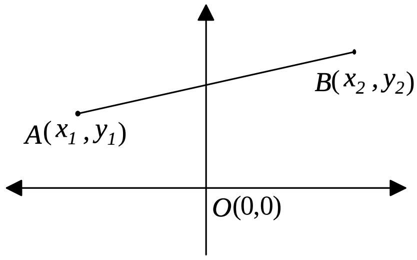
Distance Formula

Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be two points in a plane and d be a distance between A and B then



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

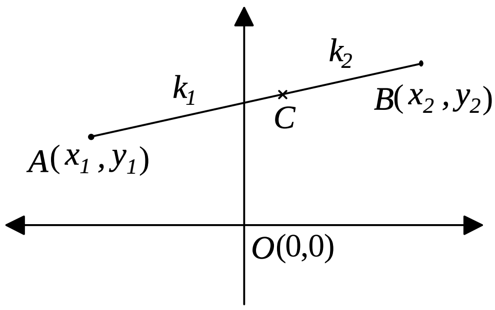
or $$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

See proof on book at page181

… …

Ratio Formula

Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be two points in a plane. The coordinates of the point C dividing the line segment AB in the ratio $k_1 : k_2$ are



$$\left(\frac{k_1x_2 + k_2x_1}{k_1 + k_2}, \frac{k_1y_2 + k_2y_1}{k_1 + k_2}\right)$$

If C be the midpoint of AB i.e. $k_1 : k_2 = 1:1$ then coordinate of C becomes

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

See proof on book at page 182

… …

Intersection of Median

Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are vertices of triangle.
Intersection of median is called centroid of triangle and can be determined as

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

See proof at page 184

… …

Centre of In-Circle (In-Centre)

Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are vertices of triangle.
And $|AB| = c$, $|BC| = a$, $|CA| = b$

Then incentre of triangle $= \left(\dfrac{ax_1 + bx_2 + cx_3}{a + b + c}, \dfrac{ay_1 + by_2 + cy_3}{a + b + c}\right)$

See proof at page 184

… …

Rotation of Axes

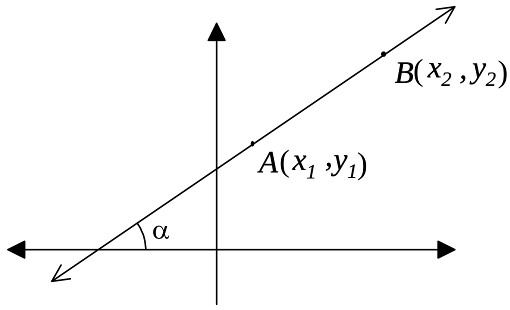
Let (x, y) be the coordinates of point P in *xy*-coordinate system. If the axes are rotated through at angle of θ and (X, Y) are coordinate of P in new *XY*-coordinate system then

$$X = x\cos\theta + y\sin\theta$$
$$Y = y\cos\theta - x\sin\theta$$

… …

3 *Inclination of a Line:*

The angle α $(0^\circ \le \alpha < 180^\circ)$ measure anti-clockwise from positive $x-$axis to the straight line l is called *inclination* of a line l.



3 *Slope or Gradient of Line*

The slope m of the line l is defined by:

$$m = \tan\alpha$$

If $A(x_1, y_1)$ and $B(x_2, y_2)$ be any two distinct points on the line l then

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$$

See proof on book at page: 191

3 *Note:*

l is horizontal, iff $m = 0$ $(\because \alpha = 0^\circ)$

l is vertical, iff $m = \infty$ i.e. m is not defined. $(\because \alpha = 90^\circ)$

If slope of AB = slope of BC, then the points A, B and C are collinear i.e. lie on the same line.

3 *Theorem*

The two lines l_1 and l_2 with respective slopes m_1 and m_2 are

(i) Parallel iff $m_1 = m_2$

(ii) Perpendicular iff $m_1 m_2 = -1$ or $m_1 = -\dfrac{1}{m_2}$

(with m_1 and m_2 non-zero)

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3 *Equation of Straight Line:*

(i) Slope-intercept form

Equation of straight line with slope m and $y-$intercept c is given by:

$$\boxed{y = mx + c}$$

See proof on book at page 194

(ii) