$
$$= \frac{2}{3}k(k+1)(2k+1)+4(k+1)^2$$

take $2(k+1)$ as common

$$= 2(k+1)\left\{\frac{k}{3}(2k+1)+2(k+1)\right\}$$
$$= 2(k+1)\left\{\frac{k(2k+1)+6(k+1)}{3}\right\}$$
$$= 2(k+1)\left\{\frac{2k^2+k+6k+6}{3}\right\}$$
$$= \frac{2}{3}(k+1)\{2k^2+7k+6\}$$
$$= \frac{2}{3}(k+1)\{2k^2+4k+3k+6\}$$
$$= \frac{2}{3}(k+1)\{2k(k+2)+3(k+2)\}$$
$$= \frac{2}{3}(k+1)(k+2)(2k+3)$$

True for $n=k+1$

R.H.S

$$= \frac{2}{3}n(n+1)(2n+1)$$

put $k+1$

$$\frac{2}{3}(k+1)(k+1+1)(2(k+1)+1)$$
$$= \frac{2}{3}(k+1)(k+2)(2k+3)$$

Hence by M.I, the formula is true for all values of n.

⑧ $1^3+2^3+3^3+\cdots+n^3 = \left\{\frac{n(n+1)}{2}\right\}^2$

Sol Step # 01: For $n=1$

$$1^3 = \left\{\frac{1(1+1)}{2}\right\}^2$$
$$1 = 1 \quad \text{True for } n=1$$

Step # 02: Suppose true for $n=k$

i.e $1^3+2^3+3^3+\cdots\cdots+k^3 = \left(\frac{k(k+1)}{2}\right)^2 \longrightarrow (i)$

Step # 03: To prove for $n=k+1$,

Add $(k+1)^3$ to b.s of (i)

$$1^3+2^3+3^3+\cdots+k^3+(k+1)^3 = \left(\frac{k(k+1)}{2}\right)^2+(k+1)^3$$
$$= \frac{k^2(k+1)^2}{4}+(k+1)^3$$

take $(k+1)^2$ as common

$$= (k+1)^2\left\{\frac{k^2}{4}+(k+1)\right\}$$
$$= (k+1)^2\left\{\frac{k^2+4(k+1)}{4}\right\}$$
$$= (k+1)^2\left\{\frac{k^2+4k+4}{4}\right\}$$
$$= (k+1)^2\,\frac{(k+2)^2}{2^2}$$
$$= \left\{\frac{(k+1)(k+2)}{2}\right\}^2$$

R.H.S

$$\left(\frac{n(n+1)}{2}\right)^2$$

put $k+1$

$$\left\{\frac{(k+1)(k+1+1)}{2}\right\}^2$$
$$= \left\{\frac{(k+1)(k+2)}{2}\right\}^2$$

Hence true for $n=k+1$

So by M.I, the formula is true for all integral values of n.

⑨ $1(1!) + 2(2!) + 3(3!) + \cdots\cdots + n(n!) = (n+1)! - 1$

Step # 01:

For $n=1$ $\quad 1(1!) = (1+1)! - 1$

$1(1) = 2! - 1$

$1 = 2-1$

$1 = 1$ True for $n=1$

Step # 02: Suppose true for $n=k$, i.e

$1(1!)+2(2!) + \cdots\cdots + k(k!) = (k+1)! - 1 \quad (i)$

Step # 03: To prove for $n=k+1$,

Add $(k+1)\{(k+1)!\}$ to b.s of equ(i)

$1(1!) + 2(2!) + \cdots\cdots + k(k!) + (k+1)(k+1)! = (k+1)! - 1 + (k+1)(k+1)!$

$= (k+1)! - 1 + (k+1)(k+1)!$

$= (k+1)(k+1)! + (k+1)! - 1$

$= (k+1)!\{(k+1) + 1\} - 1$

$= (k+1)!\,(k+2) - 1$

$= (k+2)(k+1)! - 1$

$= (k+2)! - 1$

$\Rightarrow$ True for $n = k+1$

Hence by M.I, the formula is true for all values of n (where $n \in \mathbb{Z}$).

Q:10 $1 + 2 + 2^2 + 2^3 + \cdots\cdots + 2^{n-1} = 2^n - 1$

Step # 01: For $n=1$

$2^{1-1} = 2^1 - 1$

$2^0 = 2-1$

$1 = 1$ true for $n=1$

Step # 02: Suppose true for $n=k$

$1 + 2 + 2^2 + \cdots + 2^{k-1} = 2^k - 1 \longrightarrow (i)$

Step # 03: To prove for $n = k+1$,

Add 2^k to b.s of equ(i)

$1 + 2 + 2^2 + \cdots + 2^{k-1} + 2^k = 2^k - 1 + 2^k$

$= 2^k + 2^k - 1$

$= 2\,2^k - 1$

$= 2^{k+1} - 1$ True for $n = k+1$

Hence by M.I, the formula is true for all values of n.

Q:11 $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}$

Sol

Step # 01: For $n=1$

$\frac{1}{1(1+1)} = \frac{1}{1+1}$

$\frac{1}{2} = \frac{1}{2}$

true for $n=1$

Step # 02: Suppose true for $n=k$

$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \cdots + \frac{1}{k(k+1)} = \frac{k}{k+1} \longrightarrow (i)$

Step # 03: To prove for $n = k+1$, Add $\frac{1}{(k+1)(k+2)}$ to b.s of equ(i), we get

$$\Rightarrow \frac{1}{1.2}+\frac{1}{2.3}+\cdots+\frac{1}{k(k+1)}+\frac{1}{(k+1)(k+2)} = \frac{k}{k+1}+\frac{1}{(k+1)(k+2)}$$

R.H.S

$\frac{n}{n+1}$

Put $k+1$

$\frac{k+1}{k+1+1}$

$= \frac{k+1}{k+2}$

$$= \frac{k(k+2)+1}{(k+1)(k+2)}$$

$$= \frac{k^2+2k+1}{(k+1)(k+2)}$$

$$= \frac{(k+1)^2}{(k+1)(k+2)}$$

$$= \frac{(k+1)}{k+2}$$

True for $n=k+1$

Hence by M.I, the formula is true for all integral values of n.

(12) $1.2+2.3+3.4+\cdots+n(n+1) = \frac{n(n+1)(n+2)}{3}$

Step # 01: For $n=1$

$$1(1+1) = \frac{1(1+1)(1+2)}{3}$$

$$2 = 2$$

true for $n=1$

Step # 02: Suppose true for $n=k$

$$1.2+2.3+3.4+\cdots+k(k+1) = \frac{k(k+1)(k+2)}{3} \longrightarrow (i)$$

Step # 03. To prove for $n=k+1$,

Add $(k+1)(k+2)$ to b.s of eqn(i)

$$1.2+2.3+3.4+\cdots+k(k+1)+(k+1)(k+2) = \frac{k(k+1)(k+2)}{3}+(k+1)(k+2)$$

take $(k+1)(k+2)$ as common

$$= (k+1)(k+2)\left\{\frac{k}{3}+1\right\}$$

$$= (k+1)(k+2)\left(\frac{k+3}{3}\right)$$

$$= \frac{(k+1)(k+2)(k+3)}{3}$$ True for $n=k+1$

Hence by M.I, the formula is true for all integral values of n.

(13) $\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\cdots+\frac{1}{3^n} = \frac{1}{2}\left\{1-\frac{1}{3^n}\right\}$

Step # 01: For $n=1$

$$\frac{1}{3^1} = \frac{1}{2}\left(1-\frac{1}{3^1}\right)$$

$$\frac{1}{3} = \frac{1}{2}\left(\frac{2}{3}\right)$$

$$\frac{1}{3} = \frac{1}{3} \Rightarrow \text{True for } n=1$$

Step # 02: Suppose true for $n=k+1$, ie

$$\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\cdots+\frac{1}{3^k} = \frac{1}{2}\left\{1-\frac{1}{3^k}\right\} \longrightarrow (i)$$

Step # 03: To prove for $n=k+1$, add $\frac{1}{3^{k+1}}$ to b.s of eqn (i)

$$\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\cdots+\frac{1}{3^k} = \frac{1}{2}\left\{1-\frac{1}{3^k}\right\}+\frac{1}{3^{k+1}}$$

$$= \frac{1}{2}\left\{\frac{3^k-1}{3^k}\right\}+\frac{1}{3.3^k}$$

take $\frac{1}{3^k}$ as common

$$= \frac{1}{3^k}\left\{\frac{3^k-1}{2}+\frac{1}{3}\right\}$$

$$= \frac{1}{3^k}\left\{\frac{3(3^k-1)+2}{2.3}\right\}$$

$$= \frac{3\cdot 3^k - 3 + 2}{2\cdot 3\cdot 3^k}$$

$$= \frac{3^{k+1} - 1}{2\cdot 3^{k+1}}$$

$$= \frac{1}{2}\left\{\frac{3^{k+1}}{3^{k+1}} - \frac{1}{3^{k+1}}\right\}$$

$$= \frac{1}{2}\left\{1 - \frac{1}{3^{k+1}}\right\} \quad \text{true for } n = k+1$$

Hence by M.I, the formula is true for all values of n.

(14) $\binom{3}{3} + \binom{4}{3} + \cdots + \binom{n+2}{3} = \binom{n+3}{4}$

Sol **Step # 01** For $n = 1$

$$\binom{1+2}{3} = \binom{1+3}{4}$$

$$\Rightarrow \binom{3}{3} = \binom{4}{4}$$

$$\Rightarrow 1 = 1 \quad \text{True for } n = 1$$

L.H.S $\binom{n+2}{3}$ put $k+1$: $\binom{k+1+2}{3} = \binom{k+3}{3}$

R.H.S $\binom{n+3}{4}$ put $k+1$: $\binom{k+1+3}{4}$

Step # 02: Suppose true for $n = k$

$$\binom{3}{3} + \binom{4}{3} + \cdots + \binom{k+2}{3} = \binom{k+3}{4}$$

Step # 03: To prove for $n = k+1$,
Add $\binom{k+3}{3}$ to b.s

$$\Rightarrow \binom{3}{3} + \binom{4}{3} + \cdots + \binom{k+2}{3} + \binom{k+3}{3} = \binom{k+3}{4} + \binom{k+3}{3}$$

By theorem ${}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r$

$$= \binom{k+3+1}{4} \quad \text{true for } n = k+1$$

Hence by M.I, the formula is true for all values of n.

(15) $\binom{5}{5} + \binom{6}{5} + \cdots + \binom{n+4}{5} = \binom{n+5}{6}$

Sol **Step # 01**: For $n = 1$

$$\binom{1+4}{5} = \binom{1+5}{6}$$

$$\Rightarrow \binom{5}{5} = \binom{6}{6}$$

$$\Rightarrow 1 = 1 \quad \text{True for } n = 1$$

Step # 02: Suppose true for $n = k$

$$\binom{5}{5} + \binom{6}{5} + \cdots + \binom{k+4}{5} = \binom{k+5}{6} \longrightarrow (i)$$

Step # 03: To prove for $n = k+1$,
Add $\binom{k+5}{5}$ to b.s of equ(i)

$$\binom{5}{5} + \binom{6}{5} + \cdots \binom{k+4}{5} + \binom{k+5}{5} = \binom{k+5}{6} + \binom{k+5}{5}$$

By theorem

$$= \binom{k+6}{6} \quad \text{True for } n = k+1$$

Hence by M.I, the formula is true for all values of n.

