Exercise # 4.1

Q:1 Classify the following into finite and infinite sequences.

Sol

(i) $2, 4, 6, 8, \ldots\ldots 50$ Finite

(ii) $1, 0, 1, 0, 1, \ldots$ Infinite

(iii) $\ldots, -4, 0, 4, 8, \ldots 60$ Infinite

(iv) $1, -\frac{1}{3}, \frac{1}{9}, \ldots \frac{-1}{2187}$ Finite

Q:2 Find the first four terms of the sequence with the general terms

(i) let $T_n = \dfrac{n(n+1)}{2}$

Sol

$n=1 \quad T_1 = \dfrac{1(1+1)}{2} = \dfrac{1(2)}{2} = 1$

$n=2 \quad T_2 = \dfrac{2(2+1)}{2} = \dfrac{2(3)}{2} = 3$

$n=3 \quad T_3 = \dfrac{3(3+1)}{2} = \dfrac{3(4)}{2} = 6$

$n=4 \quad T_4 = \dfrac{4(4+1)}{2} = \dfrac{4(5)}{2} = 10.$

Ans

Ex 4.1

(H-04) P-01

(ii) $(-1)^{n-1}\, 2^{n+1}$

let $a_n = (-1)^{n-1}\, 2^{n+1}$

for $n=1 \quad a_1 = (-1)^{1-1}\, 2^{1+1} = (-1)^0\, 2^2 = 1(4) = 4$

for $n=2 \quad a_2 = (-1)^{2-1}\, 2^{2+1} = (-1)^1\, 2^3 = -1(8) = -8$

for $n=3 \quad a_3 = (-1)^{3-1}\, 2^{3+1} = (-1)^2\, 2^4 = 1(16) = 16$

for $n=4 \quad a_4 = (-1)^{4-1}\, 2^{4+1} = (-1)^3\, 2^5 = -1(32) = -32$

(iii) $a_n = \left(\frac{1}{3}\right)^n$

for $n=1 \quad a_1 = \left(\frac{1}{3}\right)^1 = \frac{1}{3}$

for $n=2 \quad a_2 = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$

for $n=3 \quad a_3 = \left(\frac{1}{3}\right)^3 = \frac{1}{27}$

for $n=4 \quad a_4 = \left(\frac{1}{3}\right)^4 = \frac{1}{81}$

Ans

(iv) $a_n = \dfrac{n(n-1)(n-2)}{6}$

for $n=1 \quad a_1 = \dfrac{1(1-1)(1-2)}{6} = \dfrac{1(0)(-1)}{6} = 0$

for $n=2 \quad a_2 = \dfrac{2(2-1)(2-2)}{6} = \dfrac{2(1)(0)}{6} = 0$

for $n=3 \quad a_3 = \dfrac{3(3-1)(3-2)}{6} = \dfrac{3(2)(1)}{6} = 1$

for $n=4 \quad a_4 = \dfrac{4(4-1)(4-2)}{6} = \dfrac{4(3)(2)}{6} = 4$

Ans

Q:3 Write down the nth term of each sequence as suggested by the pattern.

(i) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots\ldots$

Sol $a_1 = \frac{1}{2}$, $a_2 = \frac{2}{3}, \ldots$ suggest that

$a_n = \frac{n}{n+1}$

(ii) $2, -4, 6, -8, 10, \ldots\ldots$

The terms suggest that

$a_n = (-1)^{n+1} 2n$

(iii) $1, -1, 1, -1, \ldots\ldots$

The terms suggest that

$a_n = (-1)^{n+1}$ or $a_n = (-1)^{n-1}$

Q:5 Write down the 1st five terms of the sequence defined recursively

(i) $a_1 = 3$, $a_{n+1} = 5 - a_n$

Sol for $n=1$ $a_{1+1} = 5 - a_1$

$\Rightarrow a_2 = 5 - 3 = 2$

for $n=2$ $a_3 = 5 - a_2$
$= 5 - 2 = 3$

for $n=3$ $a_4 = 5 - a_3$
$= 5 - 3 = 2$

for $n=4$ $a_5 = 5 - a_4$
$= 5 - 2 = 3$

(ii) $a_1 = 3$ $a_{n+1} = \frac{a_n}{n}$

Sol for $n=1$ $a_2 = \frac{a_1}{1} = \frac{3}{1} = 3$

for $n=2$ $a_3 = \frac{a_2}{2} = 3/2$

for $n=3$ $a_4 = \frac{a_3}{3} = \frac{3/2}{3} = 1/2$

for $n=4$ $a_5 = \frac{a_4}{4} = \frac{1/2}{4} = 1/8$

Q:5 Write down each series in expanded form

(i) $\sum_{j=1}^{6} (2j-3)$

Hence $a_j = 2j-3$

for $j=1$ $a_1 = 2(1)-3 = -1$
$j=2$ $a_2 = 2(2)-3 = 1$
$j=3$ $a_3 = 2(3)-3 = 3$
$j=4$ $a_4 = 2(4)-3 = 5$
$j=5$ $a_5 = 2(5)-3 = 7$
$j=6$ $a_6 = 2(6)-3 = 9$

Hence $\sum_{j=1}^{6}(2j-3) = (-1) + 1 + 3 + 5 + 7 + 9$

(ii) $\sum_{k=1}^{5} (-1)^k 2^{k-1}$

Sol Here $a_k = (-1)^k 2^{k-1}$

for $k=1$ $a_1 = (-1)^1 2^{1-1} = (-1) 2^0 = -1$

for $k=2$ $a_2 = (-1)^2 2^{2-1} = (1) 2^1 = 2$

for $k=3$ $a_3 = (-1)^3 2^{3-1} = (-1) 2^2 = -4$

for $k=4$ $a_4 = (-1)^4 2^{4-1} = (+1) 2^3 = +8$

for $k=5$ $a_5 = (-1)^5 2^{5-1} = (-1) 2^4 = -16$

Hence $\sum_{k=1}^{5} (-1)^k 2^{k-1} = (-1) + 2 + (-4) + (8) + (-16) + \cdots$

(iii) $\sum_{j=1}^{\infty} \frac{1}{2^j}$

$= \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots\cdots$

(iv) $\sum_{k=0}^{\infty} \left(\frac{3}{2}\right)^k$

$= \left(\frac{3}{2}\right)^0 + \left(\frac{3}{2}\right)^1 + \left(\frac{3}{2}\right)^2 + \cdots\cdots$

$= 1 + (3/2)^1 + (3/2)^2 + \cdots\cdots$

Ex 4.1 CH-04 P-02

Q:6 Write the following in terms of factorial.

(i) $8 \times 7 \times 6 \times 5$

Sol $\times$ and $\div$ by $4!$

$= 8 \times 7 \times 6 \times 5 \times \frac{4!}{4!}$

$= \frac{8 \times 7 \times 6 \times 5 \times 4!}{4!}$

$= \frac{8!}{4!}$

(ii) $n(n-1)(n-2)$

Sol $\times$ and $\div$ by $(n-3)!$

$= n(n-1)(n-2) \frac{(n-3)!}{(n-3)!}$

$= \frac{n!}{(n-3)!}$

Q.7 Find the Pascal sequence by using its general recursive definition.

(i) $n = 5$

Sol The recursive definition is

$$P_{r+1} = \left(\frac{n-r}{r+1}\right)\binom{n}{r} \quad \& \quad P_0 = 1$$

for $n=5$

$\Rightarrow P_{r+1} = \left(\frac{5-r}{r+1}\right)\binom{5}{r} \quad \& \quad P_0 = 1$

put $r=0$

$P_{0+1} = \left(\frac{5-0}{0+1}\right)\binom{5}{0}$

$\Rightarrow P_1 = \left(\frac{5}{1}\right)(1) \Rightarrow P_1 = 5$

put $r=1$

$\Rightarrow P_{1+1} = \left(\frac{5-1}{1+1}\right)\binom{5}{1}$

$\Rightarrow P_2 = \left(\frac{4}{2}\right)(5) \Rightarrow \boxed{P_2 = 10}$

Now put $r = 2$

$P_{2+1} = \left(\frac{5-2}{2+1}\right)\binom{5}{2}$

$\Rightarrow P_3 = \left(\frac{3}{3}\right)(10) \Rightarrow \boxed{P_3 = 10}$

Put $r=3$

$P_{3+1} = \left(\frac{5-3}{3+1}\right)\binom{5}{3}$

$\Rightarrow P_4 = \left(\frac{2}{4}\right)(10) \Rightarrow \boxed{P_4 = 5}$

Put $r=4$

$P_{4+1} = \left(\frac{5-4}{4+1}\right)\binom{5}{4}$

$P_5 = \left(\frac{1}{5}\right)(5) \Rightarrow P_5 = 1$

Put $r=5$

$P_{5+1} = \left(\frac{5-5}{5+1}\right)\binom{5}{5} \Rightarrow \boxed{P_6 = 0}$

& $P_7, P_8, \ldots = 0$

So the Pascal sequence is

$P_0, P_1, P_2, P_3, P_4, P_5, P_6, \ldots$

$1, 5, 10, 10, 5, 1, 0, 0, \ldots\ldots$

Short cut

$\binom{n}{r}$

$n=5$

$= \binom{5}{r}$

$r=0 \Rightarrow \binom{5}{0} = 1$

$r=1 \Rightarrow \binom{5}{1} = 5$

$r=2 \Rightarrow \binom{5}{2} = 10$

$r=3 \Rightarrow \binom{5}{3} = 10$

$r=4 \Rightarrow \binom{5}{4} = 5$

$r=5 \Rightarrow \binom{5}{5} = 1$

(ii) $n = 6$

$P_{r+1} = \left(\frac{6-r}{r+1}\right)\binom{6}{r} \qquad P_0 = 1$

for $r=0$

$P_1 = \left(\frac{6-0}{0+1}\right)\binom{6}{0} \Rightarrow \left(\frac{6}{1}\right)(1) = 6$

for $r=1$

$P_2 = \left(\frac{6-1}{1+1}\right)\binom{6}{1} \Rightarrow \left(\frac{5}{2}\right)(6) = 15$

for $r=2$

$P_3 = \left(\frac{6-2}{2+1}\right)\binom{6}{2} = \left(\frac{4}{3}\right)(15) = 20$

for $r=3$

$P_4 = \left(\frac{6-3}{3+1}\right)\binom{6}{3} = \left(\frac{3}{4}\right)(20) = 15$

for $r=4$

$P_5 = \left(\frac{6-4}{4+1}\right)\binom{6}{4} = \left(\frac{2}{5}\right)(15) = 6$

for $r=5$

$P_6 = \left(\frac{6-5}{5+1}\right)\binom{6}{5} = \left(\frac{1}{6}\right)(6) = 1$

for $r=6$

$P_7 = \left(\frac{6-6}{6+1}\right)\binom{6}{6} = \frac{0}{7}(1) = 0$

$P_8 = 0, \quad P_9 = 0$

Hence the Pascal sequence is

$1, 6, 15, 20, 15, 6, 1, 0, 0, \ldots$

(iii) $n = 8$

$P_{r+1} = \left(\frac{8-r}{r+1}\right)\binom{8}{r}, \ P_0 = 1$

for $r=0$

$P_1 = \left(\frac{8-0}{0+1}\right)\binom{8}{0} = 8$

for $r=1$

$P_2 = \left(\frac{8-1}{1+1}\right)\binom{8}{1} = 28$

for $r=2$

$P_3 = \left(\frac{8-2}{2+1}\right)\binom{8}{2} = 56$

for $r=3$

$P_4 = \left(\frac{8-3}{3+1}\right)\binom{8}{3} = 70$

for $r=4$

$P_5 = \left(\frac{8-4}{4+1}\right)\binom{8}{4} = 56$

for $r=5$

$P_6 = \left(\frac{8-5}{5+1}\right)\binom{8}{5} = 28$

for $r=6$

$P_7 = \left(\frac{8-6}{6+1}\right)\binom{8}{6} = 8$

Similarly

$P_8 = 1$

$P_9 = 0$

$P_{10} = 0$

Exercise # 4.2

Q:1 Find the common difference, 5th term, 10th term, and nth term of the A.P

(i) 2, 7, 12, ...

Sol common difference $= d = a_2 - a_1$
$= 7 - 2$
$\Rightarrow \boxed{d = 5}$ Ans

nth term

$a_n = a_1 + (n-1)d$ $\qquad a = 2,\ d = 5$

$\Rightarrow \boxed{a_n = 2 + (n-1)5}$ Ans

OR

$a_n = 2 + 5n - 5$

$\Rightarrow \boxed{a_n = 5n - 3}$ Ans

5th term

$n = 5$

$a_n = 2 + (n-1)5$
$a_5 = 2 + (5-1)5$
$= 2 + (4)(5)$
$\boxed{a_5 = 22}$ Ans

10th term

$a_{10} = 2 + (10-1)5$
$\boxed{a_{10} = 47}$ Ans

(ii) $-4, -2, 0, 2, \ldots$

Sol Common difference $= d = a_2 - a_1$
$\Rightarrow d = -2 - (-4)$
$\Rightarrow d = -2 + 4$
$\Rightarrow \boxed{d = 2}$ Ans

nth term

$a_n = a + (n-1)d$
$\Rightarrow \boxed{a_n = -4 + (n-1)2}$ Ans OR $\boxed{a_n = -6 + 2n}$

5th term: $a_5 = -4 + (5-1)2$
$= -4 + 8$
$\boxed{a_5 = 4}$ Ans

10th term $a_{10} = -4 + (10-1)2$
$= -4 + 18$
$\boxed{a_{10} = 14}$

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(2) If in A.P $a_1 = 43$ and $a_{10} = 7$ Find $a_{25} = ?$

Sol As $a_n = a + (n-1)d \longrightarrow (1)$
$\Rightarrow a_{10} = a + (10-1)d$
$\Rightarrow 7 = 43 + 9d$
$\Rightarrow -36 = 9d \Rightarrow \boxed{d = -4}$

Now put $n = 25$ in eqn (1)
$a_{25} = a + (25-1)d$
$= 43 + (24)(-4)$
$= 43 - 96$
$\Rightarrow \boxed{a_{25} = -53}$ Ans

Q:3 Find A.P

if $a_6 + a_4 = 6$ and $a_6 - a_4 = 2/3$

Sol As $a_6 + a_4 = 6 \longrightarrow ①$

$a_6 - a_4 = 2/3 \longrightarrow ②$

Eqn ① + Eqn ②, we get

$\Rightarrow 2a_6 = \frac{20}{3} \Rightarrow \boxed{a_6 = 10/3}$

Eqn ① $\Rightarrow \frac{10}{3} + a_4 = 6 \Rightarrow a_4 = 6 - \frac{10}{3}$

$\boxed{a_4 = \frac{8}{3}}$

Now $a_n = a + (n-1)d$

$\Rightarrow a_6 = a + (6-1)d$ and $a_4 = a + (4-1)d$

$\Rightarrow \frac{10}{3} = a + 5d \longrightarrow (i)$ $\Rightarrow \frac{8}{3} = a + 3d \longrightarrow (ii)$

Eqn (i) − Eqn (ii)

$$\begin{array}{r} a + 5d = 10/3 \\ -a \; -3d = -8/3 \\ \hline 2d = 2/3 \end{array}$$

$\frac{10}{3} - \frac{8}{3} = \frac{10-8}{3}$

$\Rightarrow \boxed{d = 1/3}$

Eqn (i) $\Rightarrow a + 5d = 10/3$

$\Rightarrow a + 5(\frac{1}{3}) = 10/3$

$\Rightarrow a = \frac{10}{3} - \frac{5}{3}$

$\Rightarrow \boxed{a = 5/3}$

Then the A.P will be

$a, \; a+d, \; a+2d, \; a+3d, \ldots\ldots$

$\Rightarrow \frac{5}{3}, \; \frac{5}{3} + \frac{1}{3}, \; 5/3 + 2(\frac{1}{3}), \ldots\ldots$

$\Rightarrow \frac{2}{3}, \; \frac{6}{3}, \; \frac{7}{3}, \ldots\ldots$ Ans

Q:4 How many terms are there in A.P in which the 1st and last terms are 33/4 and 25/2 respectively and the common difference is 1/8 ?

Sol From the statement, it is clear

that $a_1 = 33/4$

$a_n = 25/2$

$d = 1/8$

and $n = ?$

As $a_n = a + (n-1)d$

$\Rightarrow \frac{25}{2} = \frac{33}{4} + (n-1)\frac{1}{8}$

$\Rightarrow \frac{25}{2} - \frac{33}{4} = \frac{n-1}{8}$

$\Rightarrow \frac{50-33}{4} = \frac{n-1}{8}$

$\Rightarrow \frac{17}{4} = \frac{n-1}{8}$

$\frac{17}{4} \times 8 = n-1$

$\Rightarrow 17 \times 2 = n-1$

$\Rightarrow 34 = n-1$

$\Rightarrow \boxed{n = 35}$

Hence total terms are 35.

Ex 4.2

CH-04
P-04

Q:5 Which term of the A.P 4, 1, −2, ... is −77?

Sol $4, 1, -2, \ldots, -77$

Here $a = 4$ and $d = -3$ $\quad (\because d = a_2 - a_1 = 1 - 4 = -3)$

Now $a_n = a + (n-1)d$

$\Rightarrow -77 = 4 + (n-1)(-3)$

$\Rightarrow -77 - 4 = -3n + 3$

$\Rightarrow -81 = -3n + 3 \Rightarrow 3n = 84$

$\Rightarrow n = \frac{84}{3} \Rightarrow n = 28$

Hence 28th term is −77.

Q:6 A ball rolling up an incline covered 24m in 1st second, 21m in 2nd second, 18m in 3rd sec. Find how many meters it covered in the 8th sec.

Sol The distance covered makes A.P of the pattern

$24, 21, 18, \ldots$

Here $a = 24$ and $d = 21 - 24 = -3$ $\quad (a_2 - a_1)$

Now $a_n = a + (n-1)d$

$\Rightarrow a_8 = a + (8-1)d$

$= 24 + 7(-3)$

$= 24 - 21$

$a_8 = 3$

Hence in the 8th sec, distance covered is 3m.

Q:7 The population of a town is decreasing by 500 inhabitants each year. If its population at the beginning of 1960 was 20135, what was its population at the beginning of 1970?

Sol

	a_1	a_2	a_3	
Years	1960,	1961,	1962,	....
Population	20135,	19635,	19135,	...

$d = -500$

put $n = 11$ for the beginning of 1970.

$a_n = a + (n-1)d$

$a_{11} = 20135 + (11-1)(-500)$

$\Rightarrow a_{11} = 20135 + (10)(-500)$

$\Rightarrow a_{11} = 20135 - 5000$

$\Rightarrow \boxed{a_{11} = 15135}$ Ans

Q:8 Ahmad and Akram can climb 1000 ft in the 1st hour and 100 ft in each succeeding hour. When will they reach the top of a 5400 ft hill?

Sol

Hours	1st,	2nd,	3rd,	4th,	...
Distance covered	1000,	1100,	1200,	....	

Here $a = 1000$

$d = 100$

Now $a_n = a + (n-1)d$

$\Rightarrow 5400 = 1000 + (n-1)100$

$\Rightarrow 5400 - 1000 = 100n - 100$

$\Rightarrow 4400 = 100n - 100$

$\Rightarrow 4500 = 100n$

$\Rightarrow \boxed{n = 45}$

Hence they will take 45 hours.

Q:9 A man earned \$3500 in 1st year he worked. If he received a raise of \$750 at the end of each year for 20 years. What will be his salary during his twenty 1st year of work?

Sol

Salaries $3500,\ 3500+750, \ldots$

$= \underset{a_1}{3500},\ \underset{a_2}{4250},\ \underset{a_3}{5000},\ 5750, \ldots$

Sol $a = 3500,\ d = 750$

$a_n = a + (n-1)d$ for 21st yr, put $n = 21$

$a_{21} = a + (21-1)d$

$= 3500 + 20(750)$

$= 3500 + 15000$

$\boxed{a_{21} = 18500}$ Ans

Exercise # 4.3

Q:1 Find the A.M b/w

(i) 12 and 18

Sol $A = \dfrac{a+b}{2}$

$\Rightarrow A = \dfrac{12+18}{2} = \dfrac{30}{2} = 15$

$\Rightarrow \boxed{A = 15}$ Ans

(ii) $\frac{1}{3}, \frac{1}{4}$

Sol $A = \dfrac{\frac{1}{3}+\frac{1}{4}}{2} = \dfrac{\frac{4+3}{12}}{2} = \frac{7}{12} \times \frac{1}{2} = 7/24$

Hence $\boxed{A = 7/24}$ Ans

(iii) $-6, -216$

Sol: $A = \dfrac{(-6)+(-216)}{2}$

$A = \dfrac{-222}{2}$

$\Rightarrow \boxed{A = -111}$ Ans

(iv) $(a+b)^2, (a-b)^2$

Sol $A = \dfrac{(a+b)^2 + (a-b)^2}{2}$

$\Rightarrow A = \dfrac{(a^2+b^2+2ab)+(a^2+b^2-2ab)}{2}$

$\Rightarrow A = \dfrac{2a^2+2b^2}{2} = \dfrac{2(a^2+b^2)}{2}$

$\Rightarrow \boxed{A = a^2+b^2}$ Ans

Q:2 Insert

(i) Three A.Ms b/w 6 and 41.

Sol Let A_1, A_2, A_3 are the three A.Ms, then $6, A_1, A_2, A_3, 41$ is A.P

To find d

$$a_n = a + (n-1)d$$
$$\Rightarrow 41 = 6 + (5-1)d$$
$$\Rightarrow 41 - 6 = 4d$$
$$\Rightarrow 36 = 4d \Rightarrow \boxed{9 = d}$$

Now $A_n = a + nd$

$$\Rightarrow \left.\begin{aligned} A_1 &= a + 1d = 6 + 1(9) = 15 \\ A_2 &= a + 2d = 6 + 2(9) = 24 \\ A_3 &= a + 3d = 6 + 3(9) = 33 \end{aligned}\right\} \text{Ans}$$

(ii) Four A.Ms b/w 17 and 32.

Sol: Let A_1, A_2, A_3 and A_4 are the four A.Ms b/w 17 and 32.

Then $17, A_1, A_2, A_3, A_4, 32$ is A.P

As
$$a_n = a + (n-1)d$$
$$\Rightarrow 32 = 17 + (6-1)d$$
$$\Rightarrow 32 - 17 = 5d$$
$$15 = 5d$$
$$\Rightarrow \boxed{3 = d}$$

Now $A_n = a + nd$

$$\Rightarrow A_1 = a + 1d = 17 + 1(3) = 20$$
$$A_2 = a + 2d = 17 + 2(3) = 23$$
$$A_3 = a + 3d = 17 + 3(3) = 26$$
$$A_4 = a + 4d = 17 + 4(3) = 29$$

(iii) Five A.Ms b/w 9 and 33.

Sol Let A_1, A_2, A_3, A_4 and A_5 are five A.Ms b/w 9 and 33.

Then $9, A_1, A_2, A_3, A_4, A_5, 33$ is A.P

To find d:

$$a_n = a + (n-1)d$$
$$33 = 9 + (7-1)d$$
$$\Rightarrow 33 - 9 = 6d$$
$$\Rightarrow 24 = 6d$$
$$\Rightarrow \boxed{4 = d}$$

Now $A_n = a + nd$

$$\Rightarrow \left.\begin{aligned} A_1 &= a + 1d = 9 + 1(4) = 13 \\ A_2 &= a + 2d = 9 + 2(4) = 17 \\ A_3 &= a + 3d = 9 + 3(4) = 21 \\ A_4 &= a + 4d = 9 + 4(4) = 25 \\ A_5 &= a + 5d = 9 + 5(4) = 29 \end{aligned}\right\} \text{Ans}$$

Q:3 For what value of n, $\dfrac{a^{n+1} + b^{n+1}}{a^n + b^n}$ is A.M b/w a and b ?

Sol Given that

$$\frac{a^{n+1} + b^{n+1}}{a^n + b^n} = A.M$$

$$\Rightarrow \frac{a^{n+1} + b^{n+1}}{a^n + b^n} = \frac{a+b}{2}$$

$$\Rightarrow 2\left(a^{n+1} + b^{n+1}\right) = (a+b)\left(a^n + b^n\right)$$

$$\Rightarrow 2a^{n+1} + 2b^{n+1} = a.a^n + ab^n + ba^n + b.b^n$$

4-5a

116

$$\Rightarrow 2a^{n+1} + 2b^{n+1} = a^{n+1} + ab^{n} + a^{n}b + b^{n+1}$$

$$\Rightarrow 2a^{n+1} - a^{n+1} + 2b^{n+1} - b^{n+1} = a.b^{n} + a^{n}b$$

$$\Rightarrow a^{n+1} + b^{n+1} = ab^{n} + a^{n}b$$

$$\Rightarrow a.a^{n} + b.b^{n} = ab^{n} + a^{n}b$$

$$\Rightarrow a.a^{n} - a^{n}b = ab^{n} - bb^{n}$$

$$\Rightarrow (a-b)a^{n} = (a-b)b^{n}$$

$$\Rightarrow a^{n} = b^{n} \qquad \div \text{ b.s by } b^{n}$$

$$\Rightarrow \frac{a^{n}}{b^{n}} = 1 \Rightarrow \left(\frac{a}{b}\right)^{n} = 1$$

$$\Rightarrow \left(\frac{a}{b}\right)^{n} = \left(\frac{a}{b}\right)^{0}$$

$$\Rightarrow \boxed{n=0} \text{ Ans}$$

Q:4 Insert five A.Ms b/w 5 and 8. Also show that their sum is equal to 5 times A.M b/w 5 and 8.

Sol Let A_1, A_2, A_3, A_4 and A_5 are five A.Ms b/w 5 and 8.

Then $5, A_1, A_2, A_3, A_4, A_5, 8$ is A.P

To find d

$$a_n = a + (n-1)d$$

$$8 = 5 + (7-1)d$$

$$\Rightarrow 8 - 5 = 6d$$

$$\Rightarrow 3 = 6d \Rightarrow \boxed{\tfrac{1}{2} = d}$$

Now $A_n = a + nd$

$$\Rightarrow A_1 = a + 1d = 5 + 1\left(\tfrac{1}{2}\right) = 11/2$$

$$A_2 = a + 2d = 5 + 2\left(\tfrac{1}{2}\right) = 6$$

$$A_3 = a + 3d = 5 + 3\left(\tfrac{1}{2}\right) = 13/2$$

$$A_4 = a + 4d = 5 + 4\left(\tfrac{1}{2}\right) = 7$$

$$A_5 = a + 5d = 5 + 5\left(\tfrac{1}{2}\right) = 15/2.$$

Now sum of the five A.Ms

$$A_1 + A_2 + A_3 + A_4 + A_5 = \frac{5}{2}(A_1 + A_5) \qquad S_n = \frac{n}{2}(a_1 + a_n) \text{ formula}$$

$$= \frac{5}{2}\left(\frac{11}{2} + \frac{15}{2}\right)$$

$$= \frac{5}{2}\left(\frac{26}{2}\right)$$

$$= 65/2 \longrightarrow (i)$$

Now 5 times A.M of 5 and 8

$$A = \frac{5+8}{2} = \frac{13}{2}$$

$$5A = 5\left(\frac{13}{2}\right) = 65/2 \longrightarrow (ii)$$

from eqns (i) and (ii) it is proved that

$$A_1 + A_2 + A_3 + A_4 + A_5 = 5 \text{ A.M.}$$

Q.5 There are n A.Ms b/w 5 and 32 such that the ratio of 3rd and 7th means is 7 : 13. Find the value of n.

Sol

As $A_n = a + nd$

$\Rightarrow A_3 = a + 3d$ & $A_7 = a + 7d$

Now given that $A_3 : A_7 = 7 : 13$

$$\Rightarrow \frac{A_3}{A_7} = \frac{7}{13}$$

$$\Rightarrow \frac{a+3d}{a+7d} = \frac{7}{13}$$

By cross multiplication

$\Rightarrow 13a + 39d = 7a + 49d$

$\Rightarrow 13a - 7a = 49d - 39d$

$\Rightarrow 6a = 10d$

$\Rightarrow \boxed{a = \frac{5}{3}d} \longrightarrow (i)$

Now if $A_1, A_2, A_3, \ldots\ldots A_n$ are n AMs b/w 5 and 32, then

$5, A_1, A_2, A_3, \ldots\ldots\ldots, A_n, 32$ is A.P

Then $d = \frac{b-a}{n+1}$

$$\Rightarrow d = \frac{32-5}{n+1}$$

$$\Rightarrow d = \frac{27}{n+1}$$

Eq (i) $\Rightarrow$ $a = \frac{5}{3}d$

$$a = \frac{5}{3}\left(\frac{27}{n+1}\right)$$

$\Rightarrow 5 = \frac{5}{3} \times \frac{27}{n+1} \Rightarrow 1 = \frac{9}{n+1} \Rightarrow n+1 = 9$

$\Rightarrow \boxed{n = 8}$ Ans

Quote

A smart man makes a mistake, learns from it, and never makes that mistake again. But a wise man finds a smart man and learns from him how to avoid the mistake altogather

(Roy - H - williams)

Exercise 4.4

Q:1 Find the indicated term and the sum of the number of terms in the following cases.

(i) 9, 7, 5, 3, ... 20th term ; 20 terms.

Sol $a=9$ & $d=-2$ $a_{20}=?$ $S_{20}=?$

$a_n = a+(n-1)d$

$\Rightarrow a_{20} = 9+(20-1)(-2)$

$= 9-38 \Rightarrow \boxed{a_{20}=-29}$ Ans

Now sum of 1st 20 terms

$S_n = \frac{n}{2}\{2a+(n-1)d\}$

$\Rightarrow S_{20} = \frac{20}{2}\{2a+(20-1)d\}$

$= 10\{2(9)+19(-2)\}$

$= 10\{18-38\}$

$= 10(-20)$

$\boxed{S_{20}=-200}$ Ans

(ii) $3, \frac{8}{3}, \frac{7}{3}, 2, \ldots\ldots$ 11 th term ; & 11 terms

$a_{11}=?$ $S_{11}=?$

Sol $a=3$

$d = \frac{8}{3}-3 \Rightarrow \boxed{d=-\frac{1}{3}}$

As $a_n = a+(n-1)d$

$\Rightarrow a_{11} = a+(11-1)d$

$= 3+10\left(-\frac{1}{3}\right)$

$= 3-\frac{10}{3}$

$\boxed{a_{11}=\frac{-1}{3}}$ Ans

Now $S_n = \frac{n}{2}\{2a+(n-1)d\}$

$\Rightarrow S_{11} = \frac{11}{2}\left\{2(3)+(11-1)\left(-\frac{1}{3}\right)\right\}$

$= \frac{11}{2}\left\{6-\frac{10}{3}\right\}$

$= \frac{11}{2}\left\{\frac{8}{3}\right\}$

$\boxed{S_{11}=\frac{44}{3}}$ Ans

Q:2 Some of a_1, a_n, n, d, S_n are given. Find the ones that are missing

(i) $a_1=2$, $n=17$, $d=3$

Sol $a_n = a+(n-1)d$ & $S_n = \frac{n}{2}\{2a+(n-1)d\}$

$\Rightarrow a_{17} = 2+(17-1)3$

$a_{17} = 2+48$

$a_{17} = 50$ Ans

$S_{11} = \frac{17}{2}\{2(2)+(17-1)3\}$

$= \frac{17}{2}\{4+48\} = \frac{17}{2}\{52\}$

$\Rightarrow S_{11} = 442$. Ans

118

(ii) $a_1 = -40$, $S_{21} = 210$

Sol As $S_n = \frac{n}{2}\{a_1 + a_n\}$

$\Rightarrow S_{21} = \frac{21}{2}\{a_1 + a_{21}\}$

$\Rightarrow 210 = \frac{21}{2}\{-40 + a_{21}\}$

$\Rightarrow \frac{210 \times 2}{21} = -40 + a_{21} \Rightarrow 20 = -40 + a_{21}$

$\Rightarrow \boxed{a_{21} = 60}$ Ans

And $a_n = a + (n-1)d$

$\Rightarrow a_{21} = a + (21-1)d$

$\Rightarrow 60 = -40 + 20d \Rightarrow 100 = 20d \Rightarrow \boxed{d=5}$ Ans

(iii) $a_1 = -7$, $d = 8$, $S_n = 225$

Sol As $S_n = \frac{n}{2}\{2a + (n-1)d\}$

$\Rightarrow 225 = \frac{n}{2}\{2(-7) + (n-1)8\}$

$\Rightarrow 2 \times 225 = n\{-14 + 8n - 8\}$

$\Rightarrow 450 = n\{8n - 22\}$

$\Rightarrow 450 = 8n^2 - 22n$

$\Rightarrow 8n^2 - 22n - 450 = 0$

By quadratic formula

$n = \frac{-(-22) \pm \sqrt{(-22)^2 - 4(8)(-450)}}{2(8)}$

Ex 4.4 Ch-04 P-07

$\Rightarrow n = \frac{22 \pm \sqrt{484 + 14400}}{16}$

$n = \frac{22 \pm \sqrt{14884}}{16} \Rightarrow n = \frac{22 \pm 122}{16}$

$\Rightarrow n = \frac{22+122}{16}$ or $n = \frac{22-122}{16}$

$\Rightarrow n = \frac{144}{16}$ $n = -6.25$ which is not possible.

$\Rightarrow \boxed{n=9}$ Ans

Now $a_n = a + (n-1)d$

$\Rightarrow a_9 = a + (9-1)d$

$\Rightarrow a_9 = -7 + 8(8)$

$\Rightarrow \boxed{a_9 = 57}$ Ans

(iv) $a_n = 4$, $S_{15} = 30$

$n = 15$

Now $a_n = a + (n-1)d$

$a_{15} = a + 14d \rightarrow ①$

$4 = a + 14d \rightarrow ①$

$S_n = \frac{n}{2}\{2a + (n-1)d\}$

$S_{15} = \frac{15}{2}\{2a + 14d\}$

$30 = \frac{15}{2}\{2a + 14d\}$

$4 = 2a + 14d \rightarrow ②$

Eq① − Eq②

$4 = a + 14d$

$4 = 2a + 14d$

$0 = -a \Rightarrow a = 0$

$\Rightarrow 4 = 2(0) + 14d$

$4 = 14d \Rightarrow \frac{2}{7} = d$

Hence $n = 15$, $a = 0$ & $d = 2/7$

4-7a

③ Find the sum of all the nos divisible by 5 from 25 to 350.

Sol: The numbers divisible by 5 from 25 to 350 are

25, 30, 35, 40, ... 345, 350.

To find the number of terms, we have

$a_n = a + (n-1)d$ Here $a = 25$ & $d = 5$

$\Rightarrow 350 = 25 + (n-1)5$

$\Rightarrow 350 - 25 = (n-1)5$

$\Rightarrow 325 = (n-1)5$

$\Rightarrow n-1 = \frac{325}{5} \Rightarrow n-1 = 65 \Rightarrow \boxed{n = 66}$

Now sum of the terms is $25 + 30 + 35 + \cdots + 350$ is by formula

$S_n = \frac{n}{2}\{a_1 + a_n\}$

$\Rightarrow S_{66} = \frac{66}{2}\{a_1 + a_{66}\}$

$\Rightarrow S_{66} = 33\,(25 + 350)$

$\Rightarrow S_{66} = 33\,(375)$

$\Rightarrow S_{66} = 12375$ Ans

Q:4: The sum of the three numbers in an A.P is 36 and the sum of their cubes is 6336. Find the #s.

Sol: Let the numbers are

$a-d,\ a,\ a+d$

Sum = 36

$\Rightarrow (a-d) + a + (a+d) = 36$

$\Rightarrow 3a = 36 \Rightarrow \boxed{a = 12}$

Also sum of cubes = 6336

$\Rightarrow (a-d)^3 + a^3 + (a+d)^3 = 6336$

$\Rightarrow (a^3 - \cancel{d^3} - \cancel{3a^2d} + 3ad^2) + a^3 + (a^3 + \cancel{d^3} + \cancel{3a^2d} + 3ad^2) = 6336$

$\Rightarrow 3a^3 + 6ad^2 = 6336$

$\div$ by 3

$\Rightarrow a^3 + 2ad^2 = 2112$

$\Rightarrow (12)^3 + 2ad^2 = 2112$

$\Rightarrow 1728 + 2ad^2 = 2112 \Rightarrow 2ad^2 = 2112 - 1728$

$\Rightarrow 2ad^2 = 384 \Rightarrow ad^2 = 192$

$\Rightarrow \frac{ad^2}{a} = \frac{192}{a} \Rightarrow d^2 = \frac{192}{a}$

$\Rightarrow d^2 = \frac{192}{12}$

$\Rightarrow d^2 = 16$

$\Rightarrow d = \pm 4$

d

Now the terms are

if $a=12$ & $d=4$ AND if $a=12$ & $d=-4$

$a-d = 12-4 = 8$

$a = 12$

$a+d = 12+4 = 16$

$a-d = 12-(-4) = 16$

$a = 12$

$a+d = 12+(-4) = 8$

So the required terms are

$8, 12, 16$ or $16, 12, 8$

Q.5 Find $1+3-5+7+9-11+13+15-17+\dots$ upto $3n$ terms

METHOD #01

Add three, three terms separately

$\Rightarrow (1+3-5)+(7+9-11)+(13+15-17)+\dots$ upto $\frac{3n}{3}$ terms

$= -1 + 5 + 11 + \dots$ n terms

$a=-1 \quad d=6$

By formula $S_n = \frac{n}{2}\{2a+(n-1)d\}$

$\Rightarrow S_n = \frac{n}{2}\{2(-1)+(n-1)6\}$

$= \frac{n}{2}\{-2+6n-6\}$

$= \frac{n}{2}\{6n-8\}$

$= \frac{n}{2}\,2(3n-4)$

$= n(3n-4)$

$\Rightarrow S_n = 3n^2-4n$ Ans

Method #02:

We can represent the given series as sum of three series, as

$1+3-5+7+9-11+13+15-17+\dots$ upto $3n$ terms

$= (1+7+13+\dots n \text{ terms}) + (3+9+15+\dots n \text{ terms}) - (5+11+17+\dots n \text{ terms})$

$= \frac{n}{2}\{2a+(n-1)d\} + \frac{n}{2}\{2a+(n-1)d\} - \frac{n}{2}\{2a+(n-1)d\}$

$= \frac{n}{2}\{2(1)+(n-1)6\} + \frac{n}{2}\{2(3)+(n-1)6\} - \frac{n}{2}\{2(5)+(n-1)6\}$

$= \frac{n}{2}\{2+6n-6\} + \frac{n}{2}\{6+6n-6\} - \frac{n}{2}\{10+6n-6\}$

$= \frac{n}{2}\{6n-4\} + \frac{n}{2}\{6n\} - \frac{n}{2}\{6n+4\}$

take $n/2$ as common

$= \frac{n}{2}\left[(6n-4) + 6n - (6n+4)\right\}$

$= \frac{n}{2}\{6n-4+6n-6n-4\}$

$= \frac{n}{2}\{6n-8\}$

$= \frac{n}{2}\,2(3n-4)$

$= n(3n-4)$

$= 3n^2-4n$ Ans

Quote: Education is the ability to listen to almost anything without losing your temper or your self-confidence.

(Robert Frost)

Q:6 Show that the sum of 1st n positive odd integers is n^2.

Sol The sequence of 1st n positive odd numbers is

$1, 3, 5, 7, \ldots\ldots$

And the series becomes

$1+3+5+7+\ldots\ldots$

Here $a=1$, $d=2$ and terms $= n$ (given)

Then the series will be

$S_n = \frac{n}{2}\{2a+(n-1)d\}$

$\Rightarrow S_n = \frac{n}{2}\{2(1)+(n-1)2\}$

$\Rightarrow S_n = \frac{n}{2}\{2+2n-2\}$

$\Rightarrow S_n = \frac{n}{2}\{2n\} = n^2$

So the series is

$1+3+5+\ldots\ldots\ldots$ n-terms $= n^2$

Q:7 Find four numbers in A.P, whose sum is 20 and the sum of whose squares is 120.

Sol Let the #s are

$a-3d,\ a-d,\ a+d,\ a+3d$

sum $= 20$ (Given)

$\Rightarrow (a-3d)+(a-d)+(a+d)+(a+3d)=20$

$\Rightarrow 4a=20 \Rightarrow \boxed{a=5}$

Now sum of their squares $= 120$ (Given)

$\Rightarrow (a-3d)^2+(a-d)^2+(a+d)^2+(a+3d)^2=120$

$\Rightarrow a^2+9d^2-6ad+a^2+d^2-2ad+a^2+d^2+2ad+a^2+9d^2+6ad=120$

$\Rightarrow 4a^2+20d^2=120$

$\div$ by 4

$\Rightarrow a^2+5d^2=30$

$\Rightarrow 5^2+5d^2=30 \Rightarrow 25+5d^2=30$

$\Rightarrow 5d^2=5 \Rightarrow d^2=1 \Rightarrow \boxed{d=\pm 1}$

Then the required #s are

If $d=1$, $a=5$ and If $d=-1$, $a=5$

$a-3d=5-3(1)=2$	$a-3d=5-3(-1)=8$
$a-d=5-1=4$	$a-d=5-(-1)=6$
$a+d=5+1=6$	$a+d=5+(-1)=4$
$a+3d=5+3(1)=8$	$a+3d=5+3(-1)=2$

Hence the #s are 2, 4, 6, 8 or 8, 6, 4, 2 Ans

Q:8: The sum of Rs 1000 is to be divided among four people so that each person after the 1st gets Rs 20 less than the preceeding one. How much does each person receive?

Sol Total amount $=$ Rs 1000.

Let 1st person gets = Rs x
Then 2nd " gets = Rs $(x-20)$
and 3rd " gets = Rs $(x-40)$
and 4th " " = Rs $(x-60)$
So the series becomes so on

$$x + (x-20) + (x-40) + (x-60)$$

Now sum = Rs 1000

$$\Rightarrow x + (x-20) + (x-40) + (x-60) = \text{Rs } 1000$$

$$4x - 120 = 1000$$
$$4x = 1000 + 120$$
$$\Rightarrow 4x = 1120$$
$$\Rightarrow \boxed{x = 280}$$

OR $a = x$, $d = -20$

$$S_4 = \frac{4}{2}\left(2(x) + (4-1)(-20)\right)$$
$$= 2\,(2x - 60)$$
$$= 4x - 120$$

So 1st person gets = Rs 280
2nd " " = Rs 260 ($\because$ 280−20)
3rd " " = Rs 240
4th " " = Rs 220

Q.9 To dig a well a company costs \$10 for 1st ft, \$12.5 for 2nd ft, \$15 for 3rd ft and so on. What is the dept of a well that costs \$2925 to dig?

Sol

	1st ft,	2nd ft,	3rd ft,	...
Dollars	10,	12.5,	15,	...

Here $a = 10$, $d = 12.5 - 10 = 2.5$

Ex 4.4 CH-04 P-09

S_n = sum of all dollars or total cost

$S_n = \$\,2925$

$$S_n = \frac{n}{2}\left\{2a + (n-1)d\right\}$$

$$\Rightarrow 2925 = \frac{n}{2}\left\{2(10) + (n-1)2.5\right\}$$

$$\Rightarrow 5850 = n\left\{20 + 2.5n - 2.5\right\}$$

$$\Rightarrow 5850 = n\left\{2.5n + 17.5\right\}$$

$$\Rightarrow 5850 = 2.5n^2 + 17.5n$$

$$\Rightarrow 2.5n^2 - 17.5n - 5850 = 0$$

By quadratic formula

$$n = \frac{-(-17.5) \pm \sqrt{(-17.5)^2 - 4(2.5)(-5850)}}{2(2.5)}$$

$$n = \frac{17.5 \pm \sqrt{306.25 + 58500}}{5}$$

$$n = \frac{17.5 \pm \sqrt{58806.25}}{5}$$

$$n = \frac{17.5 \pm 242.5}{5}$$

$$n = \frac{17.5 + 242.5}{5}, \quad n = \frac{17.5 - 242.5}{5}$$

$$n = \frac{260}{5} \qquad n = -45 \text{ (Not possible)}$$

$$n = 52$$

Since each term represents one year also
Hence total ft = 52 Ans

Q10: The distance which an object dropped from a cliff will fall 16 ft the 1st second, 48 ft the next second, 80 ft the third second and so on. What is the total distance the object will fall in six seconds?

Sol: The distance travelled is

1st second, 2nd sec, 3rd sec, ---

16, 48, 80, ----

which is A.P with $d = 32$, $a = 16$

Now total distance of the 6 seconds is

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

$$S_6 = \frac{6}{2}\{2(16) + (6-1)32\}$$

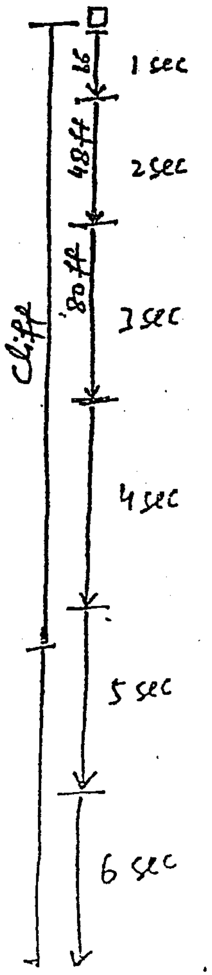
$$= 3\{32 + 160\}$$

$$= 3(192)$$

$$= 576$$

$S_6 = 576$ ft is the total distance of the 6 sec

Diagram



Q:11 Affan saves Rs1 the 1st day, Rs2 the 2nd day, Rs3 the third day and so on for thirty days. What is his total saving for the 30 days.

Sol

1st day, 2nd day, 3rd day, ---------- 30th day

Saving = 1, 2, 3, - - - - - - .

Hence $a = 1$, $d = 1$, $n = 30$

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

$$S_{30} = \frac{30}{2}\{2(1) + (30-1)1\}$$

$$= 15\{2 + 29\}$$

$$= 15(31)$$

$$\boxed{S_{30} = 465}$$ Ans

Q:12 A contest will have five cash prizes totalling Rs5000 and there will be a Rs100 difference between successive prizes. Find the 1st prize.

Sol Let the 1st prize $= x$

Then " 2nd prize $= x + 100$ $\because d = 100$ (given)

and " 3rd " $= x + 200$

⋮

5th prize

124

Here $a = x$, $d = 100$, $n = 5$ and sum $= 5000$

Now by formula

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

$$\Rightarrow S_5 = \frac{5}{2}\{2x + (5-1)100\}$$

$$\Rightarrow 5000 = \frac{5}{2}\{2x + 400\}$$

$$\Rightarrow \frac{2 \times 5000}{5} = 2x + 400$$

$$\Rightarrow 2000 = 2x + 400$$

$$\Rightarrow 2x = 1600 \Rightarrow \boxed{x = 800}$$

Hence share of 1st is Rs 800. Ans

Q:13 A theater has 40 rows with 20 seats in the 1st row, 23 in the second row, 26 in the third row and so on. How many seats are in the theater?

Sol The seats in the rows are

1st row, 2nd row, 3rd row, - - - - - 40 terms

20, 23, 26, - - - - - a_{40}

$a = 20$, $d = 3$, $n = 40$

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

$$\Rightarrow S_{40} = \frac{40}{2}\{2(20) + (40-1)3\}$$

$$= 20\{40 + 117\} = 20[157] = 3140$$

$$\Rightarrow S_{40} = 3140.$$

So the theater has 3140 seats.

Ex # 4.5

Q:1 Find the 1st five terms of the G.Ps

(i) $a_1 = 5$, $r = 3$

Sol Formula for G.P is

$$a_n = ar^{n-1}$$

$$\Rightarrow a_2 = ar^{2-1} = ar = 5(3) = 15$$

$$a_3 = ar^{3-1} = ar^2 = 5(3)^2 = 45$$

$$a_4 = ar^{4-1} = ar^3 = 5(3)^3 = 135$$

$$a_5 = ar^{5-1} = ar^4 = 5(3)^4 = 405$$

Hence the 1st five terms are

5, 15, 45, 135, 405.

(ii) $a_1 = 8$, $r = 1/2$

$$a_n = ar^{n-1}$$

$$\Rightarrow a_2 = ar^{2-1} = ar = 8(1/2) = 4$$

$$a_3 = ar^{3-1} = ar^2 = 8\left(\tfrac{1}{2}\right)^2 = 2$$

$$a_4 = ar^{4-1} = ar^3 = 8\left(\tfrac{1}{2}\right)^3 = 1$$

$$a_5 = ar^{5-1} = ar^4 = 8\left(\tfrac{1}{2}\right)^4 = 1/2.$$

Hence the 1st five terms are

8, 4, 2, 1, 1/2 Ans

(iii) $a_1 = -9/16 \quad r = -2/3$

Sol $\quad a_n = ar^{n-1}$

$\Rightarrow a_2 = ar^{2-1} = \left(\frac{-9}{16}\right)\left(\frac{-2}{3}\right)^1 = \frac{-9}{16} \times \frac{-2}{3} = \frac{3}{8}$

$a_3 = ar^{3-1} = ar^2 = \left(\frac{-9}{16}\right)\left(\frac{-2}{3}\right)^2 = \frac{-9}{16} \times \frac{4}{9} = -1/4$

$a_4 = ar^{4-1} = ar^3 = \left(\frac{-9}{16}\right)\left(\frac{-2}{3}\right)^3 = \frac{-9}{16} \times \frac{-8}{27} = 1/6$

$a_5 = ar^{5-1} = ar^4 = \left(\frac{-9}{16}\right)\left(\frac{-2}{3}\right)^4 = \frac{-9}{16} \times \frac{16}{81} = -1/9$

Hence the 1st five terms are

$\frac{-9}{16}, \frac{3}{8}, -\frac{1}{4}, \frac{1}{6}, -\frac{1}{9}$

(iv) $a_1 = \frac{x}{y}, \quad r = -\frac{y}{x}$

Sol $\quad a_n = ar^{n-1}$

$\Rightarrow a_2 = ar^{2-1} = ar^1 = \left(\frac{x}{y}\right)\left(-\frac{y}{x}\right) = -1$

$a_3 = ar^{3-1} = ar^2 = \left(\frac{x}{y}\right)\left(-\frac{y}{x}\right)^2 = \left(\frac{x}{y}\right)\left(\frac{y^2}{x^2}\right) = y/x$

$a_4 = ar^{4-1} = ar^3 = \left(\frac{x}{y}\right)\left(-\frac{y}{x}\right)^3 = \left(\frac{x}{y}\right)\left(\frac{-y^3}{x^3}\right) = -y^2/x^2$

$a_5 = ar^{5-1} = ar^4 = \left(\frac{x}{y}\right)\left(-\frac{y}{x}\right)^4 = \left(\frac{x}{y}\right)\left(\frac{y^4}{x^4}\right) = y^3/x^3$

Hence the 1st five terms are

$-1, \; y/x, \; -y^2/x^2, \; y^3/x^3$

126

Q:2 Suppose that the third term of a G.P is 27 and the 5th term is 243. Find the 1st term and the common ratio of the sequence.

Sol

$a_3 = 27 \Rightarrow ar^{3-1} = 27 \Rightarrow ar^2 = 27 \longrightarrow (1)$

$a_5 = 243 \Rightarrow ar^{5-1} = 243 \Rightarrow ar^4 = 243 \longrightarrow (2)$

From Eqn (1) $\quad a = 27/r^2$

P.T.V in Eqn (2)

$\Rightarrow \quad ar^4 = 243$

$\Rightarrow \quad \frac{27}{r^2} \cdot r^4 = 243 \Rightarrow 27 r^2 = 243$

$\Rightarrow r^2 = 243/27$

$r^2 = 9 \Rightarrow \boxed{r = \pm 3}$

Now Eqn (1) $\Rightarrow ar^2 = 27$

$\Rightarrow a(9) = 27 \Rightarrow \boxed{a = 3}$

Hence 1st term $\Rightarrow a = 3$
ratio $\Rightarrow r = \pm 3$ } Ans

Q:3: Find the seventh term of a G.P whose second and third terms are 2 & $-\sqrt{2}$.

Sol: As $\quad a_n = ar^{n-1}$

$\Rightarrow a_2 = ar^{2-1}$ and $a_3 = ar^{3-1}$

$\Rightarrow a_2 = ar \qquad \Downarrow \quad -\sqrt{2} = ar^2 \longrightarrow (2)$

$\Rightarrow 2 = ar \longrightarrow (1)$

Eqn ① $\Rightarrow a = \frac{2}{r}$ Eqn ② $\Rightarrow ar^2 = -\sqrt{2}$

$$\Rightarrow \frac{2}{r} \cdot r^2 = -\sqrt{2}$$

$$\Rightarrow 2r = -\sqrt{2} \Rightarrow r = \frac{-\sqrt{2}}{2}$$

$$\Rightarrow \boxed{r = -1/\sqrt{2}}$$

Then $a = 2/r$

$a = 2/-1/\sqrt{2} \Rightarrow \boxed{a = -2\sqrt{2}}$

Now $a_n = ar^{n-1}$

$\Rightarrow a_7 = ar^{7-1}$

$\Rightarrow a_7 = ar^6 \Rightarrow a_7 = (-2\sqrt{2})\left(\frac{-1}{\sqrt{2}}\right)^6$

$$\Rightarrow a_7 = -2.(2)^{1/2} \frac{(-1)^6}{(\sqrt{2})^6}$$

$$\Rightarrow a_7 = -2\,(2)^{1/2} \cdot \frac{1}{2^3}$$

$$\Rightarrow a_7 = -2^1 . 2^{1/2} . 2^{-3}$$

$$\Rightarrow a_7 = -\;2^{1+\frac{1}{2}-3}$$

$$a_7 = -2^{\frac{2+1-6}{2}} \Rightarrow a_7 = -2^{-3/2}$$

Hence 7th term is $a_7 = -2^{-3/2}$ Ans

Ex 4.5 CH-04 P-11

Q:4 How many terms are there in a G.P if its first term is 16, last term is 1/64 and common ratio is 1/2.

Sol

1st term $= a = 16$.

last term $= a_n = 1/64$

Ratio $= r = 1/2$

Terms $= n = ?$

As $a_n = ar^{n-1}$

$$\Rightarrow \frac{1}{64} = 16\left(\frac{1}{2}\right)^{n-1}$$

$$\Rightarrow \frac{1}{64 \times 16} = \left(\frac{1}{2}\right)^{n-1}$$

$$\Rightarrow \frac{1}{1024} = \left(\frac{1}{2}\right)^{n-1}$$

$$\Rightarrow \left(\frac{1}{2}\right)^{10} = \left(\frac{1}{2}\right)^{n-1} \Rightarrow 10 = n-1 \Rightarrow \boxed{n=11} \text{ Ans}$$

21

Q:5 Find x if $x+7, x-3, x-8$ form G.P in the given order. Also give the sequence.

Sol G.P $x+7,\ x-3,\ x-8$

$$\Rightarrow \frac{a_2}{a_1} = \frac{a_3}{a_2} \text{ for G.P}$$

$$\Rightarrow \frac{x-3}{x+7} = \frac{x-8}{x-3}$$

$$\Rightarrow (x-3)^2 = (x+7)(x-8) \Rightarrow x^2+9-6x = x^2-8x+7x-56$$

$$\Rightarrow x^2-6x+9 = x^2-x-56.$$

$$\Rightarrow -6x+9 = -x-56$$

$\Rightarrow\ 9+56 = 6x - x$

$\Rightarrow\ 65 = 5x \Rightarrow \boxed{13 = x}$ Ans

And the terms are

$$\left.\begin{aligned} x+7 &= 13+7 = 20 \\ x-3 &= 13-3 = 10 \\ x-8 &= 13-8 = 5 \end{aligned}\right\} \text{Ans}$$

Q:6 If $a_{10} = l$

$a_{13} = m$

$a_{16} = n$ Show that $ln = m^2$

Sol in G.P

$a_n = ar^{n-1}$

for $n=10$ $a_{10} = ar^{10-1} \Rightarrow l = ar^9$

for $n=13$ $a_{13} = ar^{13-1} \Rightarrow m = ar^{12}$

for $n=16$ $a_{16} = ar^{16-1} \Rightarrow n = ar^{15}$

Now L.H.S $ln = ar^9 . ar^{15}$

$= a.a.r^9.r^{15}$

$= a^{1+1}\, r^{9+15}$

$= a^2 r^{24} \longrightarrow (i)$

R.H.S

$m = ar^{12}$

squaring b.s

$\Rightarrow (m)^2 = (ar^{12})^2$

$\Rightarrow m^2 = a^2.(r^{12})^2$

$\Rightarrow m^2 = a^2 r^{24} \longrightarrow (ii)$

from eqn (i) and (ii), it is proved $ln = m^2$

128

4-11a

Q:7 Show that the reciprocals of the terms of a G.P also form a G.P.

Sol: General representation of G.P is

$a, ar, ar^2, ar^3, \ldots\ldots\ ar^{n-1}$

Then the reciprocal will be

$\frac{1}{a}, \frac{1}{ar}, \frac{1}{ar^2}, \frac{1}{ar^3}, \ldots\ldots\ \frac{1}{ar^{n-1}}$

Finding the ratio b/w consecutive term

$$\frac{A_2}{A_1} = \frac{1/ar}{1/a} = \frac{1}{ar} \times \frac{a}{1} = 1/r$$

$$\frac{A_3}{A_2} = \frac{1/ar^2}{1/ar} = \frac{1}{ar^2} . \frac{ar}{1} = 1/r$$

$$\frac{A_4}{A_3} = \frac{1/ar^3}{1/ar^2} = \frac{1}{ar^3} . \frac{ar^2}{1} = 1/r$$

$\vdots$

and so on.

Here the reciprocal of the terms also form G.P because the ratio is same.

Q:8 The yearly depreciation of a certain machine is 20% of its value at the beginning of the year. If the original cost of the machine is Rs5000, find its value after 5 years.

Sol Initial value $= a_1 =$ Rs 5000

After one years depriciation is 20% of 500

$= 20\left(\frac{1}{100}\right)(5000)$

$= 1000$

So at 2nd year, its value is $= 5000 - 1000$

$\Rightarrow a_2 = 4000$

Then the sequence of value is

$5000, 4000, \ldots$

Total 5 yrs $\quad a_1 \xrightarrow{1\,yr} a_2 \xrightarrow{1\,yr} a_3 \xrightarrow{1\,yr} a_4 \xrightarrow{1\,yr} a_5 \xrightarrow{1\,yr} a_6$

$a = 5000, \quad r = \frac{4000}{5000} = 0.8$

Now $a_n = ar^{n-1}$

$\Rightarrow a_6 = ar^{6-1}$

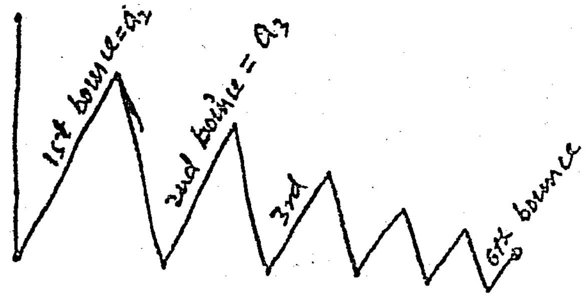
$= (5000)(0.8)^5 = 5000(0.32768) = 1638.4$

Hence after 5 yrs, value will be Rs 1638.4 Ans

Q:9 A tennis ball is dropped from a height of 10 ft and it bounces to 75% of the previous bounce. How high does the ball bounce on the sixth bounce.

Sol

$a = 10$ ft



$a = 10$ ft.

$a_2 = 75\%\, a$

$= 75\left(\frac{1}{100}\right)(10)$

$= 7.5$ ft

Then the sequence becomes

$10, 7.5, \ldots \quad \Rightarrow r = 7.5/10 \Rightarrow r = 0.75$

On sixth bounce the height will be 7th

$a_7 = ar^{7-1} \quad \because a_n = ar^{n-1}$

$\Rightarrow a_7 = 10(0.75)^6$

$a_7 = 10(0.1779)$

$\Rightarrow \boxed{a_7 = 1.78 \text{ ft}}$ Ans

Q:10 Find three numbers such that their sum is 3. The numbers form A.P but their squares form G.P.

Sol let the three numbers are $a-d, a, a+d$

sum $= 3$

$\Rightarrow (a-d) + a + (a+d) = 3$

$\Rightarrow 3a = 3 \Rightarrow \boxed{a = 1}$

squares of the #s form G.P

$\Rightarrow (a-d)^2, a^2, (a+d)^2$ form G.P

$\Rightarrow \frac{a^2}{(a-d)^2} = \frac{(a+d)^2}{a^2}$

$$\Rightarrow \quad a^2 \cdot a^2 = (a-d)^2 (a+d)^2$$

$$\Rightarrow \quad a^4 = (a-d)^2 (a+d)^2$$

$$\Rightarrow \quad 1^4 = (1-d)^2 (1+d)^2 \Rightarrow 1 = \{(1-d)(1+d)\}^2$$

$$\Rightarrow \quad 1 = (1-d^2)^2 \Rightarrow 1 = (1+d^4-2d^2)$$

$$\Rightarrow \quad d^4 - 2d^2 = 0$$

$$d^4 = 2d^2$$

$$\Rightarrow d^2 = 2 \Rightarrow d = \pm\sqrt{2}$$

Now the required numbers are

$a = 1$, $d = \sqrt{2}$ and $a = 1$, $d = -\sqrt{2}$

130 $a-d = 1-\sqrt{2}$ and $a-d = 1-(-\sqrt{2}) = 1+\sqrt{2}$

$a = 1$ $\qquad$ $a = 1$

$a+d = 1+\sqrt{2}$ $\qquad$ $a+d = 1+(-\sqrt{2}) = 1-\sqrt{2}$

Q:4 Rashid borrows Rs 2000 at 11 % interest compounded annually. If he pays off loan in full at the end of four years, how much does he pay?

Sol Initial amount $= a_1 =$ Rs 2000

Annual interest $= 11\,\%$

$$= \frac{11}{100}$$

$$= 0.11$$

The common ratio is $1 + 0.11 = 1.11$

At the end of four years we have a_5

$a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5$

1st yr $\quad$ 2nd yr $\quad$ 4th $\quad$ 5th yr

$$a_n = ar^{n-1}$$

$$\Rightarrow a_5 = ar^{5-1}$$

$$a_5 = 2000\,(1.11)^4$$

$$a_5 = 2000\,(1.51807041)$$

$$a_5 = \text{Rs } 3036.14082 \text{ Ans}$$

Quote

our prayers should be for our blessings in general, for God knows best what is good for us. (socrates).

Exercise # 4.6

Q:1 Find the Geometric Mean (G.M) b/w

(i) 3.14 and 2.71

Sol $a = 3.14$ & $b = 2.71$

Then $G.M = \sqrt{ab}$

$= \sqrt{3.14 \times 2.71}$

$= \sqrt{8.5094} = \pm 2.917$ Ans

(ii) $a = -6$ & $b = -216$

$G.M = \sqrt{-6 \times -216}$

$= \pm\sqrt{1296} = \pm 36$ Ans

(iii) $x+y$, $x-y$

Then $G.M = \sqrt{(x+y)(x-y)}$

$= \pm\sqrt{x^2-y^2}$

(iv) $a = \sqrt{2}+3$, $b = \sqrt{2}-3$

Then $G.M = \sqrt{(\sqrt{2}+3)(\sqrt{2}-3)}$

$= \pm\sqrt{(\sqrt{2})^2 - 3^2}$

$= \pm\sqrt{2-9} = \pm\sqrt{-5}$ = Imaginary

Hence G.M does not exist.

Q:2 Insert two G.Ms b/w $\sqrt{3}$ and 3.

