Exercise # 6.1

Q:1 Evaluate the following

(i) $\dfrac{9!\,0!}{5!\,4!}$

Sol $\dfrac{9!\,0!}{5!\,4!} = \dfrac{(9\times8\times7\times6\times\cancel{5!})\,(1)}{\cancel{5!}\;4\times3\times2\times1} = 126$

(ii) $\dfrac{3!+4!}{5!-4!} = \dfrac{6+24}{120-24} = \dfrac{30}{96} = \dfrac{5}{16}$ Ans

(iii) $\dfrac{(n-1)!}{(n+1)!} = \dfrac{(n-1)!}{(n+1)\,n\,(n-1)!} = \dfrac{1}{n(n+1)} = \dfrac{1}{n^2+n}$ Ans

(iv) $\dfrac{10!}{(5!)^2} = \dfrac{10!}{5!\,5!} = \dfrac{10\times9\times8\times7\times6\times\cancel{5!}}{\cancel{5!}\;5\times4\times3\times2\times1} = 252$ Ans

Q:2 Write down the following in terms of factorial.

(i) $18.17.16.15.14$

$= 18.17.16.15.14 \times \dfrac{13!}{13!} = \dfrac{18!}{13!}$ Ans

(ii) $2.4.6.8.10.12$

$= 2.(2\times2).(2\times3).(2\times4).(2\times5).(2\times6)$

$= 2^6.(6\times5\times4\times3\times2\times1) = 2^6.6!$ Ans

(iii) $n(n^2-1)$

Sol $= n(n+1)(n-1)$

$= (n+1)\,n\,(n-1)$ × and ÷ by $(n-2)!$

$= (n+1)\,n\,(n-1)\dfrac{(n-2)!}{(n-2)!}$

$= \dfrac{(n+1)!}{(n-2)!}$

(iv) $\dfrac{n(n+1)(n+2)}{3}$

$= \dfrac{(n+2)(n+1)n}{3}$ × and ÷ by $(n-1)!$

$= \dfrac{(n+2)(n+1)\,n\,(n-1)!}{3\;(n-1)!} = \dfrac{(n+2)!}{3(n-1)!}$ Ans

Q:3 Prove the following.

(i) $\dfrac{1}{6!} + \dfrac{2}{7!} + \dfrac{3}{8!} = \dfrac{75}{8!}$

Sol $\dfrac{1}{6!} + \dfrac{2}{7!} + \dfrac{3}{8!} = \dfrac{1}{6!} + \dfrac{2}{7\times6!} + \dfrac{3}{8\times7\times6!}$

$= \dfrac{1}{6!}\left\{1 + \dfrac{2}{7} + \dfrac{3}{8\times7}\right\}$

$= \dfrac{1}{6!}\left\{\dfrac{(8\times7)+(2\times8)+3}{8\times7}\right\}$

$= \dfrac{56+16+3}{8\times7\times6!} = \dfrac{75}{8!}$ Ans

(ii) $\dfrac{(n+5)!}{(n+3)!} = n^2+9n+20$

Sol $\dfrac{(n+5)!}{(n+3)!} = \dfrac{(n+5)(n+4)(n+3)!}{(n+3)!}$

$= (n+5)(n+4)$

$= n^2+4n+5n+20$

$= n^2+9n+20 = R.H.S$

Q.4 Find the values of n, where

(i) $\dfrac{n(n!)}{(n-5)!} = \dfrac{12(n!)}{(n-4)!}$

174

Sol by cross multiplication

$n(n!)(n-4)! = 12(n!)(n-5)!$

$\Rightarrow n(n-4)! = 12(n-5)!$

$\Rightarrow n(n-4)(n-5)! = 12(n-5)!$

$\Rightarrow n(n-4) = 12$

$\Rightarrow n^2-4n-12=0$

$\Rightarrow n^2-6n+2n-12=0$

$\Rightarrow n(n-6)+2(n-6)=0$

$\Rightarrow (n-6)(n+2)=0$

$n-6=0$ or $n+2=0$

$\Rightarrow \boxed{n=6}$ Ans $n=-2$ is not possible

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(ii) $\dfrac{n!}{(n-4)!} : \dfrac{(n-1)!}{(n-4)!} = 9:1$

Sol $\dfrac{n!}{(n-4)!} : \dfrac{(n-1)!}{(n-4)!} = 9:1 \Rightarrow \dfrac{\frac{n!}{(n-4)!}}{\frac{(n-1)!}{(n-4)!}} = \dfrac{9}{1}$

$\Rightarrow \dfrac{n!}{(n-1)!} = 9$

$\Rightarrow \dfrac{n(n-1)!}{(n-1)!} = 9 \Rightarrow \boxed{n=9}$ Ans

Q.5 Show that $\dfrac{(2n)!}{n!} = 2^n(1.3.5\ldots\ldots(2n-1))$

L.H.S $\dfrac{(2n)!}{n!} = \dfrac{(2n)(2n-1)(2n-2)(2n-3)(2n-4)\ldots\ldots 3\times2\times1}{n\;(n-1)\;(n-2)\;\ldots\ldots\; 3\times2\times1}$

$= \dfrac{(2n)(2n-1)\,2(n-1)(2n-3)\,2(n-2)\ldots\ldots 3\times2\times1}{n\cdot(n-1)\cdot(n-2)\ldots\ldots 3\times2\times1}$

$= 2(2n-1)\,2\,(2n-3)\,2\,(2n-5)\ldots\ldots 5\times3\times1$

$= 2^{1+1+1+\ldots\ldots+1\text{ (n times)}}\cdot(2n-1)(2n-3)\ldots 5\times3\times1$

$= 2^n\left[1.3.5\ldots\ldots(2n-1)\right]$

$= R.H.S$

Exercise # 6.2

Q:1 How many ways different batting orders are possible for a cricket team consisting of 11 players?

Sol Total batting orders $= {}^{11}P_{11}$

$= 11!$

$= 39916800$ Ans.

Q:2 How many three digit numbers can be made from the digits 1, 2, 3, 4 and 5 if repeatition is

(a) allowed (b) Not allowed

Sol (a) Repeatition is allowed.

$$= \underset{\text{1st place}}{\underline{5}} \times \underset{\text{2nd place}}{\underline{5}} \times \underset{\text{3rd place}}{\underline{5}} = 125 \text{ ways.}$$

(b) Repeatition is not allowed.

$$\frac{5}{\text{1st place}} \times \frac{4}{\text{2nd place}} \times \frac{3}{\text{3rd place}} = 60 \text{ ways.}$$

Q:3 A man has four coats, six shirts and three trousers. In how many ways can he dress himself?

Sol

$$\frac{4 \text{ ways}}{\text{coat}} \times \frac{6 \text{ ways}}{\text{shirts}} \times \frac{3 \text{ ways}}{\text{trousers}} = 72 \text{ ways}$$

Q:4 In how many ways can four french books, two English books and three German books be arranged on a shelf so that all books in the same language are togather?

Sol No of ways for French books $= 4! = 24$

" " " " English " $= 2! = 2$

" " " " German " $= 3! = 6$

Then total arrangments in which the same books will be togather will be

$$= (24 \times 2 \times 6) \times 3! = (24 \times 2 \times 6) \times 6 = 1728 \text{ Ans.}$$

Note:- 3! because if we considre the same books as one big book then they can be arranged themselves by 3! = 6 way

$\boxed{F}$ $\boxed{E}$ $\boxed{G}$

Q:5 How many different arrangments can be formed of the word "equation" if all the ~~alphabets~~ vowels are kept togather?

Sol word is equation

\# of vowels $= 5$ $\{a, e, i, o, u\}$

If we considre the five vowels as one alphabet then we get 4 alphabets $\boxed{aeiou}$ $\boxed{q}$ $\boxed{t}$ $\boxed{n}$

So $4!$

but the five vowels can change places among themselves so $5!$

Hence total words in which the vowels will be togather are $4! \times 5! = 2880$ Ans

176

Q:6 A combination lock has five wheels, each labelled with 10 digits from 0 to 9. How many five number opening combination are possible, assuming no digit is repeated? & assuming digits can be repeated?

Sol Total digits $= 10$ i.e $(0, 1, 2, \dots 9)$

Total places $= 5$

(i) <u>If no repeatition</u> then

$$\underset{1st\ place}{\underline{10\ ways}} \times \underset{2nd\ place}{\underline{9\ ways}} \times \underset{3rd}{\underline{8\ ways}} \times \underset{4th}{\underline{7\ ways}} \times \underset{5th\ place}{\underline{6\ ways}} = 30240 \text{ ways}$$

(ii) <u>If repeatition is allowed</u>

$$\underset{1st\ place}{\underline{10\ ways}} \times \underset{2nd\ place}{\underline{10\ ways}} \times \underset{3rd\ place}{\underline{10\ ways}} \times \underset{4th\ place}{\underline{10\ ways}} \times \underset{5th\ place}{\underline{10\ ways}} = 10^5 \text{ ways}$$

$= 100000$ ways

Q:7 How many signals can be given by six flags of different colours when any number of them can be used at a time?

Sol Total flags $= n = 6$

Then signals when we use one flag $= {}^6P_1 = 6$

" " " " two flags $= {}^6P_2 = 30$

" " " " three " $= {}^6P_3 = 120$

Signals when we use four flags $= {}^6P_4 = 360$

" " " " five " $= {}^6P_5 = 720$

" " " " six " $= {}^6P_6 = 720$

Now total signals are

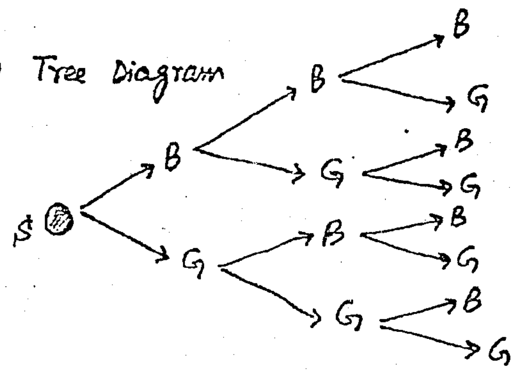
$6 + 30 + 120 + 360 + 720 + 720 = 1956$ Ans

Q:6 A couple is planning to have three children. How many boy-girl combinations are possible? Solve using (i) the multiplication principle (ii) a tree diagram.

Sol (i)

$$\underset{1st\ child}{\underline{2\ ways}} \times \underset{2nd\ child}{\underline{2\ ways}} \times \underset{3rd\ child}{\underline{2\ ways}} = 8 \text{ ways}$$

(ii) Tree Diagram



BBB
BBG
BGB
BGG
GBB
GBG
GGB
GGG

Exercise # 6.3

Q:1 Evaluate

(i) $^6P_6 = \frac{6!}{(6-6)!} = \frac{6!}{0!} = \frac{720}{1} = 720$ Ans

(ii) $^{20}P_2 = \frac{20!}{(20-2)!} = \frac{20\times19\times18!}{18!} = 380$ Ans

(iii) $^7P_0 = \frac{7!}{(7-0)!} = \frac{7!}{7!} = 1$

(iv) $^5P_7$ = Undefined

Q:2 Solve for n?

(i) $^nP_5 = 56\ ^nP_3$

Sol

$$\frac{n!}{(n-5)!} = 56\frac{n!}{(n-3)!}$$

$$\Rightarrow \frac{1}{(n-5)!} = \frac{56}{(n-3)!}$$

$$\Rightarrow (n-3)! = 56(n-5)!$$

$$\Rightarrow (n-3)(n-4)(n-5)! = 56(n-5)!$$

$$\Rightarrow (n-3)(n-4) = 56$$

$$\Rightarrow n^2-4n-3n+12 = 56$$

$$\Rightarrow n^2-7n-44 = 0$$

$$\Rightarrow n^2-11n+4n-44 = 0$$

$$\Rightarrow n(n-11)+4(n-11) = 0$$

$$\Rightarrow (n-11)(n+4) = 0$$

$n-11=0$ or $n+4=0$

$\boxed{n=11}$ Ans $\quad n=-4$ not possible

(ii) $^nP_5 = 9\ ^{n-1}P_4$

Sol

$$\frac{n!}{(n-5)!} = 9\frac{(n-1)!}{(n-1-4)!}$$

$$\Rightarrow \frac{n(n-1)!}{(n-5)!} = 9\frac{(n-1)!}{(n-5)!}$$

$$\Rightarrow \boxed{n=9}$$ Ans

(iii) $^{n^2}P_2 = 600$

Sol

$$\frac{(n^2)!}{(n^2-2)!} = 600$$

$$\Rightarrow \frac{n^2(n^2-1)(n^2-2)!}{(n^2-2)!} = 600$$

$$\Rightarrow n^2(n^2-1) = 600 \qquad \text{Let } n^2 = x$$

$$\Rightarrow x(x-1) = 600$$

$$\Rightarrow x^2-x-600 = 0$$

By quadratic formula

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-(-1)\pm\sqrt{(-1)^2-4(1)(-600)}}{2(1)} = \frac{1\pm\sqrt{1+2400}}{2}$$

$$= \frac{1\pm\sqrt{2401}}{2} = \frac{1\pm49}{2}$$

Hence $x = \frac{1+49}{2}$, $x = \frac{1-49}{2}$

$x = \frac{50}{2}$, $\boxed{x=-24}$ Not possible

$x = 25$

Hence $x = 25$

Now since $n^2 = x$

$\Rightarrow n^2 = 25$

$\Rightarrow n = \pm 5$ but $n = -5$ is not possible

$\Rightarrow \boxed{n = 5}$ Ans

Q:3: Prove the following by fundamental principle of counting.

(i) ${}^nP_r = n\left({}^{n-1}P_{r-1}\right)$

Sol L.H.S ${}^nP_r = \dfrac{n!}{(n-r)!} \longrightarrow (i)$

R.H.S $n\left({}^{n-1}P_{r-1}\right) = n \dfrac{(n-1)!}{\{(n-1)-(r-1)\}!}$

$= \dfrac{n(n-1)!}{(n-1-r+1)!}$

$= \dfrac{n!}{(n-r)!} \longrightarrow (ii)$

From eqns (i) and (ii) it is proved

${}^nP_r = n \; {}^{(n-1)}P_{(r-1)}$

(ii) ${}^nP_r = {}^{n-1}P_r + r\left({}^{n-1}P_{r-1}\right)$

L.H.S ${}^nP_r = \dfrac{n!}{(n-r)!} \longrightarrow (i)$

R.H.S ${}^{n-1}P_r + r\;{}^{n-1}P_{r-1}$

$= \dfrac{(n-1)!}{(n-1-r)!} + r \dfrac{(n-1)!}{\{(n-1)-(r-1)\}!}$

$= \dfrac{(n-1)!}{(n-r-1)!} + r \dfrac{(n-1)!}{(n-1-r+1)!}$

$= \dfrac{(n-1)!}{(n-r-1)!} + r \dfrac{(n-1)!}{(n-r)!}$

$= \dfrac{(n-1)!}{(n-r-1)!} + \dfrac{r(n-1)!}{(n-r)(n-r-1)!}$

take $\dfrac{(n-1)!}{(n-r-1)!}$ as common

$= \dfrac{(n-1)!}{(n-r-1)!}\left\{1 + \dfrac{r}{n-r}\right\}$

$= \dfrac{(n-1)!}{(n-r-1)!}\left\{\dfrac{n-r+r}{n-r}\right\}$

$= \dfrac{(n-1)!\; n}{(n-r-1)!\;(n-r)} = \dfrac{n(n-1)!}{(n-r)(n-r-1)!} = \dfrac{n!}{(n-r)!} \longrightarrow (ii)$

From eqns (i) and (ii), it is proved

${}^nP_r = {}^{n-1}P_r + r\;{}^{n-1}P_{r-1}$

Q:4 In how many ways can a police department arrange eight suspects in a line up?

Sol Total suspects $= n = 8$

suspect to be arranged $= r = 8$

Then total arrangments $= {}^8P_8 = 8! = 40320$ Ans

Q:5 How many different signal, each consisting of three flags hung one above the other, can be made from seven different flags?

Sol: Total flags $= n = 7$

Flags to be arranged $= r = 3$

Then total signals $= {}^7P_3 = \dfrac{7!}{(7-3)!} = \dfrac{7\times6\times5\times4!}{4!}$

$= 210$ Ans

Q:6 In how many ways can five students be seated in a row of eight seats if a certain two students (i) insist on sitting next to each other and (ii) refuse to sit next to each other?

Sol: Total seats $n = 8$

Total students $= r = 5$

Then total possible arrangments $= {}^8P_5$

$= \dfrac{8!}{(8-5)!} = \dfrac{8!}{3!} = 6720$

(i) If two students insist on sitting next to each other then they will be considered one

$\boxed{A}\ \boxed{B}\ \boxed{C}\ \boxed{D}\ \boxed{E}$

Then 4 boys will be there

Hence ${}^8P_4 = 1680$

But the two books can be arranged among themselves in $2! = 2$ ways

Hence total ways in which the two particular boys are together are

$= 2\times1680 = 3360$ ways.

(ii) If the two particular students are not togather are $6720 - 3360$

$= 5040$ Ans

Q:7 How many numbers each lying between 10 and 1000 can be formed with the digits 2, 3, 4, 0, 8, 9 using only once (no repetition)

Sol Total digits $= 6$

Possible two digit #s $= \dfrac{\text{1st place}}{5 \text{ ways}} \times \dfrac{\text{2nd place}}{5\text{ ways}}$

(Note 0 can't come to the left in two digit #s)

$= 25$ ways

Possible three digits #s $= \dfrac{\text{1st place}}{6\text{ ways}} \times \dfrac{\text{2nd place}}{5\text{ ways}} \times \dfrac{\text{3rd place}}{4\text{ ways}}$

$= 120$ ways

Exple Three digit #

0 4 9

2 3 8 (or)

Total possible #s $= 25 + 120$

$= 145$ Ans

Q:8 How many different words can be formed from the letters if all the letters are taken at a time?

(i) BOOKWORM

Sol. Total alphabets $= n = 8$

But $O = 3$ times

Then total possible words $= \dfrac{8!}{3!} = \dfrac{40320}{6} = 6720$ Ans

(ii) OOKKEEPER

Sol Total alphabets $= n = 9$

But $E = 3$ times

$K = 2$ times

$O = 2$ times

Then total possible words $= \dfrac{9!}{3!\,2!\,2!} = \dfrac{362880}{6\times2\times2}$

$= 15120$ Ans

(iii) ABBOTTABAD

Sol Total alphabets $= n = 10$

But $A = 3$ times

$B = 3$ times

$T = 2$ times

Then total possible words $= \dfrac{10!}{3!\,3!\,2!} = \dfrac{3628800}{6\times6\times2}$

$= 50400$ Ans

(iv) LETTER

Sol Total alphabets $= n = 6$

But $T = 2$ times

$E = $ " "

Total possible words $= \dfrac{6!}{2!\,2!} = \dfrac{720}{2\times2} = 180$ Ans

Q:9: In how many distinct ways can $x^4y^3z^5$ be expressed without exponents?

Sol As $x^4y^3z^5$ without exponents $= xxxxyyyzzzzz$

Total alphabets $= n = 12$

But $x = 4$ times

$y = 3$ times

$z = 5$ times

Total distant ways without exponents $= \dfrac{12!}{4!\,3!\,5!}$

$= \dfrac{479001600}{24\times6\times120} = 27720$ Ans

Q:10: How many different ten digit numerals can be formed from the digits 3, 3, 3, 3, 1, 1, 1, 7, 7 & 5

Sol Total digits $= 10$

But $3 = 4$ times

$1 = 3$ times

$7 = 2$ times

$5 = 1$ time

Then total possible numbers $= \dfrac{10!}{4!\,3!\,2!}$

$= \dfrac{3628800}{24\times6\times2} = 12600$ Ans

Q:11: In how many different ways can be six children seated at a round table if a certain two children (i) refuse to sit next to each other & (ii) insist on sitting next to each other?

Sol: Total number of children $= n = 6$

Total arrangment around a table $= (n-1)! = (6-1)! = 5!$

$= 120$

(ii) If the two children insist on sitting next to each other then they will be considred as one (but they can arranged themselves in $2! = 2$ ways among themselves). Also n objects can be arranged in $(n-1)!$ ways around a table.

Then total arrangment $= 2 \times (5-1)!$
$= 2 \times 4!$
$= 2 \times 24$
$= 48$ ways.

(i) The two boys refuse to sit next to each other

Then total such ways will be
$= 120 - 48$
$= 96$ ways

Q:12 If five distinct keys are placed on a key ring, how many different orders are possible?

Sol As number of possible different arrangment of keys on a ring is $\dfrac{(n-1)!}{2}$

$$= \frac{(5-1)!}{2}$$

$$= \frac{4!}{2} = \frac{24}{2} = 12 \quad \text{Ans}$$

Exercise # 6.4

Q:1 Solve the following for n?

(i) ${}^nC_2 = 36$

Sol ${}^nC_2 = 36$

$$\Rightarrow \frac{n!}{(n-2)!\,2!} = 36$$

$$\Rightarrow \frac{n(n-1)(n-2)!}{(n-2)!\,2} = 36$$

$\Rightarrow n(n-1) = 72$

Making same order to b.s

$\Rightarrow n(n-1) = 9 \times 8$

$\Rightarrow n = 9$ or $n-1 = 8$

$\Rightarrow n = 9$

Hence $\boxed{n = 9}$ Ans

(ii) ${}^{n+1}C_4 = 6 \cdot {}^{n-1}C_2$

Sol ${}^{(n+1)}C_4 = 6 \; {}^{(n-1)}C_2$

$$\Rightarrow \frac{(n+1)!}{(n+1-4)!\,4!} = 6\,\frac{(n-1)!}{(n-1-2)!\,2!}$$

$$\Rightarrow \frac{(n+1)\cdot n\,(n-1)!}{(n-3)!\,4\times3\times2\times1} = \frac{6\,(n-1)!}{(n-3)!\,2}$$

$$\Rightarrow \frac{(n+1)n}{4\times3} = 6 \quad \Rightarrow n^2+n = 6\times4\times3$$

$\Rightarrow n^2+n = 72$

$\Rightarrow n(n+1) = 8\times 9$

$\Rightarrow n=8$ or $n+1=9$
$n=8$

Hence $n=8$ Ans

(iii) ${}^{n^2}C_2 = 30\,{}^{n}C_3$

Sol
$$\frac{n^2!}{(n^2-2)!\,2!} = 30\,\frac{n!}{(n-3)!\,3!}$$

$$\Rightarrow \frac{n^2(n^2-1)(n^2-2)!}{(n^2-2)!\;2!} = 30\,\frac{n(n-1)(n-2)(n-3)!}{(n-3)!\;\;3\times 2!}$$

$$\Rightarrow n^2(n^2-1) = \frac{30\,n(n-1)(n-2)}{3}$$

$\Rightarrow n^2(n^2-1) = 10\,n(n-1)(n-2)$

$\Rightarrow n.n\,(n+1)(n-1) = 10\,n(n-1)(n-2)$

$\Rightarrow n(n+1) = 10(n-2)$

$\Rightarrow n^2+n = 10n-20$

$\Rightarrow n^2-9n+20=0$

By factorization

$\Rightarrow n^2-5n-4n+20=0$

$\Rightarrow n(n-5)-4(n-5)=0$

$\Rightarrow (n-5)(n-4)=0$

$n-5=0$ or $n-4=0$

$n=5$ $\quad n=4$

Hence $n=4;5$ Ans

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Q.2 Find n and r if
${}^{n}P_r = 840$ and ${}^{n}C_r = 35$

Sol ${}^{n}P_r = 840$ and ${}^{n}C_r = 35$

$\Rightarrow \frac{n!}{(n-r)!} = 840$ ⓘ $\quad \Rightarrow \frac{n!}{(n-r)!\,r!} = 35$

$\Rightarrow \frac{n!}{(n-r)!} = 35.\,r!$

P.T.V of $\frac{n!}{(n-r)!}$ from Equ ⓘ

$\Rightarrow 840 = 35.\,r!$

$\Rightarrow r! = \frac{840}{35}$

$\Rightarrow r! = 24$

$\Rightarrow r! = 4!$

$\Rightarrow \boxed{r=4}$

Equ ⓘ $\Rightarrow \frac{n!}{(n-4)!} = 840$

$$\Rightarrow \frac{n(n-1)(n-2)(n-3)(n-4)!}{(n-4)!} = 840$$

$\Rightarrow n(n-1)(n-2)(n-3) = 7\times 6\times 5\times 4$

$\Rightarrow n=7$ or $n-1=6$
$\therefore n=7$

Hence $\left.\begin{matrix} r=4 \\ n=7 \end{matrix}\right\}$ Ans

Q.3 Find n, when

$${}^{2n}C_3 : {}^{n}C_2 = 36 : 3$$

Sol

$${}^{2n}C_3 : {}^{n}C_2 = 36 : 3$$

$$\Rightarrow \frac{{}^{2n}C_3}{{}^{n}C_2} = \frac{36}{3}$$

$$\Rightarrow {}^{2n}C_3 = 12\,{}^{n}C_2$$

$$\Rightarrow \frac{(2n)!}{(2n-3)!\,3!} = 12\,\frac{n!}{(n-2)!\,2!}$$

$$\Rightarrow \frac{(2n)(2n-1)(2n-2)(2n-3)!}{(2n-3)!\;\;3\times 2!} = \frac{12\,n(n-1)(n-2)!}{(n-2)!\;\;2!}$$