Q.16 $\binom{2}{2} + \binom{3}{2} + \cdots + \binom{n}{2} = \binom{n+1}{3} \qquad n \geq 2$

Sol **Step # 01**: For $n = 2$

$$\binom{2}{2} = \binom{2+1}{3}$$

$$\Rightarrow 1 = \binom{3}{3}$$

$$1 = 1$$

True for $n = 1$

Step # 02: Suppose true for $n=k$, i.e

$$\binom{2}{2}+\binom{3}{2}+\cdots+\binom{k}{2}=\binom{k+1}{3} \longrightarrow (i)$$

Step # 03: To prove for $n=k+1$,

add $\binom{k+1}{2}$ to b.s of equ (i)

$$\binom{2}{2}+\binom{3}{2}+\cdots+\binom{k}{2}+\binom{k+1}{2}=\binom{k+1}{3}+\binom{k+1}{2}$$

by theorem

$$=\binom{k+1+1}{3}$$

Hence true for $n=k+1$

So by M.I, the formula is true for all values of n

Q:17 Show by Mathematical Induction that

(i) $\frac{5^{2n}-1}{24}$ is integer

Sol **Step # 01:** For $n=1$

$$\frac{5^{2n}-1}{24}=\frac{5^{2(1)}-1}{24}=\frac{5^2-1}{24}=\frac{25-1}{24}=1$$
$$= \text{Integer}$$

Hence true for $n=1$

Step # 02: Suppose true for $n=k$, i.e

$\frac{5^{2k}-1}{24}$ is integer

Step # 03: To prove for $n=k+1$

i.e $\frac{5^{2(k+1)}-1}{24}$

$$=\frac{5^{2k+2}-1}{24}$$

$$=\frac{5^{2k}.5^2-1}{24}$$

$$=\frac{5^{2k}.25-1}{24}$$

$$=\frac{25.5^{2k}-1}{24}$$

Add and subtract 25

$$=\frac{25.5^{2k}-25+25-1}{24}$$

$$=\frac{25(5^{2k}-1)+24}{24}$$

$$=25\left(\frac{5^{2k}-1}{24}\right)+\frac{24}{24}$$

from step 2

$$=25\,(\text{Integer})+1$$

$$=\text{Integer}+\text{Integer}$$

$$=\text{Integer}$$

Hence true for $n=k+1$

So by M.I, the assertion is true for all integral values of n.

(ii) $\dfrac{10^{n+1}-9n-10}{81}$ is integer

Sol

Step # 01: For $n=1$

$$\frac{10^{1+1}-9(1)-10}{81} = \frac{10^2-9-10}{81} = \frac{100-19}{81} = \frac{81}{81} = 1 = \text{integer}$$

Hence true for $n=1$

Step # 02, Suppose true for $n=k$

i.e $\dfrac{10^{k+1}-9k-10}{81}$ is integer.

Step # 03: To prove for $n=k+1$

i.e $\dfrac{10^{k+1+1}-9(k+1)-10}{81}$

$$= \frac{10^{k+1+1}-9k-9-10}{81}$$

$$= \frac{10^{k+1}\cdot 10^1-9k-19}{81}$$

Add and subtract $90.k$

$$= \frac{10\cdot 10^{k+1}-90k+90k-9k-19}{81}$$

Add and subtract 100

$$= \frac{10.10^{k+1}-90k+90k-9k+100-100-19}{81}$$

$$= \frac{10.10^{k+1}-90k-100+90k-9k+100-19}{81}$$

$$= \frac{10.10^{k+1}-90k-100+81k+81}{81}$$

$$= \frac{10(10^{k+1}-9k-10)+81(k+1)}{81}$$

$$= 10\left(\frac{10^{k+1}-9k-10}{81}\right) + 81\left(\frac{k+1}{81}\right)$$

$= 10$ (integer) from step # 02 $+ (k+1)$

$=$ Integer + Integer

$=$ Integer. So true for $n=k+1$

Hence by M.I, the assertion is true for all integral values of n.

(iii) $\dfrac{3^{2n}-2^{2n}}{5}$ is integer

Let $f(n) = \dfrac{3^{2n}-2^{2n}}{5}$

Step # 01: for $n=1$

$$f(1) = \frac{3^{2(1)}-2^{2(1)}}{5} = \frac{9-4}{5} = 1$$

True for $n=1$

Step # 02: Suppose true for $n=k$

i.e $f(k) = \dfrac{3^{2k}-2^{2k}}{5}$ is integer

Step # 03, To prove for $n=k+1$

$$f(k+1) = \frac{3^{2(k+1)}-2^{2(k+1)}}{5}$$

$\Rightarrow \quad f(k+1) = \dfrac{3^{2k+2} - 2^{2k+2}}{5}$

$= \dfrac{3^{2k}.3^2 - 2^{2k}.2^2}{5} = \dfrac{9.3^{2k} - 4.2^{2k}}{5}$

Add and subtract 9.2^{2k}

$= \dfrac{9.3^{2k} - 9.2^{2k} + 9.2^{2k} - 4.2^{2k}}{5}$

$= \dfrac{9(3^{2k} - 2^{2k}) + 5.2^{2k}}{5}$

$= 9\left(\dfrac{3^{2k} - 2^{2k}}{5}\right) + \dfrac{5.2^{2k}}{5}$

$= 9(\text{integer}) + 2^{2k}$

$= \text{integer} + \text{integer}$

$f(k+1) = \text{integer}$

$\Rightarrow$ So by M.I, the assertion is true for all values of n $(n \in \mathbb{Z})$

$\boxed{Q:18}$ (i) $2^n > n \quad \forall\, n \in N$

Sol

Step # 01: for $n = 1$

$2^1 > 1$

$\Rightarrow 2 > 1$ True

Step # 02: Suppose true for $n = k$

i.e $2^k > k \longrightarrow$ (i)

Step # 03: for $n = k+1$

Multiply b.s of (i) by 2

$\Rightarrow 2.2^k > 2k$

$\Rightarrow 2^{k+1} > k+k$

Replace k by 1

$\Rightarrow 2^{k+1} > k+1 \Rightarrow$ True for $n = k+1$

Hence by M.I $2^n > n$ for $n \in N$.

(ii) $n! > n^2$ for $n \geq 4$ & $n! > n^3$ for $n \geq 6$

Step # 01

for $n = 4$

$4! > 4^2$

$24 > 16$ true

Step # 02 Suppose true for $n = k$

$k! > k^2$ for $k \geq 4$

Step # 03

To prove for $n = k+1$

i.e $(k+1)! > (k+1)^2$

From step # 02 $k! > k^2$

xing by $(k+1)$

$(k+1)k! > (k+1)k^2$

Replace k^2 by $k+1$

$(k+1)! > (k+1)(k+1)$

$(k+1)! > (k+1)^2$

True for $n = k+1$.

Hence by M.I, $n! > n^2$ for $n \geq 4$

Step # 01 for $n = 6$

$6! > 6^3$

$720 > 216$ True

Step # 02: suppose true for $n = k$

i.e $k! > k^3$

Step # 03: To prove for $n = k+1$

Multiply by $(k+1)$

$(k+1)k! > k^3(k+1)$

Reple k^3 by $(k+1)^2$

$\Rightarrow (k+1)! > (k+1)^2(k+1)$

$\Rightarrow (k+1)! > (k+1)^3$

true for $n = k+1$.

Hence by M.I $n! > n^3$ for $n \geq 6$

Q:19 (i) Show that 5 is a factor of $3^{2n-1}+2^{2n-1}$ where n is any positive integer.

Sol

$$P(n) = 3^{2n-1} + 2^{2n-1}$$

Step # 01 : For $n=1$

$$P(1) = 3^{2(1)-1} + 2^{2(1)-1}$$
$$= 3^{2-1} + 2^{2-1}$$
$$= 3^1 + 2^1$$
$$= 5 \quad \text{true for } n=1$$

Step # 02 : Suppose true for $n=k$ i:e

5 is factor of $3^{2k-1}+2^{2k-1} = P(k)$

Step # 03

: To prove for $n=k+1$

i:e
$$P(k+1) = 3^{2(k+1)-1} + 2^{2(k+1)-1}$$
$$= 3^{2k+2-1} + 2^{2k+2-1}$$
$$= 3^{2k-1}\cdot 3^2 + 2^{2k+1}\cdot 2^2$$
$$= 9\cdot 3^{2k-1} + 4\cdot 2^{2k-1}$$

Add and subtract $9\cdot 2^{2k-1}$

$$= 9\cdot 3^{2k-1} + 9\cdot 2^{2k-1} - 9\cdot 2^{2k+1} + 4\cdot 2^{2k-1}$$
$$\Rightarrow P(k+1) = \underbrace{9(3^{2k-1}+2^{2k-1})}_{\text{factor of } 5} - \underbrace{5\cdot 2^{2k-1}}_{\text{factor of } 5}$$

$\Rightarrow P(k+1) =$ factor of 5 $\Rightarrow$ True for $n=k+1$

Hence proved.

(ii) $2^{2n}-1$ is multiple of 3

Sol
$$P(n) = 2^{2n}-1$$

Step # 01 : For $n=1$

$$P(1) = 2^{2(1)}-1 = 2^2-1 = 3$$

Hence true for $n=1$

Step # 02 : suppose true for $n=k$, i:e

$P(k) = 2^{2k}-1$ is multiple of 3

Step # 03 : To prove for $n=k+1$

$$P(k+1) = 2^{2(k+1)}-1$$
$$= 2^{2k+2}-1$$
$$= 2^{2k}\cdot 2^2 - 1$$
$$= 4\cdot 2^{2k}-1$$

Add and subtract 4

$$P(k+1) = 4\cdot 2^{2k} - 4 + 4 - 1$$
$$P(k+1) = \underbrace{4(2^{2k}-1)}_{\text{Multiple of 3}} + \underbrace{3}_{\text{Multiple of 3}}$$

$\Rightarrow P(k+1) =$ multiple of 3.

So true for $n=k+1$

Hence by M.I $2^{2n}-1$ is multiple of 3

Exercise # 7.2

Q.1 Expand the following

(i) $(2x+y)^6$

Sol By Binomial theorem

$$(2x+y)^6 = \binom{6}{0}(2x)^6 + \binom{6}{1}(2x)^5 y^1 + \binom{6}{2}(2x)^4 y^2 + \binom{6}{3}(2x)^3 y^3 + \binom{6}{4}(2x)^2 y^4 + \binom{6}{5}(2x)^1 y^5 + \binom{6}{6} y^6$$

$$= 1(64x^6) + 6(32x^5)y + 15(16x^4)y^2 + 20(8x^3)y^3 + 15(4x^2)y^4 + 6(2x)y^5 + 1y^6$$

$$= 64x^6 + 192x^5y + 240x^4y^2 + 160x^3y^3 + 60x^2y^4 + 12xy^5 + y^6$$

(ii) $(x-\frac{1}{x})^7$

Sol By Binomial theorem

$$(x-\tfrac{1}{x})^7 = (x + \tfrac{-1}{x})^7$$

$$= \binom{7}{0}x^7 + \binom{7}{1}x^6\left(\tfrac{-1}{x}\right)^1 + \binom{7}{2}x^5\left(\tfrac{-1}{x}\right)^2 + \binom{7}{3}x^4\left(\tfrac{-1}{x}\right)^3 + \binom{7}{4}x^3\left(\tfrac{-1}{x}\right)^4 + \binom{7}{5}x^2\left(\tfrac{-1}{x}\right)^5 + \binom{7}{6}x^1\left(\tfrac{-1}{x}\right)^6 + \binom{7}{7}\left(\tfrac{-1}{x}\right)^7$$

$$= 1x^7 + 7x^6\left(\tfrac{-1}{x}\right) + 21x^5\left(\tfrac{1}{x^2}\right) + 35x^4\left(\tfrac{-1}{x^3}\right) + 35x^3\left(\tfrac{1}{x^4}\right) + 21x^2\left(\tfrac{-1}{x^5}\right) + 7x\left(\tfrac{1}{x^6}\right) + 1\left(\tfrac{-1}{x^7}\right)$$

$$= x^7 - 7x^5 + 21x^3 - 35x + \frac{35}{x} - \frac{21}{x^3} + \frac{7}{x^5} - \frac{1}{x^7}$$