Sol Let G_1 and G_2 are the two G.Ms then $\sqrt{3}, G_1, G_2, 3$ is G.P

To find r, we have

$a_n = ar^{n-1}$

$\Rightarrow 3 = \sqrt{3}.\, r^{4-1}$

$\Rightarrow \frac{3}{\sqrt{3}} = r^3 \Rightarrow \sqrt{3} = r^3$

$\Rightarrow 3^{1/2} = r^3$ take $\frac{1}{3}$ power of b.s

$\Rightarrow (3^{1/2})^{1/3} = (r^3)^{1/3}$

$\Rightarrow 3^{1/6} = r$

Now $G_n = ar^n$

$G_1 = ar^1 = \sqrt{3}(3^{1/6}) = 3^{1/2}.3^{1/6} = 3^{\frac{1}{2}+\frac{1}{6}} = 3^{\frac{4}{6}} = 3^{2/3}$

$G_2 = ar^2 = \sqrt{3}(3^{1/6})^2 = \sqrt{3}.3^{\frac{1}{3}} = 3^{\frac{1}{2}+\frac{1}{3}} = 3^{\frac{5}{6}}$ Ans

Q:3 Insert three G.Ms b/w a^4 and b^4.

Sol Let G_1, G_2 and G_3 are the three G.Ms b/w a^4 and b^4.

Then a^4, G_1, G_2, G_3, b^4 is G.P

To find r, we have

$a_n = ar^{n-1}$

$\Rightarrow b^4 = a^4 r^{5-1}$

$\Rightarrow \frac{b^4}{a^4} = r^4 \Rightarrow \left(\frac{b}{a}\right)^4 = r^4 \Rightarrow \boxed{\frac{b}{a} = r}$

Now $G_n = ar^n$

$\Rightarrow G_1 = ar^1 = a^4\left(\frac{b}{a}\right) = a^3b$

$G_2 = ar^2 = a^4\left(\frac{b}{a}\right)^2 = a^4\frac{b^2}{a^2} = a^2b^2$

$G_3 = ar^3 = a^4\left(\frac{b}{a}\right)^3 = a^4\frac{b^3}{a^3} = ab^3$

Q:4 Insert four G.Ms b/w -8 and $1/4$.

Sol Let G_1, G_2, G_3 and G_4 are the four G.Ms b/w -8 and $1/4$, then

$-8, G_1, G_2, G_3, G_4, \frac{1}{4}$ is G.P

To find r,

$a_n = ar^{n-1}$

$\frac{1}{4} = -8\, r^{6-1} \Rightarrow \frac{1}{4\times -8} = r^{6-1}$

$\Rightarrow -\frac{1}{32} = r^5 \Rightarrow \left(-\frac{1}{2}\right)^5 = r^5 \Rightarrow -\frac{1}{2} = r$

Now $G_n = ar^n$

$\Rightarrow G_1 = ar^1 = -8\left(-\frac{1}{2}\right)^1 = -8\left(-\frac{1}{2}\right) = 4$

$G_2 = ar^2 = -8\left(-\frac{1}{2}\right)^2 = -8\left(\frac{1}{4}\right) = -2$

$G_3 = ar^3 = -8\left(-\frac{1}{2}\right)^3 = -8\left(-\frac{1}{8}\right) = 1$

$G_4 = ar^4 = -8\left(-\frac{1}{2}\right)^4 = -8\left(\frac{1}{16}\right) = -1/2$

Q:5 : Find two numbers, if the difference b/w them is 48 and their A.M exceeds their G.M by 18 (i.e A.M is 18 more than G.M).

Sol: Let a and b be the required numbers, then

$a - b = 48$ — (i) (given that their difference is 48)

and $G.M + 18 = A.M$ ($\because$ A.M is 18 more than G.M)

$\Rightarrow \sqrt{ab} + 18 = \left(\frac{a+b}{2}\right)$

xing by 2, we get

$\Rightarrow 2\sqrt{ab} + 36 = a + b \longrightarrow$ (ii)

From eqn (i) $b = a - 48$, put in eqn (ii)

$\Rightarrow 2\sqrt{a(a-48)} + 36 = a + (a - 48)$

$\Rightarrow 2\sqrt{a^2 - 48a} + 36 = 2a - 48$

$\Rightarrow 2\sqrt{a^2 - 48a} = 2a - 48 - 36$

$\Rightarrow 2\sqrt{a^2 - 48a} = 2a - 84 \Rightarrow 2\sqrt{a^2-48a} = 2(a-42)$

$\Rightarrow \sqrt{a^2-48a} = a - 42$

squaring b.s

$\Rightarrow \left(\sqrt{a^2-48a}\right)^2 = (a-42)^2$

$\Rightarrow a^2 - 48a = a^2 + (42)^2 - 2(a)(42)$

$\Rightarrow a^2 - 48a = a^2 + 1764 - 84a$

$\Rightarrow -48a = 1764 - 84a$

$\Rightarrow 84a - 48a = 1764$

$\Rightarrow \quad 36a = 1764 \quad \div$ b.s by 36

$\Rightarrow \quad \boxed{a = 49}$

Then $\quad a-b=48 \Rightarrow b = a-48$

$b = 49-48$

$\Rightarrow \boxed{b=1}$

Hence the required numbers are 49 and 1.

Q.6: Prove that the product of n G.Ms b/w a & b is equal to the nth power of a single G.M b/w them.

Sol:- To prove that

Product of n G.Ms b/w a and b = nth power of single G.M b/w them

i.e $\quad G_1 \cdot G_2 \cdot G_3 \cdots\cdots G_n = \sqrt{ab}$

$G.M = \sqrt{ab}$

let $G_1, G_2, G_3, \cdots\cdots G_n$ are n G.Ms b/w a and b

then $a, G_1, G_2, G_3, \cdots\cdots G_n, b$ is G.P

To find r

$$a_n = ar^{n-1}$$
$$b = a\, r^{(n+2)-1}$$
$$\frac{b}{a} = r^{n+1} \Rightarrow \left(\frac{b}{a}\right)^{\frac{1}{1+n}} = r$$

L.H.S

$$G_1 \cdot G_2 \cdot G_3 \cdots\cdots G_n$$
$$= ar^1 \cdot ar^2 \cdot ar^3 \cdots\cdots ar^n$$
$$= (a.a.a \ldots n \text{ times}) \cdot (r^1 \cdot r^2 \cdot r^3 \cdots\cdots r^n)$$
$$= a^{1+1+1+\ldots n \text{ times}} \quad r^{1+2+3+\cdots\cdots+n}$$
$$= a^n \cdot r^{n\left(\frac{n+1}{2}\right)}$$
$$= \left\{a \cdot r^{\frac{n+1}{2}}\right\}^n$$
$$= \left\{a^{\frac{2}{2}} \cdot r^{\frac{n+1}{2}}\right\}^n$$
$$= \left\{a^2 \cdot r^{n+1}\right\}^{\frac{n}{2}}$$
$$= \left\{a^2 \cdot \frac{b}{a}\right\}^{n/2}$$
$$= \{ab\}^{n/2} \longrightarrow (i)$$

R.H.S

Single G.M

$$G = \sqrt{ab}$$

taking n power of b.s

$$G^n = (\sqrt{ab})^n$$
$$G^n = \left\{(ab)^{1/2}\right\}^n$$
$$\Rightarrow G^n = (ab)^{n/2} \longrightarrow (ii)$$

From eqn (i) and (ii), we have

$$G_1 \cdot G_2 \cdot G_3 \cdots G_n = G^n$$

Q.7 For what value of n, $\dfrac{a^{n+1}+b^{n+1}}{a^n+b^n}$ is G.M b/w a and b.

Sol:

$$\frac{a^{n+1}+b^{n+1}}{a^n+b^n} = G.M$$
$$\Rightarrow \frac{a^{n+1}+b^{n+1}}{a^n+b^n} = \sqrt{ab}$$
$$\Rightarrow \frac{a^{n+1}+b^{n+1}}{a^n+b^n} = (ab)^{1/2}$$

$\therefore \Rightarrow a^{n+1} + b^{n+1} = (ab)^{1/2} \{a^n + b^n\}$

$\Rightarrow a^{n+1} + b^{n+1} = a^{1/2} b^{1/2} \{a^n + b^n\}$

$\Rightarrow a^{n+1} + b^{n+1} = a^{\frac{1}{2}} b^{\frac{1}{2}} a^n + a^{\frac{1}{2}} b^{\frac{1}{2}} b^n$

$\Rightarrow a^{n+1} + b^{n+1} = a^n a^{\frac{1}{2}} b^{\frac{1}{2}} + a^{\frac{1}{2}} b^n b^{\frac{1}{2}}$

$\Rightarrow a^{n+1} + b^{n+1} = a^{n+\frac{1}{2}} b^{\frac{1}{2}} + a^{\frac{1}{2}} b^{n+\frac{1}{2}}$

$\Rightarrow a^{n+\frac{1}{2}+\frac{1}{2}} + b^{n+\frac{1}{2}+\frac{1}{2}} = a^{n+\frac{1}{2}} b^{\frac{1}{2}} + a^{\frac{1}{2}} b^{n+\frac{1}{2}}$

$\Rightarrow a^{n+\frac{1}{2}} . a^{\frac{1}{2}} + b^{n+\frac{1}{2}} . b^{\frac{1}{2}} = a^{n+\frac{1}{2}} b^{\frac{1}{2}} + a^{\frac{1}{2}} b^{n+\frac{1}{2}}$

$\Rightarrow a^{n+\frac{1}{2}} a^{\frac{1}{2}} - a^{n+\frac{1}{2}} b^{\frac{1}{2}} = a^{\frac{1}{2}} b^{n+\frac{1}{2}} - b^{n+\frac{1}{2}} b^{\frac{1}{2}}$

take common, we get

$\Rightarrow a^{n+\frac{1}{2}} (a^{\frac{1}{2}} - b^{\frac{1}{2}}) = b^{n+\frac{1}{2}} (a^{\frac{1}{2}} - b^{\frac{1}{2}})$

$\Rightarrow a^{n+\frac{1}{2}} = b^{n+\frac{1}{2}}$

$\div$ b.s by $b^{n+\frac{1}{2}}$

$\Rightarrow \dfrac{a^{n+\frac{1}{2}}}{b^{n+\frac{1}{2}}} = 1$

$\Rightarrow \left(\dfrac{a}{b}\right)^{n+\frac{1}{2}} = \left(\dfrac{a}{b}\right)^0$

$\Rightarrow n + \frac{1}{2} = 0$

$\Rightarrow \boxed{n = -\frac{1}{2}}$ Ans

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134

4-14a

Exercise # 4.7

Q:1 Compute the sum

a) $3 + 6 + 12 + \cdots\cdots + 3.2^9$

Sol $r = \frac{6}{3} = 2 \Rightarrow r > 1$

To find n

$a_n = ar^{n-1}$

$\Rightarrow 3.2^9 = 3(2)^{n-1}$

$\Rightarrow 2^9 = 2^{n-1}$

$\Rightarrow 9 = n-1 \Rightarrow \boxed{10 = n}$

Now $S_n = \dfrac{a\{r^n - 1\}}{r-1}$

$\Rightarrow S_{10} = \dfrac{3\{2^{10}-1\}}{2-1}$

$\Rightarrow S_{10} = 3\{2^{10}-1\}$

(b) $8 + 4 + 2 + 1 + \cdots\cdots\cdots + \frac{1}{15}$ $\quad r = \frac{4}{8} = 1/2$

Sol $a_n = ar^{n-1}$

$\Rightarrow \frac{1}{15} = 8\left(\frac{1}{2}\right)^{n-1}$

$\Rightarrow \dfrac{1}{15 \times 8} = \left(\frac{1}{2}\right)^{n-1}$

$$\Rightarrow \frac{1}{2^4 \cdot 2^3} = \left(\frac{1}{2}\right)^{n-1}$$

$$\Rightarrow \frac{1}{2^7} = \left(\frac{1}{2}\right)^{n-1} \Rightarrow \left(\frac{1}{2}\right)^7 = \left(\frac{1}{2}\right)^{n-1} \Rightarrow 7 = n-1$$

$$\Rightarrow \boxed{8 = n}$$

Now $S_n = \dfrac{a\{r^n - 1\}}{r-1}$

$$S_8 = \frac{8\left\{\left(\frac{1}{2}\right)^8 - 1\right\}}{\frac{1}{2} - 1}$$

$$S_8 = \frac{8\left\{\frac{1}{256} - 1\right\}}{\frac{-1}{2}} = \frac{8\left\{\frac{-255}{256}\right\}}{-1/2}$$

$$= 8\left\{\frac{-255}{256} \times \frac{2}{-1}\right\}$$

$$= 16\left(\frac{255}{256}\right) = \frac{255}{16} \text{ Ans}$$

(c) $2^4 + 2^5 + 2^6 + \cdots\cdots + 2^{10}$

Here $a = 2^4$ and $r = 2$ and $n = 7$ $\quad \because a_n = ar^{n-1}$

$2^{10} = 2^4 \cdot 2^{n-1}$

$2^{10} = 2^{n+3}$

$10 = n+3$

$7 = n$

Now $S_n = \dfrac{a\{r^n - 1\}}{r-1}$

$$S_7 = \frac{2^4\{2^7 - 1\}}{2-1} = 2^4\{128 - 1\}$$

$$= 16(127)$$

$$= 2032 \text{ Ans}$$

(d) $\dfrac{1}{x} + \dfrac{1}{x^2} + \dfrac{1}{x^3} + \cdots\cdots + \dfrac{1}{x^6}$

Here $a = \frac{1}{x}$, $r = \frac{1}{x}$ & $n = 6$

$$S_n = \frac{a\{r^n - 1\}}{r-1}$$

$$S_6 = \frac{\frac{1}{x}\left\{\left(\frac{1}{x}\right)^6 - 1\right\}}{\frac{1}{x} - 1} = \frac{\frac{1}{x}\left\{\frac{1-x^6}{x^6}\right\}}{\frac{1-x}{x}}$$

$$= \frac{1}{x}\left(\frac{1-x^6}{x^6}\right) \cdot \left(\frac{x}{1-x}\right)$$

$$= \frac{1-x^6}{x^6(1-x)} \text{ Ans}$$

Q:2 Some of the components a_1, a_n, n, r, S_n of G.P are given. Find the ones that are missing.