$$\Rightarrow \frac{2n(2n-1)(2n-2)}{3} = 12n(n-1)$$

$$\Rightarrow 2(2n-1)(2n-2) = 36(n-1)$$

$$\Rightarrow 2(2n-1)\,2(n-1) = 36(n-1)$$

$$\Rightarrow 4(2n-1) = 36 \Rightarrow 2n-1 = 9 \Rightarrow 2n = 10 \Rightarrow \boxed{n=5}\ \text{Ans}$$

Q.4 Prove that

$${}^{n-1}C_r + {}^{n-1}C_{r-1} = {}^{n}C_r$$

Sol R.H.S

$${}^{n}C_r = \frac{n!}{(n-r)!\,r!} \longrightarrow (i)$$

L.H.S ${}^{n-1}C_r + {}^{n-1}C_{r-1}$

$$= \frac{(n-1)!}{(n-1-r)!\,r!} + \frac{(n-1)!}{\{(n-1)-(r-1)\}!\,(r-1)!}$$

$$= \frac{(n-1)!}{(n-r-1)!\,r!} + \frac{(n-1)!}{(n-1-r+1)!\,(r-1)!}$$

$$= \frac{(n-1)!}{(n-r-1)!\,r!} + \frac{(n-1)!}{(n-r)!\,(r-1)!}$$

$$= \frac{(n-1)!}{(n-r-1)!\,r(r-1)!} + \frac{(n-1)!}{(n-r)(n-r-1)!\,(r-1)!}$$

Take $\dfrac{(n-1)!}{(n-r-1)!\,(r-1)!}$ as common

$$= \frac{(n-1)!}{(n-r-1)!\,(r-1)!}\left\{\frac{1}{r} + \frac{1}{n-r}\right\}$$

$$= \frac{(n-1)!}{(n-r-1)!\,(r-1)!}\left\{\frac{1(n-r)+r}{r(n-r)}\right\}$$

$$= \frac{(n-1)!}{(n-r-1)!\,(r-1)!}\left\{\frac{n-r+r}{r(n-r)}\right\}$$

$$= \frac{(n-1)!}{(n-r-1)!\,(r-1)!}\left\{\frac{n}{r(n-r)}\right\}$$

$$= \frac{n(n-1)!}{(n-r)(n-r-1)!\,r(r-1)!}$$

$$= \frac{n!}{(n-r)!\;\;r!} \longrightarrow (ii)$$

From eqns (i) and (ii), it is proved

$${}^{n-1}C_r + {}^{n-1}C_{r-1} = {}^{n}C_r$$

(ii) $r \cdot {}^nC_r = n \cdot {}^{n-1}C_{r-1}$

L.H.S

$$r \cdot {}^nC_r$$

$$= r \cdot \frac{n!}{(n-r)!\,r!}$$

$$= r \cdot \frac{n!}{(n-r)!\,r(r-1)!}$$

$$= \frac{n!}{(n-r)!\,(r-1)!} \longrightarrow (i)$$

R.H.S

$$n \cdot {}^{n-1}C_{r-1}$$

$$= n \cdot \frac{(n-1)!}{\{(n-1)-(r-1)\}!\,(r-1)!}$$

$$= \frac{n \cdot (n-1)!}{(n-1-r+1)!\,(r-1)!}$$

$$= \frac{n!}{(n-r)!\,(r-1)!} \longrightarrow (ii)$$

From eqns (i) and (ii), it is proved that

$$r \cdot {}^nC_r = n \cdot {}^{n-1}C_{r-1}$$

Q.5 How many lines are determined by eight points if none of the three points are collinear? How many triangles are determined?

Sol Total points $= n = 8$

If we join any two lines we get a line

So select any two out of 8

Hence $r = 2$

So total possible lines $= {}^nC_r = {}^8C_2 = \frac{8!}{(8-2)!\,2!}$

$$= \frac{8 \times 7 \times 6!}{6!\;2}$$

$$= 28 \text{ Ans}$$

A triangle is formed if we join three non collinear points. So $r = 3$

Hence total triangles $= {}^nC_r$

$$= {}^8C_3$$

$$= \frac{8!}{(8-3)!\,3!} = \frac{8 \times 7 \times 6 \times 5!}{5!\;3!}$$

$$= 56 \text{ Ans}$$

Q.6 Three non-collinear points determine a circle. How many circles are determined by 5 such points?

Sol Total points $= n = 5$

Select $= r = 3$

Total possible selections

$${}^nC_r = {}^5C_3$$

$$= \frac{5!}{(5-3)!\,3!} = \frac{5 \times 4 \times 3!}{2!\;3!}$$

$$= 10 \text{ Ans}$$

Q.7: A box contains 6 red balls and 4 green balls. In how many ways 4 balls be choosen such that exactly 2 are green?

Sol Total ballz $= n = 10$
Red balls $= 6$
Green balls $= 4$

Then a selection of 4 balls out of which two balls will be green (remaining two will be red). So it is like we are selecting 2 green from 4 greens and 2 red from 6 red balls. Hence total selections are

$$= \underbrace{{}^6C_2}_{\text{for red}} \cdot \underbrace{{}^4C_2}_{\text{for green}} = \frac{6!}{(6-2)!\,2!} \cdot \frac{4!}{(4-2)!\,2!}$$

$$= \frac{6\times5\times4!}{4!\;2} \cdot \frac{4\times3\times2!}{2!\;2}$$

$$= 15 \cdot 6$$

$$= 90 \quad \text{Ans}$$

Q.8 From 12 books in how many ways can be a selection of 5 be made (i) when one specified books is alwey included (ii) when one specified book is always excluded?