(iii) $(3x-2y)^4$

Sol By Binomial theorem

$$(3x-2y)^4 = \{3x + (-2y)\}^4$$

$$= \binom{4}{0}(3x)^4 + \binom{4}{1}(3x)^3(-2y)^1 + \binom{4}{2}(3x)^2(-2y)^2 + \binom{4}{3}(3x)^1(-2y)^3 + \binom{4}{4}(-2y)^4$$

$$= 1(81x^4) + 4(27x^3)(-2y) + 6(9x^2)(4y^2) + 4(3x)(-8y^3) + 1(16y^4)$$

$$= 81x^4 - 216x^3y + 216x^2y^2 - 96xy^3 + 16y^4$$

(iv) $(\frac{3}{2}x - \frac{3}{x^2})^6$

Sol $(\frac{3}{2}x - \frac{3}{x^2})^6 = (\frac{3x}{2} + \frac{-3}{x^2})^6$

By Binomial theorem

$$= \binom{6}{0}\left(\tfrac{3x}{2}\right)^6 + \binom{6}{1}\left(\tfrac{3x}{2}\right)^5\left(\tfrac{-3}{x^2}\right)^1 + \binom{6}{2}\left(\tfrac{3x}{2}\right)^4\left(\tfrac{-3}{x^2}\right)^2 + \binom{6}{3}\left(\tfrac{3x}{2}\right)^3\left(\tfrac{-3}{x^2}\right)^3 + \binom{6}{4}\left(\tfrac{3x}{2}\right)^2\left(\tfrac{-3}{x^2}\right)^4 + \binom{6}{5}\left(\tfrac{3x}{2}\right)^1\left(\tfrac{-3}{x^2}\right)^5 + \binom{6}{6}\left(\tfrac{3x}{2}\right)^0\left(\tfrac{-3}{x^2}\right)^6$$

$$= 1\left(\tfrac{729x^6}{64}\right) + 6\left(\tfrac{243x^5}{32}\right)\left(\tfrac{-3}{x^2}\right) + 15\left(\tfrac{81x^4}{16}\right)\left(\tfrac{9}{x^4}\right) + 20\left(\tfrac{27x^3}{8}\right)\left(\tfrac{-27}{x^6}\right) + 15\left(\tfrac{9x^2}{4}\right)\left(\tfrac{81}{x^8}\right) + 6\left(\tfrac{3x}{2}\right)\left(\tfrac{-243}{x^{10}}\right) + 1\left(\tfrac{729}{x^{12}}\right)$$

$$= \frac{729}{64}x^6 - \frac{2187x^3}{16} + \frac{10935}{16} - \frac{14580}{8x^6} + \frac{10935}{4x^8} - \frac{2187}{x^9} + \frac{729}{x^{12}}$$

Q:2: Find the middle term(s) in the following

(i) $\left(\frac{a}{3} + 9b\right)^8$

Sol $n = 8$ which is even. Hence we have one middle term

$A = \frac{a}{3}$ $\quad \left(\frac{n+2}{2}\right)$th $= \left(\frac{8+2}{2}\right)$th $= 5$th

$B = 9b$

As $T_{r+1} = \binom{n}{r} A^{n-r} B^r$

put $r = 4$

$T_{4+1} = \binom{8}{4}\left(\frac{a}{3}\right)^{8-4}(9b)^4$

$\Rightarrow T_5 = 70\left(\frac{a}{3}\right)^4 (6561 b^4)$

$\Rightarrow T_5 = 70\left(\frac{a^4}{81}\right)(6561 b^4)$

$\Rightarrow T_5 = 5670 a^4 b^4$ Ans

(ii) $\left(3x + \frac{1}{2x}\right)^{10}$

Sol $n = 10$ so the one middle term will be $\left(\frac{n+2}{2}\right)$th

$a = 3x$ $\quad = \left(\frac{10+2}{2}\right)$th

$b = \frac{1}{2x}$ $\quad = 6$ th

put $r = 5$

Now $T_{r+1} = \binom{n}{r} a^{n-r} b^r$

$\Rightarrow T_{5+1} = \binom{10}{5}(3x)^{10-5}\left(\frac{1}{2x}\right)^5$

$\Rightarrow T_6 = (252)(3x)^5\left(\frac{1}{32x^5}\right)$

$\Rightarrow T_6 = (252)(243x^5)\frac{1}{32x^5}$

$\Rightarrow T_6 = 15309/16$ Ans

(iii) $\left(x^4 - \frac{1}{x^3}\right)^{11} = \left(x^4 + \frac{-1}{x^3}\right)^{11}$

Sol $n = 11$ which is odd $\Rightarrow$ we have two middle terms

$a = x^4$ $\quad \left(\frac{n+1}{2}\right)$th and $\left(\frac{n+3}{2}\right)$th

$b = \frac{-1}{x^3}$ $\quad = \left(\frac{11+1}{2}\right)$th and $\left(\frac{11+3}{2}\right)$th

$= 6$ th and 7 th

Now $T_{r+1} = \binom{n}{r} a^{n-r} b^r$

put $r = 5$

$\Rightarrow T_{5+1} = \binom{11}{5}(x^4)^{11-5}\left(-\frac{1}{x^3}\right)^5$

$\Rightarrow T_6 = 462(x^4)^6\left(-\frac{1}{x^{15}}\right)$

$\Rightarrow T_6 = 462x^{24}\left(-\frac{1}{x^{15}}\right)$

$\Rightarrow T_6 = -462x^9$ Ans

Now put $r = 6$

$T_{6+1} = \binom{11}{6}(x^4)^{11-6}\left(\frac{-1}{x^3}\right)^6$

$T_7 = 462(x^4)^5\left(\frac{1}{x^{18}}\right)$

$T_7 = 462x^{20}\left(\frac{1}{x^{18}}\right)$

$T_7 = 462x^2$ Ans

Q:3 Find the coefficient of

(i) x^9 in $\left(x + \frac{3a}{x^2}\right)^{15}$

Sol $\left(x + \frac{3a}{x^2}\right)^{15}$

$a = x$, $b = \frac{3a}{x^2}$, $n = 15$

By formula

$T_{r+1} = \binom{n}{r} a^{n-r} \cdot b^r$

$\Rightarrow T_{r+1} = \binom{15}{r}(x)^{15-r}\left(\frac{3a}{x^2}\right)^r$

$\Rightarrow T_{r+1} = \binom{15}{r} x^{15-r}\frac{3^r a^r}{x^{2r}}$

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$\Rightarrow T_{r+1} = \binom{15}{r} x^{15-r} \cdot x^{-2r} \cdot 3^r \cdot a^r$

$\Rightarrow T_{r+1} = \binom{15}{r} x^{15-r-2r} \cdot 3^r \cdot a^r$

$\Rightarrow T_{r+1} = \binom{15}{r} \cdot x^{15-3r} \cdot 3^r \cdot a^r$

put $15-3r = 9$

$\Rightarrow 15-9 = 3r$

$\Rightarrow 6 = 3r \Rightarrow \boxed{r=2}$

$\Rightarrow T_{2+1} = \binom{15}{2} x^{15-3(2)} \cdot 3^2 \cdot a^2$

$\Rightarrow T_3 = 105\, x^{15-6} \cdot 9 \cdot a^2$

$\Rightarrow T_3 = 945\, a^2 x^9$

Hence coefficient of x^9 is $945 a^2$

(ii) x^5 in $\left(2x^2 - \frac{1}{3x}\right)^{10}$

Sol

$a = 2x^2$

$b = -\frac{1}{3x}$

$n = 10$

As $T_{r+1} = \binom{n}{r} a^{n-r} b^r$

$\Rightarrow T_{r+1} = \binom{10}{r} (2x^2)^{10-r} \left(-\frac{1}{3x}\right)^r$

$\Rightarrow T_{r+1} = \binom{10}{r} 2^{10-r} (x^2)^{10-r} \frac{(-1)^r}{3^r x^r}$

$\Rightarrow T_{r+1} = \binom{10}{r} 2^{10-r} x^{20-2r} \cdot x^{-r} \cdot \frac{(-1)^r}{3^r}$

$\Rightarrow T_{r+1} = \binom{10}{r} 2^{10-r} x^{20-3r} \frac{(-1)^r}{3^r}$

put $20-3r = 5$

$\Rightarrow 15 = 3r \Rightarrow \boxed{5=r}$

$\Rightarrow T_{5+1} = \binom{10}{5} 2^{10-5} x^{20-3(5)} \frac{(-1)^5}{3^5}$

$\Rightarrow T_6 = 252\,(2^5) \cdot x^5 \frac{(-1)}{243}$

$\Rightarrow T_6 = -\frac{252 \times 32}{243} x^5$

$\Rightarrow T_6 = -33.1852 x^5$

Hence coefficient of x^5 is -33.1852.

(iii) x in $\left(2x^2 - \frac{1}{x}\right)^{12}$

Sol

$a = 2x^2, \quad b = -\frac{1}{x}, \quad n = 12$

As $T_{r+1} = \binom{n}{r} a^{n-r} b^r$

$\Rightarrow T_{r+1} = \binom{12}{r} (2x^2)^{12-r} \left(-\frac{1}{x}\right)^r$

$\Rightarrow T_{r+1} = \binom{12}{r} 2^{12-r} (x^2)^{12-r} \frac{(-1)^r}{x^r}$

$\Rightarrow T_{r+1} = \binom{12}{r} 2^{12-r} x^{24-2r} \cdot x^{-r} (-1)^r$

$\Rightarrow T_{r+1} = \binom{12}{r} 2^{12-r} x^{24-3r} (-1)^r$

put $24-3r = 1 \Rightarrow 23 = 3r \Rightarrow \boxed{r = 23/3}$ which is not possible

Hence the expansion does not contain a term having x^1. So we take $0x$

$\Rightarrow$ coefficient of x is 0.

Q:4: Find the term independent of x in

(i) $\left(x+\frac{1}{2x}\right)^8$

Sol $a=x,\ b=\frac{1}{2x},\ n=8$

As $T_{r+1}=\binom{n}{r}a^{n-r}b^r$

$\Rightarrow T_{r+1}=\binom{8}{r}(x)^{8-r}\left(\frac{1}{2x}\right)^r$

$\Rightarrow T_{r+1}=\binom{8}{r}x^{8-r}\frac{1}{2^r x^r}$

$\Rightarrow T_{r+1}=\binom{8}{r}x^{8-r}.\,x^{-r}.\,\frac{1}{2^r}$

$\Rightarrow T_{r+1}=\binom{8}{r}x^{8-2r}\left(\frac{1}{2^r}\right)$ Put $8-2r=0$

$\Rightarrow T_{4+1}=\binom{8}{4}x^{8-2(4)}\frac{1}{2^4}$ $\Rightarrow 8=2r$

$\Rightarrow r=4$

$\Rightarrow T_5=70\,x^0\,\frac{1}{16}\Rightarrow T_5=\frac{70}{16}\Rightarrow T_5=\frac{35}{8}$ Ans

(ii) $\left(2x^2-\frac{1}{x^3}\right)^{10}$

Sol $a=2x^2,\ b=-\frac{1}{x^3},\ n=10$

As $T_{r+1}=\binom{n}{r}a^{n-r}b^r$

$\Rightarrow T_{r+1}=\binom{10}{r}(2x^2)^{10-r}\left(-\frac{1}{x^3}\right)^r$

$\Rightarrow T_{r+1}=\binom{10}{r}2^{10-r}(x^2)^{10-r}\frac{(-1)^r}{x^{3r}}$

$\Rightarrow T_{r+1}=\binom{10}{r}2^{10-r}x^{20-2r}.\,x^{-3r}(-1)^r$

$\Rightarrow T_{r+1}=\binom{10}{r}2^{10-r}x^{20-5r}(-1)^r$

Put $20-5r=0$

$\Rightarrow 20=5r\Rightarrow \boxed{4=r}$

$\Rightarrow T_{4+1}=\binom{10}{4}2^{10-4}\,.\,x^{20-5(4)}(-1)^4$

$\Rightarrow T_5=210\,(2^6)\,x^0\,(+1)$

$\Rightarrow T_5=210\,(64)\Rightarrow \boxed{T_5=13440}$ Ans

(iii) $\left(2x^2+\frac{1}{x}\right)^9$

Sol $a=2x^2,\ b=\frac{1}{x},\ n=9$

$T_{r+1}=\binom{n}{r}a^{n-r}b^r$

$\Rightarrow T_{r+1}=\binom{9}{r}(2x^2)^{9-r}\left(\frac{1}{x}\right)^r$

$\Rightarrow T_{r+1}=\binom{9}{r}2^{9-r}x^{18-2r}\frac{1}{x^r}$

$\Rightarrow T_{r+1}=\binom{9}{r}2^{9-r}x^{18-2r}.\,x^{-r}$

$\Rightarrow T_{r+1}=\binom{9}{r}2^{9-r}x^{18-3r}$ Put $18-3r=0$

$\Rightarrow 18=3r\Rightarrow\boxed{6=r}$

$\Rightarrow T_{6+1}=\binom{9}{6}2^{9-6}x^{18-3(6)}$

$\Rightarrow T_7=84\,(2^3)\,.\,x^{18-18}$

$\Rightarrow T_7=84\,(8)\,x^0\Rightarrow\boxed{T_7=672}$ Ans

Q:5 Find

(i) $(1+2x-x^2)^4$

Sol $(1+2x-x^2)^4=\{(1+2x)+(-x^2)\}^4$

By Binomial theorem

$=\binom{4}{0}(1+2x)^4+\binom{4}{1}(1+2x)^3(-x^2)^1+\binom{4}{2}(1+2x)^2(-x^2)^2$
$+\binom{4}{3}(1+2x)^1(-x^2)^3+\binom{4}{4}(-x^2)^4$

$$= 1(1+2x)^4 + 4(1+2x)^3(-x^2)^1 + 6(1+2x)^2(-x^2)^2$$
$$+ 4(1+2x)(-x^2)^3 + 1(-x^2)^4$$
$$= (1+2x)^4 + 4(1+2x)^3(-x^2) + 6(1+2x)^2(x^4)$$
$$+ 4(1+2x)(-x^6) + x^8$$

Again by Binomial theorem

$$= \left\{\binom{4}{0} + \binom{4}{1}(2x)^1 + \binom{4}{2}(2x)^2 + \binom{4}{3}(2x)^3 + \binom{4}{4}(2x)^4\right\}$$
$$+ 4(1 + 8x^3 + 6x + 12x^2)(-x^2) + 6(1 + 4x^2 + 4x)x^4$$
$$+ 4(1+2x)(-x^6) + x^8$$
$$= \{1 + 4(2x) + 6(4x^2) + 4(8x^3) + 1(16x^4)\}$$
$$+ 4(-x^2 - 8x^5 - 6x^3 - 12x^4) + 6(x^4 + 4x^6 + 4x^5)$$
$$+ 4(-x^6 - 2x^7) + x^8$$
$$= (1 + 8x + 24x^2 + 32x^3 + 16x^4) + (-4x^2 - 32x^5 - 24x^3 - 48x^4)$$
$$+ (6x^4 + 24x^6 + 24x^5) + (-4x^6 - 8x^7) + x^8$$
$$= 1 + 8x + 24x^2 - 4x^2 + 32x^3 - 24x^3 + 16x^4 - 48x^4 + 6x^4 - 32x^5 + 24x^5$$
$$+ 24x^6 - 4x^6 - 8x^7 + x^8$$
$$= 1 + 8x + 20x^2 + 8x^3 - 26x^4 - 8x^5 + 20x^6 - 8x^7 + x^8$$

(ii) $(\sqrt{2}+1)^5 - (\sqrt{2}-1)^5$

Sol $(\sqrt{2}+1)^5 - (\sqrt{2}-1)^5$

$$= \left[\binom{5}{0}(\sqrt{2})^5 + \binom{5}{1}(\sqrt{2})^4 + \binom{5}{2}(\sqrt{2})^3 + \binom{5}{3}(\sqrt{2})^2 + \binom{5}{4}(\sqrt{2})^1 + \binom{5}{5}\right]$$
$$- \left[\binom{5}{0}(\sqrt{2})^5 - \binom{5}{1}(\sqrt{2})^4 + \binom{5}{2}(\sqrt{2})^3 - \binom{5}{3}(\sqrt{2})^2 + \binom{5}{4}(\sqrt{2})^1 - \binom{5}{5}\right]$$
$$= \left[1(\sqrt{2})^5 + 5(\sqrt{2})^4 + 10(\sqrt{2})^3 + 10(\sqrt{2})^2 + 5(\sqrt{2}) + 1\right]$$
$$+ \left[-1(\sqrt{2})^5 + 5(\sqrt{2})^4 - 10(\sqrt{2})^3 + 10(\sqrt{2})^2 - 5\sqrt{2} + 1\right]$$
$$= 10(\sqrt{2})^4 + 20(\sqrt{2})^2 + 2$$
$$= 10(4) + 20(2) + 2$$
$$= 82 \text{ Ans}$$

(iii) $(a+b)^5 + (a-b)^5$

Sol $(a+b)^5 + (a-b)^5$

$$= \left\{\binom{5}{0}a^5 + \binom{5}{1}a^4b^1 + \binom{5}{2}a^3b^2 + \binom{5}{3}a^2b^3 + \binom{5}{4}ab^4 + \binom{5}{5}b^5\right\}$$
$$+ \left\{\binom{5}{0}a^5 - \binom{5}{1}a^4b + \binom{5}{2}a^3b^2 - \binom{5}{3}a^2b^3 + \binom{5}{4}ab^4 - \binom{5}{5}b^5\right\}$$
$$= \{1a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + 1b^5\}$$
$$+ \{1a^5 - 5a^4b + 10a^3b - 10a^2b^3 + 5ab^4 - 1b^5\}$$
$$= 2a^5 + 20a^3b + 10ab^4 \text{ Ans}$$

Q.6 Find the numerically greatest term in $(3-2x)^{10}$ when $x = 3/4$

Sol $(3-2x)^{10}$

$$= \left\{3 - 2\left(\tfrac{3}{4}\right)\right\}^{10}$$
$$= \left(3 - \tfrac{3}{2}\right)^{10} = \left\{3\left(1 - \tfrac{1}{2}\right)\right\}^{10}$$
$$= 3^{10}\left\{\left(1 + \tfrac{-1}{2}\right)^{10}\right\}$$

By Binomial theorem

$= 3^{10}\left\{\binom{10}{0}+\binom{10}{1}\left(\frac{-1}{2}\right)^1+\binom{10}{2}\left(\frac{-1}{2}\right)^2+\binom{10}{3}\left(\frac{-1}{2}\right)^3+\binom{10}{4}\left(\frac{-1}{2}\right)^4+\binom{10}{5}\left(\frac{-1}{2}\right)^5+\cdots\cdots\right\}$

$= 3^{10}\left\{1+10\left(\frac{-1}{2}\right)+45\left(\frac{1}{4}\right)+120\left(\frac{-1}{8}\right)+210\left(\frac{1}{16}\right)+252\left(\frac{-1}{32}\right)\cdots\right\}$

$= 3^{10}\{1-5+11.25-15+13.12-7.87+\cdots\}$

It is clear that 4th term is numerically greatest.

Q:7 Find the greatest term in $\left(1+\frac{1}{2}x\right)^{12}$ when $x=1/2$

Sol $\left(1+\frac{1}{2}x\right)^{12}$ when $x=\frac{1}{2}$

$= \left(1+\frac{1}{2}\cdot\frac{1}{2}\right)^{12}$

$= \left(1+\frac{1}{4}\right)^{12}$ By Binomial expansion

$= \binom{12}{0}+\binom{12}{1}\left(\frac{1}{4}\right)^1+\binom{12}{2}\left(\frac{1}{4}\right)^2+\binom{12}{3}\left(\frac{1}{4}\right)^3+\binom{12}{4}\left(\frac{1}{4}\right)^4+\binom{12}{5}\left(\frac{1}{4}\right)^5+\cdots$

$= 1+12\left(\frac{1}{4}\right)+66\left(\frac{1}{16}\right)+220\left(\frac{1}{64}\right)+495\left(\frac{1}{256}\right)+729\left(\frac{1}{1024}\right)+\cdots$

$= 1+3+4.125+3.43+1.93+0.71+\cdots$

From the terms, it is clear that 3rd term is greatest.

Q:8 Find what is mentioned.

(i) $(\sqrt{x}+y^2)^8$; term containing $x^{5/2}$

Sol $a=\sqrt{x}$, $b=y^2$ and $n=8$

Now $T_{r+1}=\binom{n}{r}a^{n-r}b^r$

$\Rightarrow T_{r+1}=\binom{8}{r}(\sqrt{x})^{8-r}(y^2)^r$

$\Rightarrow T_{r+1}=\binom{8}{r}\left(x^{\frac{1}{2}}\right)^{8-r}y^{2r}$

$\Rightarrow T_{r+1}=\binom{8}{r}x^{\frac{8-r}{2}}y^{2r}$

put $\frac{8-r}{2}=\frac{5}{2} \Rightarrow 8-r=5 \Rightarrow r=3$

Then $T_{3+1}=\binom{8}{3}x^{\frac{8-3}{2}}y^{2(3)}$

$T_4=56x^{5/2}y^6$ Ans

(ii) $\left(x^2-\frac{y^3}{2}\right)^{17}$; term containing y^{15}.

Sol $a=x^2$

$b=\frac{-y^3}{2}$ and $n=17$

By formula $T_{r+1}=\binom{n}{r}a^{n-r}b^r$

$\Rightarrow T_{r+1}=\binom{17}{r}(x^2)^{17-r}\left(\frac{-y^3}{2}\right)^r$

$\Rightarrow T_{r+1}=\binom{17}{r}x^{34-2r}(-)^r\left(\frac{y^{3r}}{2^r}\right)$

put $3r=15 \Rightarrow r=5$

So $T_{5+1}=\binom{17}{5}x^{34-2(5)}(-)^5\frac{y^{3(5)}}{2^5}$

$\Rightarrow T_6=6188\,x^{24}(-)\frac{y^{15}}{32}$

$\Rightarrow T_6=-193.375\,x^{24}y^{15}$ Ans

(iii) $\left(3x-\frac{5}{x^3}\right)^8$; term independent of x

Sol $a=3x$, $b=\frac{-5}{x^3}$, $n=8$

As $T_{r+1}=\binom{n}{r}a^{n-r}b^r$

$\Rightarrow T_{r+1}=\binom{8}{r}(3x)^{8-r}\left(\frac{-5}{x^3}\right)^r$

$\Rightarrow T_{r+1}=\binom{8}{r}3^{8-r}x^{8-r}\frac{(-5)^r}{x^{3r}}$

$\Rightarrow T_{r+1} = \binom{8}{r} 3^{8-r} x^{8-r} x^{-3r} (-5)^r$

$\Rightarrow T_{r+1} = \binom{8}{r} 3^{8-r} x^{8-4r} (-5)^r$

put $8-4r=0 \Rightarrow 8=4r \Rightarrow \boxed{2=r}$

Then $T_{2+1} = \binom{8}{2} 3^{8-2} x^{8-4(2)} (-5)^2$

$\Rightarrow T_3 = 28(3)^6 x^{8-8} (-25)$

$\Rightarrow T_3 = 28(729) x^0 (-25)$

$\Rightarrow T_3 = 510300$ Ans

Q:9 Expand $(1.04)^5$ upto four decimal places.

Sol $(1.04)^5 = (1+0.04)^5$

By Binomial theorem.

$= \binom{5}{0} + \binom{5}{1}(0.04)^1 + \binom{5}{2}(0.04)^2 + \binom{5}{3}(0.04)^3 + \cdots$

$= 1 + 5(0.04) + 10(0.0016) + 10(0.000064) + \cdots$

$= 1 + 0.2 + 0.016 + 0.00064 + \cdots$

$= 1.21664\ldots$

Hence upto 4 decimal places $(1.04)^5 = 1.2166$ Ans

Q:10 Show that the sum of coefficients in the expansion $(1+x)^n$ is 2^n, where $n \in N$ and hence show that the sum of coefficients in the expansion $(1+x)^7$ is 128.

Sol By Binomial theorem

$(1+x)^n = \binom{n}{0} + \binom{n}{1}x^1 + \binom{n}{2}x^2 + \binom{n}{3}x^3 + \binom{n}{4}x^4 + \cdots \binom{n}{n}x^n$

put $x=1$

$\Rightarrow (1+1)^n = \binom{n}{0} + \binom{n}{1}(1)^1 + \binom{n}{2}(1)^2 + \cdots + \binom{n}{n}(1)^n$

$\Rightarrow 2^n = \binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{n}$

Hence 2^n = Sum of coefficients.

2nd part

For $(1+x)^n$ sum of coefficients $= 2^n$

$\Rightarrow$ for $(1+x)^7$ " " " $= 2^7 = 128$ Ans

Q:11 Show that the sum of odd coefficients is equal to the sum of even coefficients in the binomial expansion $(1+x)^n$ and each of them is equal to 2^{n-1}.

Sol By Binomial expansion

$(1+x)^n = \binom{n}{0} + \binom{n}{1}x^1 + \binom{n}{2}x^2 + \binom{n}{3}x^3 + \binom{n}{4}x^4 + \cdots + \binom{n}{n}x^n$

Put $x=-1$

$\Rightarrow (1-1)^n = \binom{n}{0} + \binom{n}{1}(-1)^1 + \binom{n}{2}(-1)^2 + \binom{n}{3}(-1)^3 + \cdots$

$\Rightarrow 0^n = \binom{n}{0} + \binom{n}{1}(-1) + \binom{n}{2}(1) + \binom{n}{3}(-1) + \cdots$

$\Rightarrow 0 = \binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + \cdots$

$\Rightarrow \binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \cdots = \binom{n}{0} + \binom{n}{2} + \binom{n}{4} + \cdots$

sum of odd coefficients = sum of even coefficients

2nd part,

As $\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \binom{n}{3} + \cdots = 2^n$

$\Rightarrow$ Sum of coefficients $= 2^n$

$\Rightarrow$ (Sum of Even coefficients) + (sum of odd coefficients) $= 2^n$

$\Rightarrow$ sum of odd coefficients + sum of odd coefficients $= 2^n$

$\Rightarrow$ 2 sum of odd coefficients $= 2^n$

$\Rightarrow$ sum of odd coefficients $= \frac{2^n}{2}$

$\Rightarrow$ sum of odd coefficients $= 2^n \cdot 2^{-1}$

$= 2^{n-1}$

$=$ sum of even coefficients.