(i) $a_1 = 1$, $r = -2$, $a_n = 64$

As $a_n = ar^{n-1}$

$\Rightarrow 64 = 1(-2)^{n-1}$

$\Rightarrow (-2)^6 = (-2)^{n-1}$

$\Rightarrow 6 = n-1 \Rightarrow \boxed{7 = n}$ Ans

And $S_7 = \dfrac{1\{(-2)^7 - 1\}}{(-2) - 1}$ $\quad \because S_n = \dfrac{a\{r^n - 1\}}{r-1}$

$$S_7 = \frac{1\{-128-1\}}{-2-1} = \frac{-129}{-3} = 43 \text{ Ans}$$

Hence $n=7$

$S_7 = 43$

(ii) $r = \frac{1}{2}$ and $a_9 = 1$

Sol

$a_n = ar^{n-1}$

$\Rightarrow a_9 = ar^{9-1}$

$\Rightarrow 1 = a(\frac{1}{2})^8 \Rightarrow 1 = a(\frac{1}{256}) \Rightarrow \boxed{256 = a}$ A

Now $S_9 = \frac{a\{r^9-1\}}{r-1}$ $\because S_n = \frac{a\{r^n-1\}}{r-1}$

$$\Rightarrow S_9 = \frac{256\{(\frac{1}{2})^9-1\}}{\frac{1}{2}-1} = \frac{256\{\frac{1}{512}-1\}}{-\frac{1}{2}}$$

$$= 256\left\{\frac{1-512}{512}\right\} \times \frac{2}{-1}$$

$$= \frac{-511}{-1} = 511 \text{ Ans}$$

(iii) $r = -2$, $S_n = -63$, $a_6 = -96$

Sol

$a_n = ar^{n-1}$

$\Rightarrow a_6 = ar^{6-1}$

$\Rightarrow -96 = a(-2)^5$

$\Rightarrow -96 = a(-32) \Rightarrow a = \frac{-96}{-32}$

$\Rightarrow \boxed{a=3}$ Ans

Now $S_n = \frac{a\{r^n-1\}}{r-1}$

$$\Rightarrow -63 = \frac{3\{(-2)^n-1\}}{-2-1}$$

$$\Rightarrow -63 = \frac{3\{(-2)^n-1\}}{-3}$$

$\Rightarrow 63 = (-2)^n - 1 \Rightarrow 64 = (-2)^n \Rightarrow (-2)^6 = (-2)^n$

$\Rightarrow \boxed{n=6}$

$\left.\begin{array}{l} a=3 \\ n=6 \end{array}\right\}$ Ans

Q3: Find r, such that

$S_{10} = 244 S_5$

Sol As $S_n = \frac{a\{r^n-1\}}{r-1} \Rightarrow S_{10} = \frac{a\{r^{10}-1\}}{r-1}$

& $S_5 = \frac{a\{r^5-1\}}{r-1}$

P.T.V in $S_{10} = 244 S_5$

$$\Rightarrow \frac{a\{r^{10}-1\}}{r-1} = 244\frac{a\{r^5-1\}}{r-1}$$

$\Rightarrow r^{10} - 1 = 244r^5 - 244$

$\Rightarrow r^{10} - 244r^5 + 243 = 0$

$\Rightarrow r^{10} - 243r^5 - r^5 + 243 = 0$

$\Rightarrow r^5(r^5 - 243) - 1(r^5 - 243) = 0$

$\Rightarrow (r^5 - 243)(r^5 - 1) = 0$

$\Rightarrow r^5 - 243 = 0$ or $r^5 - 1 = 0$

$\Rightarrow r^5 = 243 \qquad r^5 = 1$

$\Rightarrow r^5 = 3^5 \qquad \Rightarrow r^5 = 1^5$

$\Rightarrow \boxed{r = 3} \qquad \Rightarrow \boxed{r = 1}$ Not possible

Hence $r = 3$ Ans

Q:4 Prove that

$$S_n\{S_{3n} - S_{2n}\} = (S_n - S_{2n})^2$$

Sol: Sum of n terms $= S_n = \dfrac{a\{r^n - 1\}}{r-1}$

" " $2n$ " $= S_{2n} = \dfrac{a\{r^{2n} - 1\}}{r-1}$

" " $3n$ " $= S_{3n} = \dfrac{a\{r^{3n} - 1\}}{r-1}$

L.H.S $\quad S_n\{S_{3n} - S_{2n}\}$

$$= \frac{a\{r^n - 1\}}{r-1}\left\{\frac{a\{r^{3n}-1\}}{r-1} - \frac{a\{r^{2n}-1\}}{r-1}\right\}$$

take $\dfrac{a}{r-1}$ as common

$$= \frac{a\{r^n-1\}}{r-1}\,\frac{a}{r-1}\left\{(r^{3n}-1) - (r^{2n}-1)\right\}$$

$$= \frac{a^2(r^n-1)}{(r-1)^2}\left\{r^{3n} - 1 - r^{2n} + 1\right\}$$

$$= \frac{a^2(r^n-1)}{(r-1)^2}\left\{r^{3n} - r^{2n}\right\}$$

Ex 4.7 CH-04 P-16

$$= \frac{a^2(r^n-1)}{(r-1)^2}\left\{r^{2n}r^n - r^{2n}\right\}$$

$$= \frac{a^2(r^n-1)}{(r-1)^2}\, r^{2n}(r^n - 1)$$

$$= \frac{a^2 r^{2n}(r^n-1)^2}{(r-1)^2}$$

$$= \left\{\frac{a r^n (r^n-1)}{r-1}\right\}^2$$

$$= \left\{\frac{a r^n r^n - a r^n}{r-1}\right\}^2$$

$$= \left\{\frac{a r^{2n} - a r^n}{r-1}\right\}^2$$ Add and subtract a

$$= \left\{\frac{a r^{2n} - a r^n + a - a}{r-1}\right\}^2$$

$$= \left\{\frac{a r^{2n} - a - a r^n + a}{r-1}\right\}^2$$

$$= \left\{\frac{a(r^{2n}-1) - a(r^n-1)}{r-1}\right\}^2$$

$$= \left\{\frac{a(r^{2n}-1)}{r-1} - \frac{a(r^n-1)}{r-1}\right\}^2$$

$$= \{S_{2n} - S_n\}^2 = \left(-\{-S_{2n} + S_n\}\right)^2 = +(S_n - S_{2n})^2$$

$= R.H.S$

138

Q.5 Find the sum S_n of the 1st n terms of the sequence $\{(\frac{1}{2})^n\}$

Sol

$a_n = (\frac{1}{2})^n$

$\Rightarrow a_1 = (\frac{1}{2})^1 = \frac{1}{2^1}$

$a_2 = (\frac{1}{2})^2 = \frac{1}{2^2}$

$\vdots$

$a_n = (\frac{1}{2})^n = \frac{1}{2^n}$

The series will be

$\frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots\cdots + \frac{1}{2^n}$ which is G.P

Here $a = \frac{1}{2}$ and $r = 1/2$

$S_n = a\left\{\frac{r^n - 1}{r - 1}\right\}$

$\Rightarrow S_n = \dfrac{(\frac{1}{2})\{(\frac{1}{2})^n - 1\}}{\frac{1}{2} - 1}$

$\Rightarrow S_n = \dfrac{(\frac{1}{2})\{(\frac{1}{2})^n - 1\}}{\frac{1-2}{2}} = \dfrac{(\frac{1}{2})\ (-1)\{-(\frac{1}{2})^n + 1\}}{-(\frac{1}{2})}$

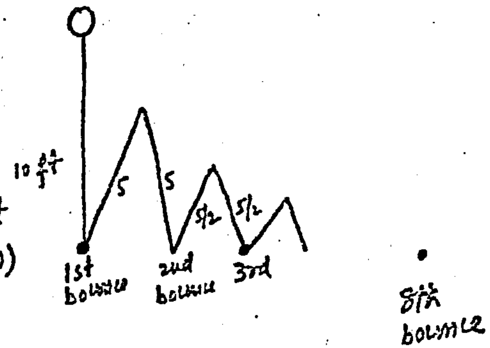
$= -(\frac{1}{2})^n + 1$

$= 1 - (\frac{1}{2})^n$

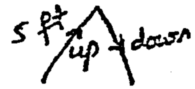
$\Rightarrow S_n = 1 - \frac{1}{2^n}$ Ans

Q.6 A ball rebounces to half the height from which it is dropped. If it is dropped from 10 ft, how far does it travel from the moment it is dropped untill the moment of its eighth bounce?

Sol



When the ball is dropped from 10 ft height. 10 ft is only once. But when it bounces, it covers that distance twice. (up & down)



So the sequence becomes

$10 + \{5 + 5 + 5/2 + 5/2 \cdots\}$

$= 10 + 2\{5 + 5/2 + \cdots\cdots a_7\}$

$= 10 + 2\left\{\dfrac{a(r^7 - 1)}{r - 1}\right\}$

$= 10 + 2\left\{\dfrac{5\{(\frac{1}{2})^7 - 1\}}{\frac{1}{2} - 1}\right\}$

$= 10 + 2\left\{\dfrac{5(\frac{1}{128} - 1)}{\frac{1-2}{2}}\right\}$

$= 10 + 2\left\{\dfrac{5(\frac{1-128}{128})}{\frac{-1}{2}}\right\} = 10 + 10\left(\frac{-127}{128}\right)\left(\frac{2}{-1}\right)$

$= 10 + 19.84375$

$= 29.8437$ Ans

or $29\frac{27}{32}$ ft

Q:7 A man wishes to save money by setting aside Rs 1 the 1st day, Rs 2 the 2nd day, Rs 4 the third day and so on, doubling the amount each day. If this continued, how much must be set aside on the 15th day? What is the total amount saved at the end of 30 days?

Sol

	Day 1	Day 2	Day 3	Day 4	Day 30
Saving →	Rs 1,	Rs 2,	Rs 4,	....	a_{30}

Amount of 15th day $\quad a=1, \ r=2$

$a_n = ar^{n-1}$

$\Rightarrow a_{15} = 1\{2\}^{15-1}$

$\Rightarrow a_{15} = (2)^{14}$

$\Rightarrow a_{15} = 16384$ Ans

Total Amount of the 30 days:

$1 + 2 + 4 + \cdots\cdots + a_{30}$

$$S_n = \frac{a\{r^n - 1\}}{r-1}$$

$$S_{30} = \frac{1\{2^{30}-1\}}{2-1} = \frac{1(1073741824-1)}{1}$$

$\Rightarrow S_{30} =$ Rs. 1073741823 Ans

Q:8 The population of an insect is found to tripple each week in the summer months. If there are twenty insects in the colony at the beginning of the summer, how many are present at the end of 11 weeks assuming no deaths are there?

Sol

Weeks	1st	2nd	3rd	......	a_{12}
Insects	20,	60,	180,	....	a_{12}

because starting insects = 20 and each week they triple.

Total insects at 11 weeks (end of 11 weeks we have a_{12})

$20 + 60 + 180 + \cdots\cdots a_{12}$

$a = 20$ & $r = 3$

$$S_n = \frac{a\{r^n - 1\}}{r-1}$$

$$S_{12} = \frac{20\{3^{12}-1\}}{3-1}$$

$$S_{12} = \frac{20\{3^{12}-1\}}{2}$$

$S_{12} = 10\{3^{12}-1\}$

$= 10\{531441 - 1\}$

$= 10\{531440\}$

$S_{12} = 5314400$ Ans

Exercise # 4.8

Q:1 Find the sum of each of the given infinite G.P

(i) 16, 12, 9, ...

The geometric series will be

$16 + 12 + 9 + \cdots\cdots$

$a = 16 \quad r = \frac{12}{16} \Rightarrow r = 3/4 \quad |r| < 1$

$$S_\infty = \frac{a}{1-r}$$

$$\Rightarrow S_\infty = \frac{16}{1-\frac{3}{4}} = \frac{16}{\frac{4-3}{4}} = \frac{16}{\frac{1}{4}} = 16 \times \frac{4}{1} = 64 \text{ Ans}$$

(ii) $\frac{1}{25}, \frac{1}{5}, 1, 5, \cdots$

Sol $r = \frac{a_4}{a_3}$

$r = \frac{5}{1} = 5 \Rightarrow |r| > 1$

$\Rightarrow S_\infty$ is not possible

(iii) $2, \frac{2}{\sqrt{2}}, 1, \frac{1}{\sqrt{2}}, \frac{1}{2}, \cdots\cdots$

Sol $r = \frac{a_2}{a_1} = \frac{2/\sqrt{2}}{2} = \frac{1}{\sqrt{2}} = 0.707\cdots < 1$

$$S_\infty = \frac{a}{1-r} = \frac{2}{1-\frac{1}{\sqrt{2}}} = \frac{2}{\frac{\sqrt{2}-1}{\sqrt{2}}} = \frac{2}{\sqrt{2}-1} \times \sqrt{2} = \frac{2\sqrt{2}}{\sqrt{2}-1} \text{ Ans}$$

(iv) $15, 1.5, .015, 0.015, \cdots$

Sol $a = 15 \quad r = \frac{1.5}{15} = 0.1 \Rightarrow |r| < 1$

Now $S_\infty = \frac{a}{1-r}$

$$\Rightarrow S_\infty = \frac{15}{1-0.1} = \frac{15}{0.9} = \frac{150}{9} = \frac{50}{3} \text{ Ans}$$

Q:2 Find the 1st five terms of the following infinite geometric sequence.

(i) $a_1 = 25 \quad S_\infty = 125$

As $S_\infty = \frac{a}{1-r}$

$\Rightarrow 125 = \frac{25}{1-r}$

$\Rightarrow 1-r = \frac{25}{125} \Rightarrow 1-r = \frac{1}{5} \Rightarrow 1-\frac{1}{5} = r \Rightarrow \frac{4}{5} = r$

$r = 4/5 \;\&\; a = 25$

$a_1 = a = 25$

$a_2 = ar = 25\left(\frac{4}{5}\right) = 20$

$a_3 = ar^2 = 25\left(\frac{4}{5}\right)^2 = 25\left(\frac{16}{25}\right) = 16$

$a_4 = ar^3 = 25\left(\frac{4}{5}\right)^3 = 25\left(\frac{64}{125}\right) = 64/5$

$a_5 = ar^4 = 25\left(\frac{4}{5}\right)^4 = 25\left(\frac{256}{625}\right) = \frac{256}{25}$ Ans

(ii) $a_1 = 4$ and $S = -7$

Sol As $S_\infty = \frac{a}{1-r}$

$\Rightarrow -7 = \frac{4}{1-r} \Rightarrow 1-r = -\frac{4}{7} \Rightarrow 1+\frac{4}{7} = r$

$\Rightarrow \frac{11}{7} = r$

Since $|r| > 1$

$\Rightarrow$ Such infinite geometric series does not exist.

Q:3 Find the 1st five terms and the sum of an infinite geometric sequence having $a_2 = 2$ and $a_3 = 1$.

Sol $a_2 = ar^{2-1}$ and $a_3 = ar^{3-1}$ $\because a_n = ar^{n-1}$

$\Rightarrow 2 = ar$ and $1 = ar^2 \longrightarrow (i)$

$\Rightarrow a = \frac{2}{r}$, P.T.V in eqn (i)

$1 = ar^2$

$\Rightarrow 1 = \frac{2}{r}.r^2 \Rightarrow 1 = 2r \Rightarrow \boxed{r = 1/2}$

Now $a = \frac{2}{r}$

$\Rightarrow a = \frac{2}{1/2} \Rightarrow \boxed{a=4}$

Then the terms are

$a_1 = a = 4$

$a_2 = ar = 4(\frac{1}{2}) = 2$

$a_3 = ar^2 = 4(\frac{1}{2})^2 = 1$

$a_4 = ar^3 = 4(\frac{1}{2})^3 = 1/2$

$a_5 = ar^4 = 4(\frac{1}{2})^4 = 1/4$

and $S_\infty = \frac{a}{1-r}$

$= \frac{4}{1-\frac{1}{2}} = \frac{4}{\frac{2-1}{2}}$

$S_\infty = 8$ Ans

Q:4 Convert each decimal to common fraction. Ex 4.8 CH-04 P-13

(i) $0.\overline{8}$

Sol $0.\overline{8} = 0.8888.....$

$= 0.8 + 0.08 + 0.008....$

$a = 0.8$ and $r = \frac{0.08}{0.8} = 0.1$ $|r| < 1$

Then $0.\overline{8} = \frac{a}{1-r}$

$= \frac{0.8}{1-0.1}$

$= \frac{0.8}{0.9} = 8/9$

Hence $0.\overline{8} = 8/9$

(ii) $1.\overline{63}$

Sol $1.\overline{63} = 1.63636363.....$

$= 1 + 0.636363.....$

$= 1 + \{0.63 + 0.6363 + 0.636363....\}$

$= 1 + \frac{a}{1-r}$ where $a = 0.63$

and $r = \frac{a_2}{a_1} = \frac{0.6363}{0.63}$

$= 1 + \frac{0.63}{1-0.01}$

$r = 0.01$

$= 1 + \frac{0.63}{0.99}$

$= 1 + \frac{63}{99} = \frac{99+63}{99} = \frac{162}{99}$

Hence $1.\overline{63} = \frac{162}{99}$ Ans

(ii) $2.\overline{15}$

Sol $2.\overline{15} = 2.151515....$

$= 2 + (0.151515.....)$

$= 2 + (0.15 + 0.0015 + ...)$

$= 2 + \frac{a}{1-r}$ where $a = 0.15$

$r = \frac{a_2}{a_1} = \frac{0.0015}{0.15} = 0.01$

$= 2 + \frac{0.15}{1-0.01}$

$= 2 + \frac{0.15}{0.99}$

$= 2 + \frac{15}{99}$

$= \frac{198+15}{99}$

$2.\overline{15} = \frac{213}{99}$

Self question

(ii) $2.1\overline{5}$

Sol $2.1\overline{5} = 2.15555....$

$= 2.1 + 0.05555....$

$= 2.1 + \{0.05 + 0.005 + 0.0005.....\}$

$= 2.1 + \{\frac{a}{1-r}\}$ $r = \frac{0.005}{0.05}$

$= 2.1 + \frac{0.05}{1-0.1}$ $r = 0.1$

$= 2.1 + \frac{0.05}{0.9}$

$= 2.1 + \frac{5}{90}$

$= \frac{21}{10} + \frac{5}{90} = \frac{189+5}{90} = \frac{194}{90}$

$\Rightarrow 2.1\overline{5} = \frac{194}{90} = \frac{97}{45}$

(iv) $0.\overline{123}$

Sol $0.\overline{123} = 0.123123123...$

$= 0.123 + 0.000123 + 0.000000123 + ...$

$0.\overline{123} = \frac{a}{1-r}$ $a = 0.123$ & $r = \frac{0.000123}{0.123}$

$r = 0.001$

$= \frac{0.123}{1-0.001}$

$= \frac{0.123}{0.999} = \frac{123}{999}$

$\Rightarrow 0.\overline{123} = \frac{123}{999} = \frac{41}{333}$

Q:5 The sum of infinite geometric series is 15 and the sum of their squares is 15. Find the series.