Sol Total books $= n = 12$
5 books will be selected $\Rightarrow r = 5$

(i) If we book must be selected $= {}^1C_1$

Then 4 other books will be selected from the remaining 11 books. Then total possible selections are

$${}^1C_1 \cdot {}^{11}C_4$$

$$= 1 \cdot (330) = 330 \quad \text{Ans}$$

(ii) If one book is always excluded

$\Rightarrow$ we will be selecting 5 books from 11 books
 ($r = 5$, $n = 11$)

$$\Rightarrow \boxed{{}^{11}C_5 = 462} \quad \text{Ans}$$

Q.9 How many diagnals can be drawn in a plane figure of 8 sides.

Sol Total sides $= n = 8$
Select any two $\Rightarrow r = 2$, we will get total lines out of which 8 will be sides
Then diagnals $=$ Total $-$ sides
$= {}^nC_2 - n$

$\Rightarrow$ Total diagnals

$$= {}^8C_2 - 8$$

$$= \frac{8!}{(8-2)!\,2!} - 8$$

$$= \frac{8\times7\times6!}{6!\;2!} - 8 = 28 - 8 = 20 \quad \text{Ans}$$

Q.10 A committe of seven persons is to be choosen from 10 men and 8 women. How many of these will have

(a) exactly four men

(b) At the most four men

(c) At least four men

For solution P.T.O

(i) Total persons $= n = 18$
because men $= 10$
and women $= 8$
Persons to be selected $= 7$.

(i) Exactly four men (Then three women will be selected out 8)

$= {}^{10}C_4 \cdot {}^8C_3$

$= \frac{10!}{(10-4)!\,4!} \times \frac{8!}{(8-3)!\,3!}$

$= \frac{10\times9\times8\times7\times6!}{6!\;4\times3\times2\times1} \times \frac{8\times7\times6\times5!}{5!\;3\times2\times1}$

$= 210 \times 56$

$= 11760$ Ans

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(ii) At most four men
Then possibilities

- ${}^{10}C_0 \cdot {}^8C_7 = 1 \times 8 = 8$
- ${}^{10}C_1 \cdot {}^8C_6 = 10 \times 28 = 280$
- ${}^{10}C_2 \cdot {}^8C_5 = 45 \times 56 = 2520$
- ${}^{10}C_3 \cdot {}^8C_4 = 120 \times 70 = 8400$
- ${}^{10}C_4 \cdot {}^8C_3 = 210 \times 56 = 11760$

Total possibilities $= 8 + 280 + 2520 + 8400 + 11760 = 22968$

(iii) At least four Meen

- ${}^{10}C_4 \cdot {}^8C_3 = 210 \times 56 = 11760$
- ${}^{10}C_5 \cdot {}^8C_2 = 252 \times 28 = 7056$
- ${}^{10}C_6 \cdot {}^8C_1 = 210 \times 8 = 1680$

${}^{10}C_7 \cdot {}^8C_0 = 120 \cdot 1 = 120$

Total $= 11760 + 7056 + 1680 + 120$

$= 20616$

Exercise # 6.5

Q.1 $S = \{1,2,3,4,5,6\}$

(i) Let $A = \{5\} \Rightarrow n(A) = 1$

$P(A) = \frac{\text{No of elements in } A}{\text{No of elements in } S} = \frac{n(A)}{n(S)} = \frac{1}{6}$

(ii) $B =$ less than 1

$\Rightarrow B = \{\ \} \Rightarrow n(B) = 0$

$P(B) = \frac{n(B)}{n(S)} = \frac{0}{6} = 0$

(iii) Let $C =$ Greater than 0

$\Rightarrow C = \{1, 2, 3, 4, 5, 6\} \Rightarrow n(C) = 6$

So $P(C) = \frac{n(C)}{n(S)} = \frac{6}{6} = 1$

(iv) Let $D =$ Multiple of 3

$\Rightarrow D = \{3, 6\} \Rightarrow n(D) = 2$

$\Rightarrow P(D) = \frac{n(D)}{n(S)} = \frac{2}{6} = \frac{1}{3}$

(v) Let $E =$ Greater or equal to 4.

$\Rightarrow E = \{4,5,6\}$

$\Rightarrow P(E) = \frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

Q:2 Give the sample space of rolling a pair of DICE.
(a) what is probability of
(i) Rolling a total of 7?

Sol The sample space of rolling a pair of DICE is

$$S = \begin{Bmatrix} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6) \\ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6) \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6) \\ (4,1), (4,2), (4,3), (4,4), (4,5), (4,6) \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6) \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \end{Bmatrix} \Rightarrow n(S) = 36$$

(i) let A = total is 7

$\Rightarrow A = \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\} \Rightarrow n(A) = 6$

$\Rightarrow P(A) = \dfrac{n(A)}{n(S)} = \dfrac{6}{36} = \dfrac{1}{6}$

(ii) Let B = total is 11

$\Rightarrow B = \{(5,6), (6,5)\} \Rightarrow n(B) = 2$

so $P(B) = \dfrac{n(B)}{n(S)} = \dfrac{2}{36} = \dfrac{1}{18}$ Ans

(iii) let C = Total is than or equal to 12.

$\Rightarrow C = \{(6,6)\} \Rightarrow n(C) = 1$

$\Rightarrow P(C) = \dfrac{n(C)}{n(S)} = \dfrac{1}{36}$

(b) Which is more likely
(i) Rolling a total 7 or total 9? why?

Sol let m_1 = Rolling a total 7

$\Rightarrow m_1 = \{(3,4), (5,2), (6,1), (1,6), (2,5), (4,3)\}$

$\Rightarrow n(m_1) = 6$

$n(S) = 36$

Then probability of rolling a total 7

$P(m_1) = \dfrac{n(m_1)}{n(S)} = \dfrac{6}{36} = \dfrac{1}{6}$

now Let m_2 = Rolling a total 9

$\Rightarrow m_2 = \{(3,6), (4,5), (5,4), (6,3)\}$

$\Rightarrow n(m_2) = 4$

Then probability of rolling a total 9 will be

$P(m_2) = \dfrac{n(m_2)}{n(S)} = \dfrac{4}{36} = \dfrac{1}{9}$

Now $\dfrac{1}{6} > \dfrac{1}{9}$

$\Rightarrow P(m_1) > P(m_2) \Rightarrow$ m_1 has more chances of occurance than m_2

Hence a total of 7 is more likely than a total of 9.