Q:12 Considre $(1+x)^n$ and take $\binom{n}{r} = C_r$, show that

$C_1 + 2C_2x + 3C_3x^2 + \cdots + nC_nx^{n-1} = n(1+x)^{n-1}$

Sol L.H.S $C_1 + 2C_2x + 3C_3x^2 + \cdots + nC_nx^{n-1}$

$\Rightarrow {}^nC_1 + 2\,{}^nC_2x + 3\,{}^nC_3x^2 + \cdots + n\,{}^nC_nx^{n-1}$

$\Rightarrow \frac{n!}{(n-1)!1!} + 2\frac{n!}{(n-2)!2!}x + 3\frac{n!}{(n-3)!3!}x^2 + \cdots + n\frac{n!}{(n-n)!n!}x^{n-1}$

$= \frac{n(n-1)!}{(n-1)!} + \frac{2n(n-1)(n-2)!x}{(n-2)!\,2\times1} + \frac{3n(n-1)(n-2)(n-3)!\,x^2}{(n-3)!\,3\times2\times1} + \cdots\; n\,x^{n-1}$

$= n + n(n-1)x + \frac{n(n-1)(n-2)}{2}x^2 + \cdots + nx^{n-1}$

$= n\left\{1 + (n-1)x + \frac{(n-1)(n-2)}{2!}x^2 + \cdots + x^{n-1}\right\}$

$= n(1+x)^{n-1}$

$=$ R.H.S

Exercise # 7.3

Q:1: Find the 1st four terms in the expansion of

(i) $(1-x)^{-\frac{1}{2}}$

Sol By Binomial expansion

$\{1+(-x)\}^{-\frac{1}{2}} = 1 + \frac{-1}{2}(-x) + \frac{-1}{2}\left(\frac{-1}{2}-1\right)\frac{1}{2!}(-x)^2 + \frac{-1}{2}\left(\frac{-1}{2}-1\right)\left(\frac{-1}{2}-2\right)\frac{1}{3!}(-x)^3 + \cdots$

$= 1 + \frac{x}{2} + \frac{-1}{2}\left(\frac{-3}{2}\right)\frac{1}{2}x^2 + \frac{-1}{2}\left(\frac{-3}{2}\right)\left(\frac{-5}{2}\right)\frac{1}{3\times2\times1}(-x^3) \cdots$

$= 1 + \frac{x}{2} + \frac{3x^2}{8} + \frac{5x^3}{16} + \cdots$

(ii) $(1-x)^{3/2}$

Sol $(1-x)^{3/2} = 1 + \frac{3}{2}(-x) + \frac{3}{2}\left(\frac{3}{2}-1\right)\frac{1}{2!}(-x)^2 + \frac{3}{2}\left(\frac{3}{2}-1\right)\left(\frac{3}{2}-2\right)\frac{1}{3!}(-x)^3 + \cdots$

$= 1 - \frac{3}{2}x + \frac{3}{2}\left(\frac{1}{2}\right)\frac{1}{2}(+x^2) + \frac{3}{2}\left(\frac{1}{2}\right)\left(\frac{-1}{4}\right)\frac{1}{3\times2\times1}(-x^3) + \cdots$

$= 1 - \frac{3}{2}x + \frac{3x^2}{8} + \frac{x^3}{16} + \cdots$

(iii) $(8+12x)^{2/3}$

Sol $(8+12x)^{2/3} = \left\{8 + \frac{8}{8}(12x)\right\}^{2/3} = \left\{8\left(1 + \frac{12}{8}x\right)\right\}^{2/3} = 8^{2/3}\left(1+\frac{3}{2}x\right)^{2/3}$

$= 2^2\left\{1 + \frac{2}{3}\left(\frac{3}{2}x\right) + \frac{2}{3}\left(\frac{2}{3}-1\right)\frac{1}{2!}\left(\frac{3}{2}x\right)^2 + \frac{2}{3}\left(\frac{2}{3}-1\right)\left(\frac{2}{3}-2\right)\frac{1}{3!}\left(\frac{3}{2}x\right)^3 + \cdots\right\}$

$= 4\left\{1 + x + \frac{2}{3}\left(-\frac{1}{3}\right)\frac{9x^2}{4} + \frac{2}{3}\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)\frac{1}{3\times2\times1}\frac{27x^3}{8} + \cdots\right\}$

$= 4\left\{1 + x - \frac{x^2}{2} + \frac{x^3}{6} + \cdots\right\}$

$= 4 + 4x - \frac{4x^2}{12} + \frac{4x^3}{6} + \cdots$

$= 4 + 4x - \frac{x^2}{3} + \frac{2x^3}{3} + \cdots$

(iv) $(4-8x)^{-3/2}$

Sol $(4-8x)^{-3/2} = \{4(1-2x)\}^{-3/2} = (4)^{-3/2}(1-2x)^{-3/2}$

$= 2^{-3}\left(1 + \frac{-3}{2}(-2x) + \frac{-3}{2}\left(\frac{-3}{2}-1\right)\frac{1}{2!}(-2x)^2 + \frac{-3}{2}\left(\frac{-3}{2}-1\right)\left(\frac{-3}{2}-2\right)\frac{1}{3!}(-2x)^3 \cdots\cdots\right\}$

$= \frac{1}{8}\left(1 + 3x + \frac{-3}{2}\left(\frac{-5}{2}\right)\frac{1}{2}(4x^2) + \frac{-3}{2}\left(\frac{-5}{2}\right)\left(\frac{-7}{2}\right)\left(\frac{1}{3\times2\times1}\right)(-8x^3) + \cdots\right\}$

$= \frac{1}{8}\left\{1 + 3x + \frac{15x^2}{2} + \frac{35x^3}{2} + \cdots\cdots\right\}$

$= \frac{1}{8} + \frac{3}{8}x + \frac{15}{16}x^2 + \frac{35}{16}x^3 + \cdots\cdots$ Ans

(v) $(1-x)^{-3}$

Sol $(1-x)^{-3} = 1 + (-3)(-x) + (-3)(-3-1)\frac{1}{2!}(-x)^2 + (-3)(-3-1)(-3-2)\frac{1}{3!}(-x)^3 + \cdots$

$= 1 + 3x + (-3)(-4)\frac{1}{2}(x^2) + (-3)(-4)(-5)\frac{1}{3\times2\times1}(-x^3) + \cdots$

$= 1 + 3x + 6x^2 + 10x^3 + \cdots\cdots$ Ans

(vi) $\sqrt[3]{1+x}$

Sol $(1+x)^{1/3}$

$= 1 + \frac{1}{3}(x) + \frac{1}{3}\left(\frac{1}{3}-1\right)\frac{1}{2!}(x)^2 + \frac{1}{3}\left(\frac{1}{3}-1\right)\left(\frac{1}{3}-2\right)\frac{1}{3!}(x)^3 + \cdots\cdots$

$= 1 + \frac{x}{3} + \frac{1}{3}\left(\frac{-2}{3}\right)\frac{1}{2}x^2 + \frac{1}{3}\left(\frac{-2}{3}\right)\left(\frac{-5}{3}\right)\frac{1}{3\times2\times1}x^3 + \cdots\cdots$

$= 1 + \frac{x}{3} - \frac{9x^2}{2} + \frac{5x^3}{81} + \cdots\cdots$ Ans

Q:2 Find $\sqrt{26}$ correct to 3 decimal places

Sol $\sqrt{26} = (26)^{1/2}$

$= (25+1)^{1/2}$

$= \left\{25\left(1+\frac{1}{25}\right)\right\}^{1/2}$

$= (25)^{1/2}\left(1+\frac{1}{25}\right)^{1/2}$

$= 5\left(1 + \frac{1}{2}\left(\frac{1}{25}\right) + \frac{1}{2}\left(\frac{1}{2}-1\right)\frac{1}{2!}\left(\frac{1}{25}\right)^2 + \frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)\frac{1}{3!}\left(\frac{1}{25}\right)^3 + \cdots\right)$

$= 5\left(1 + \frac{1}{50} + \frac{1}{2}\left(\frac{-1}{2}\right)\frac{1}{2}\left(\frac{1}{625}\right) + \frac{1}{2}\left(\frac{-1}{2}\right)\left(\frac{-3}{2}\right)\frac{1}{3\times2}\frac{1}{15625} + \cdots\right)$

(ii) $\frac{1}{\sqrt{0.938}}$ to four significant figures

(iii) Find the cube roots of 126 correct upto five decimal places.

Sol $\sqrt[3]{126} = (126)^{1/3}$

$= (125+1)^{1/3}$

$= \left\{125\left(1+\frac{1}{125}\right)\right\}^{1/3}$

$= (125)^{\frac{1}{3}}\left(1+\frac{1}{125}\right)^{1/3}$

$= (5^3)^{\frac{1}{3}}\left\{1+\frac{1}{3}\left(\frac{1}{125}\right)+\frac{1}{3}\left(\frac{1}{3}-1\right)\frac{1}{2!}\left(\frac{1}{125}\right)^2+\frac{1}{3}\left(\frac{1}{3}-1\right)\left(\frac{1}{3}-2\right)\frac{1}{3!}\left(\frac{1}{125}\right)^3\cdots\right\}$

$= 5\left\{1+\frac{1}{375}+\frac{1}{3}\left(\frac{-2}{3}\right)\frac{1}{2}\,\frac{1}{15625}+\frac{1}{3}\left(\frac{-2}{3}\right)\left(\frac{-5}{3}\right)\frac{1}{6}\left(\frac{1}{1953125}\right)\cdots\right\}$

$= 5\left\{1+\frac{1}{375}-\frac{1}{140625}+\frac{10}{316406250}\cdots\right\}$

$= 5\{1+0.002666-0.000007111\cdots\}$

$= 5\{1.002658\cdots\} = 5.01329$ Ans

(iv) Evaluate $\sqrt[4]{65}$ upto four decimal places.

Sol $\sqrt[4]{65} = (65)^{1/4}$

$= (64+1)^{1/4} = \left\{64\left(1+\frac{1}{64}\right)\right\}^{1/4} = (64)^{1/4}\left(1+\frac{1}{64}\right)^{1/4}$

By Binomial expansion:

$= (2^6)^{\frac{1}{4}}\left\{1+\frac{1}{4}\left(\frac{1}{64}\right)+\frac{1}{4}\left(\frac{1}{4}-1\right)\frac{1}{2!}\left(\frac{1}{64}\right)^2+\cdots\cdots\right\}$

$= 2^{3/2}\left\{1+\frac{1}{256}+\frac{1}{4}\left(\frac{-3}{4}\right)\frac{1}{2}\,\frac{1}{4096}+\cdots\cdots\right\}$

$= (2.8284\ldots)\left\{1+\frac{1}{256}-\frac{3}{131072}\cdots\right\}$

$= (2.8284\ldots)(1+0.00390625-0.000022888\cdots\}$

$= (2.8284\ldots)(1.003883362\ldots)$

$= 2.8393$ Ans

Q.3 Expand $\sqrt{\frac{1-x}{1+x}}$ upto x^3.

Sol $\sqrt{\frac{1-x}{1+x}} = \left(\frac{1-x}{1+x}\right)^{1/2} = \frac{(1-x)^{1/2}}{(1+x)^{1/2}} = (1-x)^{1/2}(1+x)^{-1/2}$

$= (1-x)^{1/2}(1+x)^{-1/2}$ Expand by Binomial expansion upto x^3

$= \left\{1+\frac{1}{2}(-x)+\frac{1}{2}\left(\frac{1}{2}-1\right)\frac{1}{2!}(-x)^2+\frac{1}{2}\left(\frac{1}{2}-1\right)\frac{1}{3!}(-x)^3+\cdots\right\}\left\{1+\frac{-1}{2}x+\frac{-1}{2}\left(\frac{-1}{2}-1\right)\frac{1}{2!}(+x)^2 + \frac{-1}{2}\left(\frac{-1}{2}-1\right)\left(\frac{-1}{2}-2\right)\frac{1}{3!}x^3+\cdots\right\}$

$= \left\{1-\frac{x}{2}+\frac{1}{2}\left(\frac{-1}{2}\right)\frac{1}{2}x^2+\frac{1}{2}\left(\frac{-1}{2}\right)\frac{1}{6}(-x^3)\cdots\right\}\left\{1-\frac{x}{2}+\frac{-1}{2}\left(\frac{-3}{2}\right)\frac{1}{2}x^2+\frac{-1}{2}\left(\frac{-3}{2}\right)\left(\frac{-5}{2}\right)\frac{1}{6}x^3+\cdots\right\}$

$= \left(1-\frac{x}{2}-\frac{x^2}{8}+\frac{x^3}{24}\right)\left(1-\frac{x}{2}+\frac{3x^2}{8}-\frac{5x^3}{16}\right)$

Now multiplying by ignoring terms having powers of x greater than 3

$= 1-\frac{x}{2}+\frac{3x^2}{8}-\frac{5x^3}{16}-\frac{x}{2}+\frac{x^2}{4}-\frac{3x^3}{16}-\frac{x^2}{8}+\frac{x^3}{16}-\frac{x^3}{16}$

Rearraning the terms, we get

$= 1-\frac{x}{2}-\frac{x}{2}+\frac{3x^2}{8}+\frac{x^2}{4}-\frac{x^2}{8}-\frac{5x^3}{16}-\frac{3x^3}{16}$

$= 1-\frac{2x}{2}+\frac{3x^2+2x^2-x^2}{8}-\frac{8x^3}{16}$

$= 1-x+\frac{4x^2}{8}-\frac{x^3}{2}$

$= 1-x+\frac{x^2}{2}-\frac{x^3}{2}$ Ans

Q.4 If x is such that x^2 and higher powers may be neglected, show that

$$\sqrt{\frac{1-3x}{1+4x}} = 1-\frac{7x}{2}$$

Sol L.H.S

$$\sqrt{\frac{1-3x}{1+4x}} = \left(\frac{1-3x}{1+4x}\right)^{1/2} = \frac{(1-3x)^{1/2}}{(1+4x)^{1/2}}$$

$$= (1-3x)^{1/2}(1+4x)^{-1/2}$$

Expand by Binomial theorem but neglecting $x^2, x^3, \ldots$

$$= \left\{1+\frac{1}{2}(-3x) + \text{Neglected term}\right\}\left\{1+\frac{-1}{2}(4x)+\text{Neglected terms}\right\}$$

$$= \left(1-\frac{3}{2}x\right)(1-2x)$$

Multiplying, we get

$$= 1-2x-\frac{3x}{2} + \text{Neglected term}$$

$$= 1+\frac{-4x-3x}{2} = 1+\frac{-7x}{2}$$

$$= 1-\frac{7x}{2} = \text{R.H.S}$$

Q.5 If x is so small that its square and high powers can be neglected, show that

(i) $$\frac{(8+3x)^{2/3}}{(2+3x)\sqrt{4-5x}} = 1-\frac{5x}{8}$$

L.H.S

$$\frac{(8+3x)^{2/3}}{(2+3x)(4-5x)^{1/2}} = \frac{\left\{8\left(1+\frac{3}{8}x\right)\right\}^{2/3}}{2\left(1+\frac{3}{2}x\right)\left\{4\left(1-\frac{5}{4}x\right)\right\}^{1/2}}$$

$$= \frac{8^{2/3}\left(1+\frac{3}{8}x\right)^{2/3}}{2\left(1+\frac{3}{2}x\right)\,4^{\frac{1}{2}}\left(1-\frac{5}{4}x\right)^{1/2}}$$

$$= \frac{4\left(1+\frac{3}{8}x\right)^{2/3}}{2\left(1+\frac{3}{2}x\right)2\left(1-\frac{5}{4}x\right)^{1/2}} = \frac{4\left(1+\frac{3}{8}x\right)^{2/3}}{4\left(1+\frac{3}{2}x\right)\left(1-\frac{5}{4}x\right)^{1/2}}$$

$$= \left(1+\frac{3}{8}x\right)^{2/3}\cdot\left(1+\frac{3}{2}x\right)^{-1}\left(1-\frac{5}{4}x\right)^{-1/2}$$

Expand and neglect $x^2, x^3, \ldots$

$$= \left\{1+\frac{2}{3}\left(\frac{3}{8}x\right)\right\}\left\{1+(-1)\left(\frac{3}{2}x\right)\right\}\left\{1+\left(\frac{-1}{2}\right)\left(-\frac{5}{4}x\right)\right\}$$

$$= \left(1+\frac{x}{4}\right)\left(1-\frac{3}{2}x\right)\left(1+\frac{5}{8}x\right)$$

$$= \left(\frac{4+x}{4}\right)\left(\frac{2-3x}{2}\right)\left(\frac{8+5x}{8}\right)$$

$$= \frac{1}{64}\left\{(4+x)(2-3x)(8+5x)\right\}$$

Again multiply and neglect $x^2, x^3, \ldots$

$$= \frac{1}{64}\left\{(8-12x+2x)\cdot(8+5x)\right\}$$

$$= \frac{1}{64}(8-10x)(8+5x)$$

$$= \frac{1}{64}(64+40x-80x+\text{Neglected term})$$

$$= \frac{1}{64}(64-40x)$$

$$= \frac{64}{64}-\frac{40}{64}x = 1-\frac{5x}{8} = \text{R.H.S}$$

Q.6 If x is large enough if $\frac{1}{x^3}$ may be neglected then find the approximated value of $\frac{x\sqrt{x^2-2x}}{(x+1)^2}$

Sol

$$\frac{x\sqrt{x^2-2x}}{(x+1)^2} = \frac{x\sqrt{x^2\left(1-\frac{2}{x}\right)}}{\left\{x\left(1+\frac{1}{x}\right)\right\}^2}$$

$$= \frac{x \cdot x\sqrt{1-\frac{2}{x}}}{x^2\left(1+\frac{1}{x}\right)^2} = \frac{x^2\sqrt{1-\frac{2}{x}}}{x^2\left(1+\frac{1}{x}\right)^2}$$

$$= \left(1-\frac{2}{x}\right)^{\frac{1}{2}}\left(1+\frac{1}{x}\right)^{-2}$$

Expand but neglect $\frac{1}{x^3}, \frac{1}{x^4}, \ldots$

$$= \left(1+\frac{1}{2}\left(\frac{-2}{x}\right)+\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2!}\right)\left(\frac{-2}{x}\right)^2 + \text{Neglected terms}\right)$$

$$\left(1+(-2)\left(\frac{1}{x}\right)+(-2)(-2-1)\frac{1}{2!}\left(\frac{1}{x}\right)^2 + \text{Neglected terms}\right)$$

$$= \left\{1-\frac{1}{x}+\frac{1}{2}\left(\frac{-1}{2}\right)\frac{1}{2}\left(\frac{4}{x^2}\right)\right\}\left\{1-\frac{2}{x}-(2)(-3)\frac{1}{2}\,\frac{1}{x^2}\right\}$$

$$= \left(1-\frac{1}{x}-\frac{1}{2x^2}\right)\left(1-\frac{2}{x}+\frac{3}{x^2}\right)$$

multiply we get (Neglecting the useless terms)

$$= 1-\frac{2}{x}+\frac{3}{x^2}-\frac{1}{x}+\frac{2}{x^2}-\frac{1}{2x^2}$$

$$= 1-\frac{2}{x}-\frac{1}{x}+\frac{3}{x^2}+\frac{2}{x^2}-\frac{1}{2x^2}$$

$$= 1-\frac{3}{x}+\frac{6+4-1}{2x^2}$$

$$= 1-\frac{3}{x}+\frac{9}{2x^2} \quad \text{Ans}$$

Q.7 If x^4 and high powers are neglected show that $(1+x)^{1/4}+(1-x)^{\frac{1}{4}} = a-bx^2$ and find a and b.

Sol L.H.S $(1+x)^{\frac{1}{4}}+(1-x)^{\frac{1}{4}}$

$$= \left(1+\frac{1}{4}x+\frac{1}{4}\left(\frac{1}{4}-1\right)\frac{1}{2!}x^2+\frac{1}{4}\left(\frac{1}{4}-1\right)\left(\frac{1}{4}-2\right)\frac{1}{3!}x^3+\text{Neglected}\right)$$

$$+\left(1+\frac{1}{4}(-x)+\frac{1}{4}\left(\frac{1}{4}-1\right)\frac{1}{2!}(-x)^2+\frac{1}{4}\left(\frac{1}{4}-1\right)\left(\frac{1}{4}-2\right)\frac{1}{3!}(-x)^3+\text{Neglected}\right)$$

$$= \left(1+\frac{x}{4}+\frac{1}{4}\left(\frac{-3}{4}\right)\frac{1}{2}x^2+\frac{1}{4}\left(\frac{-3}{4}\right)\left(\frac{-7}{4}\right)\frac{1}{3\times2\times1}x^3\right)$$

$$+\left(1-\frac{x}{4}+\frac{1}{4}\left(\frac{-3}{4}\right)\frac{1}{2}x^2+\frac{1}{4}\left(\frac{-3}{4}\right)\left(\frac{-7}{4}\right)\frac{1}{3\times2\times1}(-x^3)\right)$$

$$= \left(1+\frac{x}{4}-\frac{3}{32}x^2+\frac{7x^3}{128}\right)+\left(1-\frac{x}{4}-\frac{3x^2}{32}-\frac{7x^3}{128}\right)$$

$$= 2-\frac{6}{32}x^2 = 2-\frac{3}{16}x^2$$

$$= a-bx^2 \text{ form} \quad \text{Hence proved}$$

where $a=2$ and $b=\frac{3}{16}$

Q.8: If x is of the size that its value are considered upto x^3, show that

$$\frac{\left(1+\frac{1}{2}x\right)^3-(1+3x)^{1/2}}{\left(1-\frac{5}{6}x\right)} = \frac{15x^2}{8}$$

Sol L.H.S $\dfrac{\left(1+\frac{x}{2}\right)^3-(1+3x)^{1/2}}{\left(1-\frac{5}{6}x\right)}$

$$=\left\{\left(1+\frac{x}{2}\right)^3-(1+3x)^{1/2}\right\}\left\{1-\frac{5x}{6}\right\}^{-1}$$

Expand upto x^3

$$=\left(\left\{1+\frac{3x}{2}+\frac{3(3-1)}{2!}\left(\frac{x}{2}\right)^2+\frac{3(3-1)(3-2)}{3!}\left(\frac{x}{2}\right)^3\right\}-\left\{1+\frac{1}{2}(3x)+\frac{1}{2}\left(\frac{1}{2}-1\right)\frac{1}{2!}(3x)^2+\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)\frac{1}{3!}(3x)^3\right\}\right)$$

$$\times\left(1+(-1)\left(-\frac{5x}{6}\right)+(-1)(-1-1)\frac{1}{2!}\left(-\frac{5x}{6}\right)^2+\frac{(-1)(-1-1)(-1-2)}{3!}\left(-\frac{5x}{6}\right)^3\right)$$

$$=\left\{\left(1+\frac{3x}{2}+\frac{3x^2}{4}+\frac{x^3}{8}\right)-\left(1+\frac{3x}{2}-\frac{9x^2}{8}+\frac{27x^3}{16}\right)\right\}\times\left\{1+\frac{5x}{6}+\frac{25x^2}{36}+\frac{125x^3}{216}\right\}$$

$$=\left\{1+\frac{3x}{2}+\frac{3x^2}{4}+\frac{x^3}{8}-1-\frac{3x}{2}+\frac{9x^2}{8}-\frac{27x^3}{16}\right\}\left\{1+\frac{5x}{6}+\frac{25x^2}{36}+\frac{125x^3}{6}\right\}$$

$$=\left\{\frac{3x^2}{4}+\frac{9x^2}{8}+\frac{x^3}{8}-\frac{27x^3}{16}\right\}\left[1+\frac{5x}{6}+\frac{25x^2}{36}+\frac{125x^3}{6}\right)$$

$$=\left[\frac{15x^2}{8}-\frac{25x^3}{16}\right]\left[1+\frac{5x}{6}+\frac{25x^2}{36}+\frac{125x^3}{6}\right)$$

$$=\frac{15x^2}{8}+\frac{75x^3}{48}-\frac{25x^3}{16}=\frac{15x^2}{8}+\frac{75x^3-75x^3}{48}=\frac{15x^2}{8}=R.H.S$$

⑨ Find the coefficient of x^n in $\left(\frac{1+x}{1-x}\right)^2$

Sol

$$\left(\frac{1+x}{1-x}\right)^2=\frac{(1+x)^2}{(1-x)^2}$$

$$=(1+x)^2(1-x)^{-2}$$

$$=(1+2x+x^2)\left\{1+(-2)(-x)+\frac{(-2)(-2-1)}{2!}(-x)^2+\dots\right\}$$

$$=(1+2x+x^2)\left(1+2x+3x^2+\dots\right)$$

$$=(1+2x+x^2)\left(1+2x+3x^2+\dots(n-1)x^{n-2}+nx^{n-1}+(n+1)x^n\right)$$

Multiply those terms that give x^n

$$=1(n+1)x^n+2x\cdot n(x)^{n-1}+x^2(n-1)x^{n-2}$$

$$=(n+1)x^n+2n\,x^{1+n-1}+(n-1)x^{2+n-2}$$

$$=(n+1)x^n+2nx^n+(n-1)x^n$$

take x^n as common

$$=\{(n+1)+2n+(n-1)\}x^n$$

$$=\{n+2n+n\}x^n$$

$$=4nx^n.$$

Hence coefficient of x^n is $4n$

$\boxed{Q.10}$ Find the coefficients of x^{3n} in $\frac{(1+x^2)^3}{(1-x^3)^2}$

Sol

$$\frac{(1+x^2)^3}{(1-x^3)^2}=(1+x^2)^3\cdot(1-x^3)^{-2}$$

$$=(1+3x^2+3x^4+x^6)\left(1+(-2)(-x^3)+\frac{(-2)(-2-1)}{2!}(-x^3)^2\dots\right)$$

$$=(1+3x^2+3x^4+x^6)\left(1+2x^3+\frac{(-2)(-3)}{2}x^6+\dots\right)$$

$$=(1+3x^2+3x^4+x^6)\left(1+2x^3+3x^6+\dots\right)$$

$$=(1+3x^2+3x^4+x^6)\left(1+2x^3+3x^6+\dots(n-1)x^{3n-6}+nx^{3n-3}+(n+1)x^{3n}\right)$$

Multiply those terms that give x^{3n}

$$=1.(n+1)x^{3n}+(n-1)x^{3n-6}x^6$$

$$=(n+1)x^{3n}+(n-1)x^{3n}=(n+1+n-1)x^{3n}=2nx^{3n}$$

Hence coefficient of x^{3n} is $2n$

Q:11 Identify the following series and find the sum of each.

(i) $1-\frac{1}{2^2}+\frac{1.3}{2!}\cdot\frac{1}{2^4}+\cdots\cdots$

Sol Let $(1+x)^n$ is the given series

i.e $(1+x)^n = 1-\frac{1}{2^2}+\frac{1.3}{2!}\cdot\frac{1}{2^4}+\cdots\cdots$

$\Rightarrow 1+nx+\frac{n(n-1)}{2!}x^2+\cdots = 1-\frac{1}{2^2}+\frac{1.3}{2!}\cdot\frac{1}{2^4}+\cdots$

compare the 2nd and third terms of b.s, we get

$nx = -\frac{1}{2^2}$ & $\frac{n(n-1)}{2!}x^2 = \frac{1.3}{2!}\cdot\frac{1}{2^4}$

$\Rightarrow x = -\frac{1}{4n}$

P.T.V of x

$\frac{n(n-1)}{2!}\left(-\frac{1}{4n}\right)^2 = \frac{1.3}{2!}\cdot\frac{1}{16}$

$\Rightarrow \frac{n(n-1)}{2}\cdot\frac{1}{16n^2} = \frac{3}{2}\cdot\frac{1}{16}$

$\Rightarrow \frac{n-1}{n} = 3 \Rightarrow n-1 = 3n$

$\Rightarrow -1 = 2n \Rightarrow \boxed{n=-\frac{1}{2}}$

Now $x = -\frac{1}{4n}$

$x = \frac{-1}{4(-\frac{1}{2})} \Rightarrow \boxed{x=\frac{1}{2}}$

Hence the given series is $(1+x)^n = \left(1+\frac{1}{2}\right)^{-1/2}$ Ans

i.e $1-\frac{1}{2^2}+\frac{1.3}{2!}\cdot\frac{1}{2^4}+\cdots = \left(1+\frac{1}{2}\right)^{-\frac{1}{2}}$

Now Sum of the series:

$1-\frac{1}{2^2}+\frac{1.3}{2!}\cdot\frac{1}{2^4}+\cdots = \left(\frac{3}{2}\right)^{-\frac{1}{2}} = \left(\frac{2}{3}\right)^{1/2}$

$= \sqrt{2/3}$ Ans

(ii) $1+\frac{1}{3}+\frac{1}{2!}\cdot\frac{1}{3}+\frac{1.5}{3!}\cdot\frac{1}{3^2}+\cdots\cdots$

Sol Let $(1+x)^n = 1+\frac{1}{3}+\frac{1}{2!}\cdot\frac{1}{3}+\frac{1.5}{3!}\cdot\frac{1}{3^2}+\cdots$

$\Rightarrow 1+nx+\frac{n(n-1)}{2!}x^2+\cdots = 1+\frac{1}{3}+\frac{1}{2!}\cdot\frac{1}{3}+\cdots$

compare b.s, we get

$nx = \frac{1}{3}$ & $\frac{n(n-1)}{2!}x^2 = \frac{1}{2!}\cdot\frac{1}{3}$

$\Rightarrow x = \frac{1}{3n}$

P.T.V of x

$\Rightarrow \frac{n(n-1)}{2!}\left(\frac{1}{3n}\right)^2 = \frac{1}{2!}\cdot\frac{1}{3}$

$\Rightarrow \frac{n(n-1)}{2!}\cdot\frac{1}{9n^2} = \frac{1}{2!}\cdot\frac{1}{3}$

$\Rightarrow \frac{n-1}{3n} = 1 \Rightarrow n-1 = 3n$

$\Rightarrow -1 = 2n \Rightarrow \boxed{\frac{-1}{2}=n}$

Now $x = \frac{1}{3n}$

$x = \frac{1}{3(\frac{-1}{2})} \Rightarrow \boxed{x=-\frac{2}{3}}$

Hence the given series is $(1+x)^n = \left(1-\frac{2}{3}\right)^{-1/2}$

i.e $1+\frac{1}{3}+\frac{1}{2!}\cdot\frac{1}{3}+\cdots = \left(1-\frac{2}{3}\right)^{-1/2}$

Now Sum of the series

$1+\frac{1}{3}+\frac{1}{2!}\cdot\frac{1}{3}+\cdots = \left(\frac{3-2}{3}\right)^{-1/2} = \left(\frac{1}{3}\right)^{-1/2}$

$= \left(\frac{3}{1}\right)^{1/2} = \sqrt{3}$ Ans

(iii) $1+\frac{2}{9}+\frac{2.5}{9.18}+\frac{2.5.8}{9.8.27}+\cdots\cdots$

Sol: let $(1+x)^n = 1+\frac{2}{9}+\frac{2.5}{9.18}+\frac{2.5.8}{9.8.27}+\cdots$

$\Rightarrow 1+nx+\frac{n(n-1)}{2!}x^2+\cdots = 1+\frac{2}{9}+\frac{2.5}{9.18}+\cdots$

compare b.s, we get

$\Rightarrow nx = \frac{2}{9}$ & $\frac{n(n-1)}{2!}x^2 = \frac{2.5}{9.18}$

$\Rightarrow x = \frac{2}{9n}$ $\qquad \Rightarrow \frac{n(n-1)}{2!}\left(\frac{2}{9n}\right)^2 = \frac{2.5}{9.18}$

$\Rightarrow \frac{n(n-1)}{2}\cdot\frac{2^2}{9^2n^2} = \frac{2.5}{9.18}$

$\Rightarrow \frac{n-1}{n} = \frac{10}{4} \Rightarrow \frac{n-1}{n} = \frac{5}{2}$

$\Rightarrow 2n-2 = 5n$

$\Rightarrow \boxed{-\frac{2}{3} = n}$

Now P.T.V in x

$x = \frac{2}{9n}$

$= \frac{2}{9\left(-\frac{2}{3}\right)}$

$\boxed{x = -\frac{1}{3}}$

Hence the given series is $(1+x)^n = \left(1-\frac{1}{3}\right)^{-\frac{2}{3}}$ Ans

i.e $1+\frac{2}{9}+\frac{2.5}{9.18}+\cdots = \left(1-\frac{1}{3}\right)^{-\frac{2}{3}}$

Sum: $1+\frac{2}{9} \div \frac{2.5}{9.18}+\cdots = \left(\frac{3-1}{3}\right)^{-\frac{2}{3}} = \left(\frac{2}{3}\right)^{-2/3}$

$= (3/2)^{2/3}$. Ans

(iv) $1+\frac{5}{8}+\frac{5.8}{8.12}+\frac{5.8.11}{8.12.16}+\cdots\cdots$

Sol let $(1+x)^n$ is the series.

$(1+x)^n = 1+\frac{5}{8}+\frac{5.8}{8.12}+\cdots$

$\Rightarrow 1+nx+\frac{n(n-1)}{2!}x^2+\cdots = 1+\frac{5}{8}+\frac{5.8}{8.12}+\cdots$

compare b.s, we get

$nx = \frac{5}{8}$ & $\frac{n(n-1)}{2!}x^2 = \frac{5.8}{8.12}$

$\Rightarrow x = \frac{5}{8n}$ & P.T.V of x

$\frac{n(n-1)}{2!}\left(\frac{5}{8n}\right)^2 = \frac{5.8}{8.12}$

$\frac{n(n-1)}{2!}\cdot\frac{25}{64n^2} = \frac{5}{12}$

$\Rightarrow \frac{n(n-1)5}{64n^2} = \frac{1}{6} \Rightarrow \frac{5n(n-1)}{32n^2} = \frac{1}{3}$

$\Rightarrow \frac{15(n-1)}{32n} = 1 \Rightarrow 15n-15 = 32n$

$\Rightarrow -15 = 17n \Rightarrow \boxed{n = \frac{-15}{17}}$

Now $x = \frac{5}{8n}$

$x = \frac{5}{8\left(\frac{-15}{17}\right)} \Rightarrow \boxed{x = \frac{-17}{24}}$

Hence the series is $(1+x)^n = \left(1-\frac{17}{24}\right)^{\frac{-15}{17}}$

Sum: $\left(1-\frac{17}{24}\right)^{\frac{-15}{17}} = \left(\frac{24-17}{24}\right)^{\frac{-15}{17}} = \left(\frac{7}{24}\right)^{-15/17}$

$= \left(\frac{24}{7}\right)^{15/17}$

Q.12. Find the sum to infinity.