Sol $S_\infty = 15$

$\Rightarrow \frac{a}{1-r} = 15 \Rightarrow a = 15(1-r) \longrightarrow ①$

And given that the sum of their squares is 45

$\Rightarrow a^2 + (ar)^2 + (ar^2)^2 + \text{——} = 45$

$\Rightarrow a^2 + a^2r^2 + a^4r^4 + ... = 45$

$\Rightarrow a^2(1 + r^2 + r^4 + ...) = 45$

$a = 1$
ratio $= r^2$

$\Rightarrow a^2\left(\frac{1}{1-r^2}\right) = 45 \Rightarrow \frac{a^2}{1-r^2} = 45$

$\Rightarrow a^2 = 45(1-r^2)$

$\Rightarrow a^2 = 45(1-r)(1+r)$

$\Rightarrow a^2 = 3 \times 15(1-r)(1+r)$ from eqn (i)

$\Rightarrow a^2 = 3 \quad a \quad (1+r) \qquad a = 15(1-r)$

$\Rightarrow a = 3(1+r)$

$\Rightarrow 15(1-r) = 3(1+r)$

$\Rightarrow 5(1-r) = 1+r$

$\Rightarrow 5-5r = 1+r \Rightarrow 6r = 4 \Rightarrow r = 2/3$

Now $a = 15(1-r)$

$a = 15\left(1-\frac{2}{3}\right)$

$a = 15\left(\frac{3-2}{3}\right) \Rightarrow \boxed{a=5}$

Now the series will be

$a + ar + ar^2 + ar^3 + \cdots\cdots$

$= 5 + 5\left(\frac{2}{3}\right) + 5\left(\frac{2}{3}\right)^2 + \cdots\cdots\cdots$

$= 5 + \frac{10}{3} + \frac{20}{9} + \cdots\cdots\cdots$ Ans.

Q.6: The sum of 1st 6 terms of a geometric series is 9 times the sum of its 1st three terms. Find the common ratio.

Sol Sum of 1st 6 terms $= S_6$

" " " 3 terms $= S_3$

According to the condition

Sum of 1st 6 terms is 9 times the sum of 1st 3 terms

$\Rightarrow S_6 = 9\,S_3$

$\Rightarrow \frac{a\{r^6-1\}}{r-1} = 9\,\frac{a\{r^3-1\}}{r-1}$

$\Rightarrow r^6 - 1 = 9r^3 - 9$

$\Rightarrow r^6 - 9r^3 + 8 = 0$

$\Rightarrow (r^3)^2 - 9r^3 + 8 = 0 \qquad$ Let $r^3 = x$

$\Rightarrow x^3 - 9x + 8 = 0$

$\Rightarrow x^2 - 8x - x + 8 = 0$

$\Rightarrow x(x-8) - 1(x-8) = 0$

$\Rightarrow (x-8)(x-1) = 0$

$\Rightarrow x-8=0 \quad$ or $\quad x-1=0$

$x = 8 \quad$ or $\quad x = 1$

$\Rightarrow r^3 = 8 \quad$ or $\quad r^3 = 1$

$\Rightarrow r^3 = 2^3 \quad$ or $\quad r^3 = 1^3$.

$\Rightarrow \boxed{r=2}$ Ans $\qquad r = 1$ is not possible.

Q:7 How many terms of the series $1+\sqrt{3}+3+\cdots$ be added to get the sum $40+13\sqrt{3}$.

Sol Geometric series is

$1+\sqrt{3}+3+\cdots$

Here $a=1$ and $r=\sqrt{3}$, $S_n=40+13\sqrt{3}$, $n=?$

$$S_n = \frac{a\{r^n-1\}}{r-1}$$

$$\Rightarrow 40+13\sqrt{3} = \frac{1\{(\sqrt{3})^n-1\}}{\sqrt{3}-1}$$

$$\Rightarrow (40+13\sqrt{3})(\sqrt{3}-1) = (\sqrt{3})^n-1$$

$$\Rightarrow 40\sqrt{3}-40+13(\sqrt{3})^2-13\sqrt{3} = (\sqrt{3})^n-1$$

$$\Rightarrow 40\sqrt{3}-40+39-13\sqrt{3} = (\sqrt{3})^n-1$$

$$\Rightarrow 40\sqrt{3}-1-13\sqrt{3} = (\sqrt{3})^n-1$$

$$\Rightarrow 40\sqrt{3}-13\sqrt{3} = (\sqrt{3})^n$$

$$\Rightarrow 27\sqrt{3} = (\sqrt{3})^n$$

$$\Rightarrow 3^3\cdot 3^{1/2} = (3^{1/2})^n$$

$$\Rightarrow 3^{3+\frac{1}{2}} = 3^{n/2}$$

$$\Rightarrow 3^{7/2} = 3^{n/2}$$

compare the powers

$$\frac{7}{2} = \frac{n}{2} \Rightarrow \boxed{7=n} \text{ Ans}$$

Q:8: Find an infinite geometric series whose sum is 6 and such that each term is four times the sum of all the terms that follow it.

Sol Let $a+ar+ar^2+\cdots$ be the infinite geometric series.

Given that $S_\infty = 6$

$$\Rightarrow \frac{a}{1-r} = 6$$

$$\Rightarrow a = 6-6r \longrightarrow (i)$$

Also each term is 4 times the sum of all the terms that follow it.

$a_1 = 4(a_2+a_3+a_4+\cdots)$ or $a_2 = 4(a_3+a_4+\cdots)$

$$\Rightarrow a = 4(ar^1+ar^2+ar^3+\cdots)$$

$$\Rightarrow \cancel{a} = 4\cancel{a}r(1+r+r^2+\cdots)$$

$$\Rightarrow 1 = 4r\left(\frac{a_1}{1-r}\right)$$

$$\Rightarrow 1 = 4r\left(\frac{1}{1-r}\right)$$

$$\Rightarrow 1-r=4r \Rightarrow 1=5r \Rightarrow \boxed{r=1/5}$$

Eqn (i) $\Rightarrow$ $a=6-6r$

$$a = 6-6\left(\tfrac{1}{5}\right) \Rightarrow a = \frac{30-6}{5} \Rightarrow \boxed{a=24/5}$$

Then the series becomes

$$a+ar+ar^2+\cdots$$

$$= \frac{24}{5}+\frac{24}{5}\left(\frac{1}{5}\right)+\cdots$$

$$= 24/5 + 24/25 + 24/125 + \cdots \text{ Ans}$$

Q:9 If $y = \frac{x}{3} + \frac{x^2}{3^2} + \frac{x^3}{3^3} + \cdots\cdots$ where $0 < x < 3$

Show that $x = \frac{3y}{1+y}$

Sol

$$y = \frac{x}{3} + \frac{x^2}{3^2} + \frac{x^3}{3^3} + \cdots\cdots$$

It is infinite geometric series with $a = \frac{x}{3}$ & $r = \frac{x}{3}$

$$y = \frac{a}{1-r}$$

$$y = \frac{x/3}{1-\frac{x}{3}} \Rightarrow y = \frac{x/3}{\frac{3-x}{3}} \Rightarrow y = \frac{x/3}{3-x/3} \Rightarrow y = \frac{x}{3-x}$$

Then $(3-x)y = x \Rightarrow 3y - yx = x$

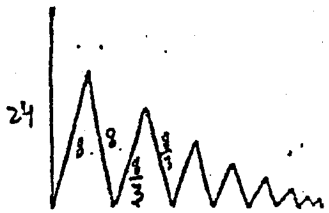
$\Rightarrow 3y = x + yx$

$\Rightarrow 3y = x(1+y)$

$\Rightarrow \boxed{x = \frac{3y}{1+y}}$ proved.

Q:10 Find how far a ball travels before coming to rest, if it is dropped from a heigth of 24 meters and each time it hits the ground, it rebounds one third of the distance from which it fell?

Sol Diagram



Sol Total distance travelled is

$24 + \{8 + 8 + \frac{8}{3} + \frac{8}{3} + \cdots\}$

$= 24 + 2\{8 + \frac{8}{3} + \frac{8}{24} + \cdots\}$

$= 24 + 2\{\frac{a}{1-r}\}$ where $a = 8$ & $r = 1/3$

$= 24 + 2\{\frac{8}{1-\frac{1}{3}}\}$

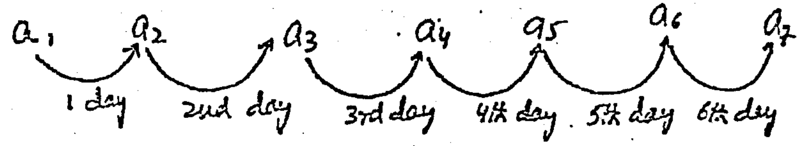
$= 24 + 2\{\frac{8}{2/3}\}$

$= 24 + 24$

$= 48$ m Ans

Q:11 The number of bacteria in a culture increased geometrically from 64000 to 729000 in 6 days. Find the daily rate of increase if the rate is assumed to be constant.

Sol



start $= a_1 = 64000$

Total days $= n = 7$

End $= a_n = 729000$

To find r: $\quad a_n = ar^{n-1}$

$\Rightarrow 729000 = 64000 \cdot r^{7-1}$

$$\Rightarrow \frac{729000}{64000} = r^6 \quad \text{or} \quad \frac{729}{64} = r^6$$

$$\Rightarrow 11.390625 = r^6 \qquad \frac{3^6}{2^6} = r^6$$

$$\Rightarrow r^6 = 11.390625 \qquad (3/2)^6 = r^6$$

take $\frac{1}{6}$ power of b.s

$$\Rightarrow \boxed{r = 3/2}$$

$$(r^6)^{1/6} = (11.390625)^{1/6}$$

$$r = 1.5$$

Now common ratio of 1.5 implies that quantity 1 becomes 1.5 after the increase.

So the rate of increase = 50%

$1 + 0.5$
$= 1.5$
0.5 is half of 1

or Increase $= 0.5$

$$= 0.5 \times \frac{100}{100}$$

$$= 50 \times \frac{1}{100}$$

$$= 50\ \%$$

Q:12: If the population of a town increases geometrically at the rate of 5% per year and the present population is 300,000. What will be the population after 6 years from now.

Sol Starting population $= 300,000 = a_1$

Increase $= 5\% = r$

So rate $= 5\%$

$$= 5\left(\frac{1}{100}\right) = 0.05$$

Then the common ratio $= 1 + 0.05 = 1.05$

After 6 years term will be a_7.

$$a_n = ar^{n-1} \Rightarrow a_7 = ar^{7-1}$$

$$= (300,000)(1.05)^6$$

$$= (300,000)(1.340095)$$

$$= 402028.68$$

$$a_7 = 402029 \quad \text{Ans}$$

Q:13: A machine which contains 16000 litters of water. Each day one half of the water in the tank is removed with out replacement. How much water remains in the tank at the end of the 8th day?

Sol Initial water $= a_1 = 16000$ liters

Removal per day $= \frac{1}{2}$ or half

So water at 2nd day $= \frac{1}{2}(16000) = 8000$

$$\Rightarrow a_2 = 8000$$

So the sequence becomes

$$16,000,\ 8000,\ 4000, \ldots \Rightarrow r = 1/2$$

$$a_1 \xrightarrow{\text{1st day}} a_2 \xrightarrow{\text{2nd day}} a_3 \xrightarrow{\text{3rd day}} a_4 \ldots$$

End of 8th day implies a_9.

$a_n = ar^{n-1}$

$\Rightarrow a_9 = ar^{9-1} = 16000(1/2)^8$

$= 16000\left(\frac{1}{256}\right)$

$= 62.5$

$a_9 = 62.5$ litters.

Exercise # 4.9

Q:1 Find the indicated term of the H.P

(i) $\frac{1}{2}, \frac{1}{5}, \frac{1}{8}, \ldots$ 9th term.

Sol H.P is $\frac{1}{2}, \frac{1}{5}, \frac{1}{8}, \ldots$

$\Rightarrow$ A.P will be $2, 5, 8, \ldots$

$a = 2$ $d = 3$, then

$a_n = a+(n-1)d$

$a_9 = 2+(9-1)3$

in A.P. $a_9 = 26$

$\Rightarrow$ In H.P $\boxed{a_9 = 1/26}$ Ans

(ii) $6, 2, \frac{6}{5}, \ldots$ 20th term

Sol H.P is $6, 2, 6/5, \ldots$

A.P will be $\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \ldots$

$\Rightarrow a = 1/6$ & $d = \frac{1}{2} - \frac{1}{6}$

$d = \frac{3-1}{6} \Rightarrow \boxed{d = 1/3}$

$a_n = a+(n-1)d$.

$\Rightarrow a_{20} = \frac{1}{6} + (20-1)\frac{1}{3}$

$a_{20} = \frac{1}{6} + \frac{19}{3} = \frac{1+38}{6} = 39/6 = 13/2$

in A.P $a_{20} = 13/2$

in H.P $a_{20} = 2/13$ Ans

(iii) $5\frac{2}{3}, 3\frac{2}{5}, 2\frac{3}{7}, \ldots$ 8th term

$\Rightarrow \frac{17}{3}, \frac{17}{5}, \frac{17}{7}, \ldots$ is H.P

$\Rightarrow \frac{3}{17}, \frac{5}{17}, \frac{7}{17}, \ldots$ is A.P

$a = 3/17$ & $d = \frac{5}{17} - \frac{3}{17} \Rightarrow d = \frac{2}{17}$

$a_n = a+(n-1)d$

$a_8 = \frac{3}{17} + (8-1)\frac{2}{17} = \frac{3}{17} + \frac{14}{17} = \frac{3+14}{17} = \frac{17}{17} = 1$

$a_8 = 1$ in A.P

in H.P $a_8 = \frac{1}{1}$

$\Rightarrow a_8 = 1$ Ans

Q:2 Find five more terms of the H.P

$\frac{1}{3}, 1, -1, \ldots\ldots$

Sol H.P is $\frac{1}{3}, 1, -1, \ldots$

A.P will be $\frac{3}{1}, \frac{1}{1}, \frac{1}{-1}, \ldots$

$3, 1, -1, \ldots$

$\Rightarrow a = 3 \quad d = -2 \quad (\because d = a_2 - a_1 = 1 - 3)$

as $a_n = a + (n-1)d$ in A.P

$a_4 = a + (4-1)d = 3 + 3(-2) = -3$

$a_5 = a + (5-1)d = 3 + 4(-2) = -5$

$a_6 = a + (6-1)d = 3 + 5(-2) = -7$

$a_7 = a + (7-1)d = 3 + 6(-2) = -9$

$a_8 = a + (8-1)d = 3 + 7(-2) = -11$

In H.P $a_4 = \frac{-1}{3}$, $a_5 = \frac{-1}{5}$, $a_6 = \frac{-1}{7}$, $a_7 = \frac{-1}{9}$, $a_8 = \frac{-1}{11}$

Q:3 The second term of H.P is 1/2 and fifth term is $-\frac{1}{4}$. Find the 12th term.

Sol $a_2 = \frac{1}{2}$ and $a_5 = -\frac{1}{4}$ in H.P

In A.P $a_2 = 2$ and $a_5 = -4$

$a + 1d = 2$ (i) $\quad a + 4d = -4$ (ii) $\quad \because a_n = a + (n-1)d$

Eqn (ii) − Eqn (i)

$$\begin{array}{r} a + 4d = -4 \\ \underline{-a \mp 1d = -2} \\ 3d = -6 \end{array} \Rightarrow \boxed{d = -2}$$

Eqn (i) $\Rightarrow a + d = 2$

$a - 2 = 2 \Rightarrow \boxed{a = 4}$

Now $a_n = a + (n-1)d$

$a_{12} = a + (12-1)d$

$= 4 + 11(-2) = 4 - 22 = -18$

So in A.P. $a_{12} = -18$

$\Rightarrow$ in H.P $a_{12} = -\frac{1}{18}$ Ans

Q:4 Find the A.M, H.M and G.M of each of the following. Also verify that

$AH = G^2$

(i) 3.14, 2.71

Sol $a = 3.14, \quad b = 2.71$

$A = \frac{a+b}{2} = \frac{3.14 + 2.71}{2} = \frac{5.85}{2} = 2.925 \Rightarrow \boxed{A = 2.925}$

$G = \sqrt{ab} = \sqrt{3.14 \times 2.71} = \sqrt{8.5094} = 2.917 \Rightarrow \boxed{G = \pm 2.92}$

$H = \frac{2ab}{a+b} = \frac{2 \times 3.14 \times 2.71}{3.14 + 2.71} = \frac{17.0188}{5.85} = 2.909 \Rightarrow \boxed{H = 2.91}$

To verify $A.M \times H.M = (G.M)^2$

$AH = G^2$

L.H.S $AH = (2.925)(2.91) = 8.5117 \Rightarrow A.H = 8.52$ (i)

R.H.S $G = \pm 2.92$

$\Rightarrow (G)^2 = (\pm 2.92)^2$

$\Rightarrow G^2 = 8.52 \longrightarrow$ (ii)

From eqns (i) and (ii), it is verified that

$AH = G^2$

(ii) -6 and -216

Sol $a = -6$ and $b = -216$

$A.M = \frac{a+b}{2} = \frac{(-6)+(-216)}{2} = \frac{-222}{2} = -111$

$H.M = \frac{2ab}{a+b} = \frac{2(-6)(-216)}{(-6)+(-216)} = \frac{2592}{-222} = -11.68$

$G.M = \sqrt{ab} = \pm\sqrt{-6 \times -216} = \pm\sqrt{1296} = \pm 36$

Now $A.H = (-111)(-11.68) = 1296$

$G^2 = (\pm 36)^2 = 1296$

Hence $A.H = G^2$

(iii) $x+y$, $x-y$

Sol $a = x+y$ $b = x-y$

$A = \frac{a+b}{2} = \frac{(x+y)+(x-y)}{2} = \frac{2x}{2} = x$

$H = \frac{2ab}{a+b} = \frac{2(x+y)(x-y)}{(x+y)+(x-y)} = \frac{2(x^2-y^2)}{2x} = \frac{x^2-y^2}{x}$

$G = \sqrt{ab} = \sqrt{(x+y)(x-y)} = \pm\sqrt{x^2-y^2}$

Now L.H.S $AH = x\left(\frac{x^2-y^2}{x}\right) = x^2-y^2$

R.H.S $G = \pm\sqrt{x^2-y^2}$

$\Rightarrow G^2 = x^2-y^2$

Hence $AH = G^2$

(iv) $\sqrt{2}+3$, $\sqrt{2}-3$

Sol let $a = \sqrt{2}+3$

$b = \sqrt{2}-3$

$A = \frac{a+b}{2} = \frac{(\sqrt{2}+3)+(\sqrt{2}-3)}{2} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

$G = \sqrt{ab} = \sqrt{(\sqrt{2}+3)(\sqrt{2}-3)} = \sqrt{(\sqrt{2})^2-3^2} = \sqrt{2-9} = \sqrt{-5}$

$G =$ Undefined

Now $H = \dfrac{2ab}{a+b} = \dfrac{2(\sqrt{2}+3)(\sqrt{2}-3)}{(\sqrt{2}+3)+(\sqrt{2}-3)}$

$H = \dfrac{2\{(\sqrt{2})^2 - 3^2\}}{2\sqrt{2}}$

$= \dfrac{2-9}{\sqrt{2}} = \dfrac{-7}{\sqrt{2}}$ Ans

Since $G =$ Undefined, so $AH = G^2$ is also not verified

Q:5 For what value of n, $\dfrac{a^{n+1}+b^{n+1}}{a^n+b^n}$ is H.M b/w a and b.

Sol

$\dfrac{a^{n+1}+b^{n+1}}{a^n+b^n} = H.M$

$\Rightarrow \dfrac{a^{n+1}+b^{n+1}}{a^n+b^n} = \dfrac{2ab}{a+b}$

$\Rightarrow (a+b)(a^{n+1}+b^{n+1}) = 2ab(a^n+b^n)$

$\Rightarrow a.a^{n+1} + ab^{n+1} + ba^{n+1} + b.b^{n+1} = 2ab.a^n + 2ab.b^n$

$\Rightarrow a.a^{n+1} + ab^{n+1} + ba^{n+1} + b.b^{n+1} = 2ba^{n+1} + 2ab^{n+1}$

$\Rightarrow a.a^{n+1} + b.b^{n+1} = 2ba^{n+1} - ba^{n+1} + 2ab^{n+1} - ab^{n+1}$

$\Rightarrow a.a^{n+1} + b.b^{n+1} = ba^{n+1} + ab^{n+1}$

$\Rightarrow a.a^{n+1} - b.a^{n+1} = ab^{n+1} - b.b^{n+1}$

$\Rightarrow a^{n+1}(a-b) = b^{n+1}(a-b)$

$\Rightarrow a^{n+1} = b^{n+1}$

$\div$ by b^{n+1}

$\Rightarrow \dfrac{a^{n+1}}{b^{n+1}} = 1$

$\Rightarrow \left(\dfrac{a}{b}\right)^{n+1} = \left(\dfrac{a}{b}\right)^0$

$\Rightarrow n+1 = 0$

$\Rightarrow \boxed{n=-1}$ Ans

Q:6 Insert two H.Ms b/w 12 and 48?

Sol Let H_1 and H_2 are two H.Ms b/w 12 & 48

then $12, H_1, H_2, 48$ is H.P

$\Rightarrow \dfrac{1}{12}, \dfrac{1}{H_1}, \dfrac{1}{H_2}, \dfrac{1}{48}$ is A.P

To find d

$a_n = a + (n-1)d$

$\dfrac{1}{48} = \dfrac{1}{12} + (4-1)d$

$\Rightarrow \dfrac{1}{48} - \dfrac{1}{12} = 3d$

$\Rightarrow \dfrac{1-4}{48} = 3d \Rightarrow \dfrac{-3}{48} = 3d \Rightarrow \boxed{d = \dfrac{-1}{48}}$

Now $H = \dfrac{2ab}{a+b} = \dfrac{2(\sqrt{2}+3)(\sqrt{2}-3)}{(\sqrt{2}+3)+(\sqrt{2}-3)}$

$$H = \frac{2\{(\sqrt{2})^2-3^2\}}{2\sqrt{2}}$$

$$= \frac{2-9}{\sqrt{2}} = \frac{-7}{\sqrt{2}} \text{ Ans}$$

Since G = Undefined, so $AH = G^2$ is also not verified

Q:5 For what value of n, $\dfrac{a^{n+1}+b^{n+1}}{a^n+b^n}$ is H.M b/w a and b.

Sol

$$\frac{a^{n+1}+b^{n+1}}{a^n+b^n} = H.M$$

$$\Rightarrow \frac{a^{n+1}+b^{n+1}}{a^n+b^n} = \frac{2ab}{a+b}$$

$$\Rightarrow (a+b)(a^{n+1}+b^{n+1}) = 2ab(a^n+b^n)$$

$$\Rightarrow a.a^{n+1}+ab^{n+1}+ba^{n+1}+b.b^{n+1} = 2ab.a^n+2ab.b^n$$

$$\Rightarrow a.a^{n+1}+ab^{n+1}+ba^{n+1}+b.b^{n+1} = 2ba^{n+1}+2ab^{n+1}$$

$$\Rightarrow a.a^{n+1}+b.b^{n+1} = 2ba^{n+1}-ba^{n+1}+2ab^{n+1}-ab^{n+1}$$

$$\Rightarrow a.a^{n+1}+b.b^{n+1} = ba^{n+1}+ab^{n+1}$$

$$\Rightarrow a.a^{n+1}-b.a^{n+1} = ab^{n+1}-b.b^{n+1}$$

$$\Rightarrow a^{n+1}(a-b) = b^{n+1}(a-b)$$

$$\Rightarrow a^{n+1} = b^{n+1}$$

$\div$ by b^{n+1}

$$\Rightarrow \frac{a^{n+1}}{b^{n+1}} = 1$$

$$\Rightarrow \left(\frac{a}{b}\right)^{n+1} = \left(\frac{a}{b}\right)^0$$

$$\Rightarrow n+1 = 0$$

$$\Rightarrow \boxed{n=-1} \text{ Ans}$$

Q:6 Insert two H.Ms b/w 12 and 48?

Sol Let H_1 and H_2 are two H.Ms b/w 12 & 48

then $12, H_1, H_2, 48$ is H.P

$$\Rightarrow \frac{1}{12}, \frac{1}{H_1}, \frac{1}{H_2}, \frac{1}{48} \text{ is A.P}$$

To find d

$$a_n = a+(n-1)d$$

$$\frac{1}{48} = \frac{1}{12}+(4-1)d$$

$$\Rightarrow \frac{1}{48}-\frac{1}{12} = 3d$$

$$\Rightarrow \frac{1-4}{48} = 3d \Rightarrow \frac{-3}{48} = 3d \Rightarrow \boxed{d = \frac{-1}{48}}$$

Now $H_n = \frac{1}{a+nd}$

$H_1 = \frac{1}{a+1d} = \frac{1}{\frac{1}{12}+\frac{-1}{48}} = \frac{1}{\frac{4-1}{48}} = \frac{1}{3/48} = \frac{1}{\frac{1}{16}} = 16$

$H_2 = \frac{1}{a+2d} = \frac{1}{\frac{1}{12}+2\left(\frac{-1}{48}\right)} = \frac{1}{\frac{1}{12}-\frac{1}{24}} = \frac{1}{\frac{2-1}{24}} = 24$ Ans

Hence $\left.\begin{matrix} H_1 = 16 \\ H_2 = 24 \end{matrix}\right\}$ Ans

Q.7 Insert four H.Ms b/w 7/3 and 7/11 ?

Sol Let H_1, H_2, H_3, H_4 are four H.Ms b/w $\frac{7}{3}$ and $\frac{7}{11}$

Then $\frac{7}{3}, H_1, H_2, H_3, H_4, \frac{7}{11}$ is H.P

$\Rightarrow \frac{3}{7}, \frac{1}{H_1}, \frac{1}{H_2}, \frac{1}{H_3}, \frac{1}{H_4}, \frac{11}{7}$ is A.P.

$a_n = a + (n-1)d$

$\frac{11}{7} = \frac{3}{7} + (6-1)d$

$\frac{11}{7} - \frac{3}{7} = 5d$

$\frac{8}{7} = 5d \Rightarrow \boxed{d = 8/35}$

$H_n = \frac{1}{a+nd}$

$H_1 = \frac{1}{a+1d} = \frac{1}{\frac{3}{7}+1\left(\frac{8}{35}\right)} = \frac{1}{\frac{15+8}{35}} = \frac{35}{23}$

$H_2 = \frac{1}{a+2d} = \frac{1}{\frac{3}{7}+2\left(\frac{8}{35}\right)} = \frac{1}{\frac{15+16}{35}} = \frac{35}{31}$

$H_3 = \frac{1}{a+3d} = \frac{1}{\frac{3}{7}+3\left(\frac{8}{35}\right)} = \frac{1}{\frac{15+24}{35}} = \frac{35}{39}$

$H_4 = \frac{1}{a+4d} = \frac{1}{\frac{3}{7}+4\left(\frac{8}{35}\right)} = \frac{1}{\frac{15+32}{35}} = \frac{35}{47}$ A

Q.8 Prove that the square of the G.M of two numbers equal the product of A.M and H.M of the two numbers.

Sol Let a and b are the two numbers. To prove $AH = G^2$

Then $A = \frac{a+b}{2}$, $G = \sqrt{ab}$, $H = \frac{2ab}{a+b}$

L.H.S $AH = \left(\frac{a+b}{2}\right)\left(\frac{2ab}{a+b}\right) = ab \longrightarrow$ (i)

R.H.S $G = \pm\sqrt{ab}$

$G^2 = ab \longrightarrow$ (ii)

From (i) and (ii) it is proved

$G^2 = AH$

Q.9 The A.M of two numbers is 8 and the H.M is 6. Find the numbers.

Sol Let a and b are the numbers.

Then $A = \frac{a+b}{2}$ & $H = \frac{2ab}{a+b}$

$\Rightarrow 8 = \frac{a+b}{2}$

$\Rightarrow a+b = 16$

$\Rightarrow a = 16-b$

$6 = \frac{2ab}{a+b}$

$\Rightarrow 6a+6b = 2ab \longrightarrow$ (i)

P.T.V of a

$\Rightarrow 6(16-b) + 6b = 2(16-b)b$

$\Rightarrow 96 - 6b + 6b = 32b - 2b^2$

$\Rightarrow 96 = 32b - 2b^2$

$\Rightarrow 2b^2 - 32b + 96 = 0$

$\div$ by 2

$\Rightarrow b^2 - 16b + 48 = 0$

By factorization

$\Rightarrow b^2 - 12b - 4b + 48 = 0$

$\Rightarrow b(b-12) - 4(b-12) = 0$

$\Rightarrow (b-12)(b-4) = 0$

$\Rightarrow b - 12 = 0$ or $b - 4 = 0$

$b = 12$ or $b = 4$

Now

$a = 16 - b$

$b=4 \Rightarrow a = 16 - 4 = 12$

$b=12 \Rightarrow a = 16 - 12 = 4$

Hence the two #s are $12, 4$ or $4, 12$ Ans

Q:10 The H.M b/w two numbers is $4\frac{4}{5}$ and their G.M is 6. What are the numbers?

Sol Let a and b are the numbers.

Then $H.M = \dfrac{2ab}{a+b}$ & $G.M = \sqrt{ab}$

$\Rightarrow 4\frac{4}{5} = \dfrac{2ab}{a+b}$ & $6 = \sqrt{ab}$

$\Rightarrow \dfrac{24}{5} = \dfrac{2ab}{a+b}$ & $6^2 = ab$

$36 = ab$

$\Rightarrow 24a + 24b = 10ab$ $\Rightarrow a = \dfrac{36}{b}$

put $a = 36/b$

$\Rightarrow 24\left(\dfrac{36}{b}\right) + 24b = 10\left(\dfrac{36}{b}\right)b$

$\Rightarrow \dfrac{864}{b} + 24b = 360$

Xing by b.

$\Rightarrow 864 + 24b^2 = 360b$

$\Rightarrow 24b^2 - 360b + 864 = 0$, Divide by 24

$\Rightarrow b^2 - 15b + 36 = 0$

$\Rightarrow b^2 - 12b - 3b + 36 = 0$

$\Rightarrow b(b-12) - 3(b-12) = 0$

$\Rightarrow (b-12)(b-3) = 0$

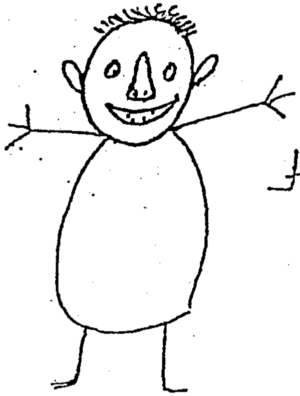
$\Rightarrow b = 12$ or $b = 3$

Now $a = \dfrac{36}{b}$

for $b=3$ $a = \dfrac{36}{3} = 12$

for $b=12$ $a = \dfrac{36}{12} = 3$

Hence the numbers are $3, 12$ or $12, 3$ A



Hurrah! Thats the end of chapter # 04