(ii) Rolling a total of 11 or total of 3. why.

Sol Let m_1 = Rolling a total of 11.

$\Rightarrow m_1 = \{(5,6), (6,5)\} \Rightarrow n(m_1) = 2$

$\Rightarrow P(m_1) = \dfrac{2}{36} = \dfrac{1}{18}$

and m_2 = Rolling a total of 3

$\Rightarrow m_2 = \{(1,2), (2,1)\}$

$\Rightarrow n(m_2) = 2$

$\Rightarrow P(m_2) = \dfrac{2}{36} = \dfrac{1}{18}$

Since $P(m_1) = P(m_2) \Rightarrow$ Chances of occurance of m_1 is the same as that of m_2

$\Rightarrow$ Both are equally likely to occur.

Q:3 A true or false test contains eight questions. If a student guesses the answer for each question, find the probability

(a) 8 answers are correct.

Sol Since each question can have two answer i:e True or False and we have eight such questions. Then by fundamental principle of counting total outcomes will be

$S = 2\times2\times2\times2\times2\times2\times2\times2$ or $(2\times2\times \cdots$ 8 times$)$

$= 2^8$

$= 256$ outcomes.

(a) Let $A = 8$ answers are correct

$= {}^8C_8 = 1$

$\Rightarrow n(A) = 1$

Then $P(A) = \dfrac{n(A)}{n(S)} = \dfrac{1}{256}$ Ans

(b) 7 answers are correct

Let $B = 7$ answers are correct

$\Rightarrow n(B) = {}^8C_7 = 8$

Then $P(B) = \dfrac{8}{256} = \dfrac{1}{32}$ Ans

(c) 6 answers correct & 2 incorrect. Let C is the case

$\Rightarrow {}^8C_6 = \dfrac{8!}{(8-6)!\,6!} = \dfrac{8\times7\times6!}{2!\;6!} = 28$

$\Rightarrow P(C) = \dfrac{n(C)}{n(S)} = \dfrac{28}{256} = \dfrac{7}{64}$ Ans

(d) Let $D =$ Atleast 6 answers are correct.

$\Rightarrow D = {}^8C_6 + {}^8C_7 + {}^8C_8$

$= 28 + 8 + 1 = 37$

$\Rightarrow P(D) = \dfrac{n(D)}{n(S)}$

$\Rightarrow P(D) = \dfrac{37}{36}$ Ans

Q:4 A golf ball is selected at random from a container. If the container has 9 white, 8 green and ba . Find the probability that the golf ball is

(a) white

Sol White balls $= 9$

Green balls $= 8$

orange balls $= 3$

$\Rightarrow$ Total balls $= 20$.

(a) $P(\text{white}) = \dfrac{9}{20}$

(b) $P(\text{Green}) = 8/20$

(c) $P(\text{white or Green})$

Let $X_1 =$ white or Green

$X_1 = 9 + 8$

$\Rightarrow X_1 = 17$

$\Rightarrow P(X_1) = \dfrac{17}{20}$

(d) Not white

Sol Let $X_2 =$ not white $=$ Green or orange $= 8 + 3 = 11$

$\Rightarrow P(X_2) = \dfrac{11}{20}$ Ans

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Q.5 A committe of 5 persons is to be selected at random from 6 men and 4 women. Find the probability that the committe will consist of (i) 3 men & 2 women

Sol

Men = 6

Women = 4

Total = 6 + 4 = 10

Total possible selections = ${}^{10}C_5 = 252$

(i) 3 men and 2 women are selected

$= {}^6C_3 \cdot {}^4C_2$

$= 20 \cdot 6 = 120$

Then $P(3 \text{ men and } 2 \text{ women}) = \frac{120}{252} = \frac{10}{21}$ Ans

(ii) 2 men and 3 women

$= {}^6C_2 \cdot {}^4C_3$

$= 15 \cdot 4 = 60$

$\Rightarrow P(2 \text{ men and } 3 \text{ women}) = \frac{60}{252} = \frac{5}{21}$ Ans

Exercise # 6.6

Q.1 Suppose events A and B are such that $P(A) = 2/5$ and $P(B) = 2/5$ and $P(A \cup B) = 1/2$ find $P(A \cap B) = ?$

Sol As $P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$\Rightarrow \frac{1}{2} = \frac{2}{5} + \frac{2}{5} - P(A\cap B)$

$\Rightarrow \frac{1}{2} = \frac{4}{5} - P(A\cap B)$

$\Rightarrow P(A\cap B) = \frac{4}{5} - \frac{1}{2}$

$\Rightarrow P(A\cap B) = \frac{8-5}{10} \Rightarrow \boxed{P(A\cap B) = 3/10}$ Ans

Q.2 If $P(A) = \frac{1}{3}$, $P(A\cup B) = \frac{1}{2}$ and $P(A\cap B) = \frac{1}{4}$. Find $P(B)$.

Sol As $P(A\cup B) = P(A) + P(B) - P(A\cap B)$, Put the values

$\Rightarrow \frac{1}{2} = \frac{1}{3} + P(B) - \frac{1}{4}$

$\Rightarrow \frac{1}{2} = \frac{1}{3} - \frac{1}{4} + P(B)$

$\Rightarrow \frac{1}{2} = \frac{4-3}{12} + P(B)$

$\Rightarrow \frac{1}{2} = \frac{1}{12} + P(B)$

$\Rightarrow P(B) = \frac{1}{2} - \frac{1}{12}$

$\Rightarrow P(B) = \frac{6-1}{12}$

$\Rightarrow \boxed{P(B) = 5/12}$ Ans

Q.3 If $S = (A \cup B)$, $P(A) = 0.75$, $P(B) = 0.65$. Find $P(A \cap B)$

Sol As $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

As $P(S) = 1$

$\Rightarrow 1 = 0.75 + 0.65 - P(A \cap B)$

$\Rightarrow 1 = 1.4 - P(A \cap B)$

$\Rightarrow P(A \cap B) = 1.4 - 1$

$\Rightarrow P(A \cap B) = 0.4$ Ans

Q.4: A bag contains 30 tickets numbered from 1 to 30. One ticket is selected at random. Find the probability that its number is either odd or the square of an integer?