(i) $1+2\cdot\frac{1}{3}+3\cdot\frac{1}{3^2}+4\cdot\frac{1}{3^3}+\cdots$

Sol Let $(1+x)^n$ is the series

$(1+x)^n = 1+\frac{2}{3}+3\cdot\frac{1}{3^2}+4\cdot\frac{1}{3^3}+\cdots$

$\Rightarrow 1+nx+\frac{n(n-1)}{2!}x^2+\cdots = 1+\frac{2}{3}+\frac{1}{3}+\frac{4}{3^3}+\cdots$

compare the terms, we get

$nx=\frac{2}{3}$ & $\frac{n(n-1)}{2!}x^2=\frac{1}{3}$

$\Rightarrow x=\frac{2}{3n}$ P.T.V of x

$\frac{n(n-1)}{2!}\left(\frac{2}{3n}\right)^2=\frac{1}{3}$

$\Rightarrow \frac{n(n-1)}{2}\cdot\frac{4}{9n^2}=\frac{1}{3} \Rightarrow \frac{2(n-1)}{3n}=1$

$\Rightarrow 2n-2=3n \Rightarrow -2=n$

Now $x=\frac{2}{3n}$

$\Rightarrow x=\frac{2}{3(-2)} \Rightarrow \boxed{x=-\frac{1}{3}}$

Hence the given series is $(1+x)^n=\left(1-\frac{1}{3}\right)^{-2}$

i.e $1+2\cdot\frac{1}{3}+3\cdot\frac{1}{3^2}+\cdots=\left(1-\frac{1}{3}\right)^{-2}$

So the sum is $\left(1-\frac{1}{3}\right)^{-2}=\left(\frac{3-1}{3}\right)^{-2}=\left(\frac{2}{3}\right)^{-2}$

$=\left(\frac{3}{2}\right)^2$

$=9/4$ Ans

(ii) $1+\frac{1}{6}+\frac{1.3}{1.2}\cdot\frac{1}{6^2}+\cdots$

Sol Let $(1+x)^n$ is the series

$(1+x)^n = 1+\frac{1}{6}+\frac{1.3}{1.2}\cdot\frac{1}{6^2}+\cdots$

$\Rightarrow 1+nx+\frac{n(n-1)}{2!}x^2+\cdots = 1+\frac{1}{6}+\frac{1.3}{1.2}\cdot\frac{1}{6^2}+\cdots$

compare the terms

$nx=\frac{1}{6}$ & $\frac{n(n-1)}{2!}x^2=\frac{3}{2}\cdot\frac{1}{6^2}$

$\Rightarrow x=\frac{1}{6n}$ P.T.V of x

$\frac{n(n-1)}{2!}\left(\frac{1}{6n}\right)^2=\frac{3}{2}\cdot\frac{1}{36}$

$\Rightarrow \frac{n(n-1)}{2!}\cdot\frac{1}{36n^2}=\frac{3}{2\cdot 36}$

$\Rightarrow \frac{n-1}{n}=3 \Rightarrow n-1=3n \Rightarrow -1=2n$

$\Rightarrow \boxed{n=-\frac{1}{2}}$

Now $x=\frac{1}{6n}$

$x=\frac{1}{6(-\frac{1}{2})} \Rightarrow \boxed{x=-\frac{1}{3}}$

Then the series is $(1+x)^n=\left(1-\frac{1}{3}\right)^{-\frac{1}{2}}$

and the sum is $\left(1-\frac{1}{3}\right)^{-\frac{1}{2}}=\left(\frac{3-1}{3}\right)^{-1/2}$

$=\left(\frac{2}{3}\right)^{-\frac{1}{2}}$

$=\left(\frac{3}{2}\right)^{\frac{1}{2}}$ Ans

$=\sqrt{3/2}$

(13) $y = \frac{1}{2^2} + \frac{1.3}{2!}\cdot\frac{1}{2^4} + \frac{1.3.5}{3!}\cdot\frac{1}{2^6} + \cdots\cdots$

Show that $y^2 + 2y - 1 = 0$

Sol: Add 1 to b.s of the given eqn

$1 + y = 1 + \frac{1}{2^2} + \frac{1.3}{2!}\cdot\frac{1}{2^4} + \cdots\cdots \quad$ (A)

Let $(1+x)^n$ is the series on R.H.S

i.e $(1+x)^n = 1 + \frac{1}{2^2} + \frac{1.3}{2!}\cdot\frac{1}{2^4} + \cdots\cdots$

$\Rightarrow 1 + nx + \frac{n(n-1)}{2!}x^2 + \cdots = 1 + \frac{1}{2^2} + \frac{1.3}{2!}\cdot\frac{1}{2^4} + \cdots$

compare b.s.

$\Rightarrow nx = \frac{1}{2^2}$ & $\frac{n(n-1)}{2!}x^2 = \frac{1.3}{2!}\cdot\frac{1}{2^4}$

$\Rightarrow x = \frac{1}{4n}$

P.T.V of x

$\frac{n(n-1)}{2!}\left(\frac{1}{4n}\right)^2 = \frac{3}{2}\cdot\frac{1}{16}$

$\Rightarrow \frac{n(n-1)}{2}\cdot\frac{1}{16n^2} = \frac{3}{2\times16}$

$\Rightarrow \frac{n-1}{n} = 3 \Rightarrow n - 1 = 3n$

$\Rightarrow -1 = 2n \Rightarrow \boxed{-\frac{1}{2} = n}$

P.T.V of $x = \frac{1}{4n}$

$x = \frac{1}{4(-\frac{1}{2})} \Rightarrow x = \frac{-1}{2}$

So the given series is $(1+x)^n = \left(1 - \frac{1}{2}\right)^{-\frac{1}{2}}$

i.e $1 + \frac{1}{2^2} + \frac{1.3}{2!}\cdot\frac{1}{2^4} + \cdots = \left(1 - \frac{1}{2}\right)^{-\frac{1}{2}}$

P.T.V in eqn (A)

$1 + y = \left(1 - \frac{1}{2}\right)^{-\frac{1}{2}}$

$\Rightarrow 1 + y = \left(\frac{2-1}{2}\right)^{-\frac{1}{2}}$

$\Rightarrow 1 + y = \left(\frac{1}{2}\right)^{-\frac{1}{2}} \Rightarrow 1 + y = \left(\frac{2}{1}\right)^{1/2}$

$\Rightarrow 1 + y = \sqrt{2}$

squaring b.s

$\Rightarrow (1+y)^2 = (\sqrt{2})^2$

$\Rightarrow 1 + y^2 + 2y = 2$

$\Rightarrow \boxed{y^2 + 2y - 1 = 0}$ Hence proved

(14) If $2y = \frac{1}{2^2} + \frac{1.3}{2!}\cdot\frac{1}{2^4} + \frac{1.3.5}{3!}\cdot\frac{1}{2^6} + \cdots$

Prove that $4y^2 + 4y - 1 = 0$

Sol: Add 1 to b.s of the given eqn

$1 + 2y = 1 + \frac{1}{2^2} + \frac{1.3}{2!}\cdot\frac{1}{2^4} + \cdots\cdots \longrightarrow$ (A)

Let $(1+x)^n$ is the series on R.H.S of eqn (A)

i.e $(1+x)^n = 1 + \frac{1}{2^2} + \frac{1.3}{2!}\cdot\frac{1}{2^4} + \cdots\cdots$

$\Rightarrow 1 + nx + \frac{n(n-1)}{2!}x^2 + \cdots = 1 + \frac{1}{2^2} + \frac{1.3}{2!}\cdot\frac{1}{2^4} + \cdots$

compare b.s, we get

$nx = \frac{1}{2^2}$ & $\frac{n(n-1)}{2!}x^2 = \frac{1.3}{2!}\cdot\frac{1}{2^4}$

$\Rightarrow x = \frac{1}{4n}$

P.T.V of x

$$\frac{n(n-1)}{2}\left(\frac{1}{4n}\right)^2 = \frac{1.3}{2!}\cdot\frac{1}{16}$$

$$\Rightarrow \frac{n(n-1)}{2}\,\frac{1}{16n^2} = \frac{3}{2}\times\frac{1}{16}$$

$$\Rightarrow \frac{n-1}{n} = 3 \Rightarrow n-1 = 3n \Rightarrow -1 = 2n$$

$$\Rightarrow \boxed{-\frac{1}{2} = n}$$

Now $x = \frac{1}{4n}$

$$x = \frac{1}{4(-\frac{1}{2})} \Rightarrow \boxed{x = -\frac{1}{2}}$$

Hence the given series is

$$(1+x)^n = \left(1-\frac{1}{2}\right)^{-\frac{1}{2}}$$

i.e $1 + nx + \frac{n(n-1)}{2!}x^2 + \cdots = 1 + \frac{1}{2^2} + \frac{1.3}{2!}\cdot\frac{1}{2^4} + \cdots$

$$\Rightarrow \left(1-\frac{1}{2}\right)^{-\frac{1}{2}} = 1 + \frac{1}{2^2} + \frac{1.3}{2!}\cdot\frac{1}{2^4} + \cdots$$

P.T.V in Equ (A)

$$1+2y = \left(1-\frac{1}{2}\right)^{-\frac{1}{2}}$$

$$\Rightarrow 1+2y = \left(\frac{2-1}{2}\right)^{-\frac{1}{2}}$$

$$\Rightarrow 1+2y = \left(\frac{1}{2}\right)^{-\frac{1}{2}}$$

$$\Rightarrow 1+2y = \left(\frac{2}{1}\right)^{1/2}$$

$$\Rightarrow 1+2y = \sqrt{2}$$

squaring b.s

$$\Rightarrow (1+2y)^2 = (\sqrt{2})^2$$

$$\Rightarrow 1+4y+4y^2 = 2$$

$$\Rightarrow \boxed{4y^2+4y-1=0}$$ Hence proved.

Q.15 If x is so small that x^3 and high powers of x can be ignored, show that the nth ~~power~~ root of $1+x$ is equal to $\frac{2n+(n+1)x}{2n+(n-1)x}$

Sol To show that

$$(1+x)^{\frac{1}{n}} = \frac{2n+(n+1)x}{2n+(n-1)x}$$

R.H.S $\frac{2n+(n+1)x}{2n+(n-1)x}$

take $2n$ as common

$$= \frac{2n\left[1+\frac{n+1}{2n}x\right]}{2n\left[1+\frac{n-1}{2n}x\right]}$$

$$= \left\{1+\left(\frac{n+1}{2n}\right)x\right\}\left\{1+\left(\frac{n-1}{2n}\right)x\right\}^{-1}$$

Expand by Binomial theorem

$$= \left\{1+\left(\frac{n+1}{2n}\right)x\right\}\left\{1+(-1)\left(\frac{n-1}{2n}\right)x + \frac{(-1)(-1-1)}{2!}\left(\frac{n-1}{2n}x\right)^2 \cdots\right\}$$

$$= \left\{1+\frac{n+1}{2n}x\right\}\left\{1-\frac{n-1}{2n}x+\frac{(n-1)^2}{4n^2}x^2+\text{Neglected terms}\right\}$$

Multiply upto x^2

$$= 1-\frac{(n-1)}{2n}x+\frac{(n-1)^2x^2}{4n^2}+\frac{(n+1)}{2n}x-\frac{(n+1)(n-1)}{4n^2}x^2+\text{Neglected}$$

= Rearranging the terms

$$= 1+\left(\frac{n+1}{2n}\right)x-\left(\frac{n-1}{2n}\right)x+\frac{(n-1)^2x^2}{4n^2}-\frac{(n+1)(n-1)}{4n^2}x^2$$

$$= 1+x\left\{\frac{n+1}{2n}-\frac{n-1}{2n}\right\}+x^2\left\{\frac{(n-1)^2}{4n^2}-\frac{(n+1)(n-1)}{4n^2}\right\}$$

$$= 1+x\left\{\frac{n+1-n+1}{2n}\right\}+x^2\left\{\frac{(n-1)^2-(n^2-1)}{4n^2}\right\}$$

$$= 1+x\left(\frac{2}{2n}\right)+x^2\left\{\frac{n^2+1-2n-n^2+1}{4n^2}\right\}$$

$$= 1+\frac{x}{n}+x^2\left(\frac{2-2n}{4n^2}\right)$$

$$= 1+\frac{1}{n}x+x^2\,2\left(\frac{1-n}{4n^2}\right)$$

$$= 1+\frac{1}{n}x+x^2\left(\frac{1-n}{2n^2}\right)$$

$$= 1+\frac{1}{n}x+\frac{x^2}{2!}\left(\frac{1}{n^2}-\frac{n}{n^2}\right)$$

$$= 1+\frac{1}{n}x+\frac{x^2}{2!}\left(\frac{1}{n^2}-\frac{1}{n}\right)$$ take $\frac{1}{n}$ as common

$$= 1+\frac{1}{n}x+\frac{x^2}{2!}\,\frac{1}{n}\left(\frac{1}{n}-1\right)$$

$$= 1+\frac{1}{n}x+\frac{1}{n}\left(\frac{1}{n}-1\right)\frac{1}{2!}x^2 = (1+x)^{1/n} = \text{L.H.S}$$

Q.16 If x is nearly equal to unity then show that
$$px^p-qx^q=(p-q)x^{p+q}$$

Sol As $x\approx 1$

let $x=1+h$ where h is very small and $h^2, h^3, \ldots$ are negligible

L.H.S $\quad px^p-qx^p$

$$= p(1+h)^p-q(1+h)^q$$

Expand and neglect $h^2, h^3, \ldots$ etc

$$= p(1+ph+\text{Neglected})-q(1+qh^2+\text{Neglected})$$

$$= p(1+ph)-q(1+qh)$$

$$= p+p^2h-q-q^2h$$

$$= p-q+p^2h-q^2h$$

$$= (p-q)+(p^2-q^2)h$$

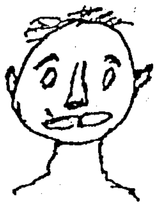
$$= (p-q)+(p+q)(p-q)h$$

take $p-q$ as common

$$= (p-q)\{1+(p+q)h\}$$

$$= (p-q)\{1+h\}^{(p+q)}$$

$$= (p-q)\,x^{p+q} = \text{R.H.S}$$



Hurrah! Its the end of chapter # 07