Sol
$S = \{1, 2, 3, \ldots\ldots, 29, 30\}$

$n(S) = 30$

let A = The number selected is odd

$\Rightarrow A = \{1, 3, 5, \ldots\ldots, 29\}$

$\Rightarrow n(A) = 15$

Now probability that the number is odd will be

$\Rightarrow P(A) = \dfrac{n(A)}{n(S)} = \dfrac{15}{30} = 1/2$

let B = the number selected is square of an integer

$\Rightarrow B = \{4, 9, 16, 25\}$

$\Rightarrow n(B) = 4$

Probability of a number whose is square of an integer

$\Rightarrow P(B) = \dfrac{n(B)}{n(S)} = \dfrac{4}{30}$

$= \dfrac{2}{15}$

Now $A \cap B = \{9, 25\}$ i.e $A \cap B$ will show both A & B i.e odd and square of an integer

$\Rightarrow n(A \cap B) = 2$

$\Rightarrow P(A \cap B) = \dfrac{2}{30} = \dfrac{1}{15}$

As $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A \cup B) = \dfrac{1}{2} + \dfrac{2}{15} - \dfrac{1}{15}$

$= \dfrac{15 + 4 - 2}{30} = 17/30$ Ans

Q:5 Given $P(A) = 0.5$ $P(A \cup B) = 0.6$
Find $P(B)$ if A and B are mutually exclusive.

Sol
As $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

and if A and B are mutually exclusive (or disjoint)

$\Rightarrow A \cap B = \{\} \Rightarrow P(A \cap B) = 0$

Hence $P(A \cup B) = P(A) + P(B)$

Put the values, we get

$\Rightarrow 0.6 = 0.5 + P(B)$

$\Rightarrow P(B) = 0.6 - 0.5$

$\Rightarrow \boxed{P(B) = 0.1}$

Q:6: Suppose that each of the letters of the word MATHEMATICS are written on scrapes of paper of the same size, dropped into a bag and mixed thoroughly. Find the probability of drawing an M or an N.

Sol S. $S = \{M, A, T, H, E, M, A, T, I, C, S\}$

$\Rightarrow n(S) = 11$

But M is twice. Also A is Twice

$\Rightarrow P(M) = \frac{2}{11}$ & $P(A) = \frac{2}{11}$

Now $P(M \text{ or } N) = P(M) + P(N)$ Because both events are disjoints.

$\Rightarrow P(M \text{ or } N) = \frac{2}{11} + \frac{2}{11}$

$= \frac{4}{11}$ Ans

Q:7 In a class of 100 students 50 have taken physics and 80 have taken Mathematics. One student is selected at random. Show that the probality that he has taken both subjects is ≥ 0.30.

Sol Total student $= 100$

$\Rightarrow n(S) = 100$

Let X = student which has taken Physics

$\Rightarrow n(X) = 50$

and Y = student which has taken mathematics

$\Rightarrow n(Y) = 80$

Now $P(X) = \frac{n(X)}{n(S)} \Rightarrow P(X) = \frac{50}{100} = \frac{1}{2}$

and $P(Y) = \frac{n(Y)}{n(S)}$

$= \frac{80}{100}$

$= \frac{4}{5}$

$P(X \cap Y)$
$\Rightarrow$ both X & Y
$P(X \cup Y)$
$=$ X or Y

we will show that $P(X \cap Y) \leq 1$

As $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$

$\Rightarrow P(X \cap Y) = P(X) + P(Y) - P(X \cup Y)$

$\Rightarrow P(X \cap Y) = \frac{1}{2} + \frac{4}{5} - P(X \cup Y)$

$\Rightarrow P(X \cap Y) = \frac{13}{10} - P(X \cup Y) \Rightarrow P(X \cap Y) = 1.3 - P(X \cup Y)$

put $P(X \cup Y) \leq 1$

$\Rightarrow P(X \cap Y) \geq 1.3 - 1 \Rightarrow P(X \cap Y) \geq 0.3$ Ans

Q:8 A student figures that the probability of passing a test is 8/9. What is the probability of failing that test?

Sol Let X shows passing the test

$\Rightarrow X'$ shows not passing the test.

As $P(X) + P(X') = 1$

$\Rightarrow P(X') = 1 - P(X)$

$\Rightarrow P(X') = 1 - \frac{8}{9}$

$\Rightarrow P(X') = \frac{1}{9}$ Ans

Q.9 In a two dice experiment, given that the 1st dice shows 4.. what is the probability that the second dice shows a number greater than 4?

Sol. Sample space for a two dice experiment is

$$S = \begin{Bmatrix} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6) \\ (2,1), (2,2), \text{- - - - -} (2,6) \\ \vdots \\ (6,1), (6,2), \text{- - - - - - - -} (6,6) \end{Bmatrix}$$

$\Rightarrow n(S) = 36$

Let A shows that the 1st dice outcome is 4

$\Rightarrow A = \{(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)\}$

$\Rightarrow n(A) = 6$

$P(A) = \dfrac{n(A)}{n(S)} = \dfrac{6}{36} = \dfrac{1}{6}$

Let B shows that the outcome of 2nd dice is greater than 4.

$$\Rightarrow B = \begin{Bmatrix} (1,5), (1,6), (2,5), (2,6), (3,5), (3,6) \\ (4,5), (4,6), (5,5), (5,6), (6,5), (6,6) \end{Bmatrix}$$

$\Rightarrow n(B) = 12 \quad \Rightarrow P(B) = \dfrac{n(B)}{n(S)} \quad \Rightarrow P(B) = \dfrac{12}{36} = \dfrac{1}{3}$

As $(4,5)$ and $(4,6)$ is common in both

$\Rightarrow A \cap B = \{(4,5), (4,6)\}$

So $n(A \cap B) = 2 \quad \Rightarrow P(A \cap B) = \dfrac{2}{36} = \dfrac{1}{18}$

As $P(A \cap B) = P(A) \cdot P(B/A)$

$\Rightarrow P(B/A) = \dfrac{P(A \cap B)}{P(A)}$

$\Rightarrow P(B/A) = \dfrac{1/18}{1/6}$

$= \dfrac{6}{18}$

$\boxed{P(B/A) = \dfrac{1}{13}}$ Ans

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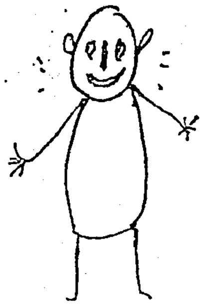

Hurrah! we have finished chapter # 06.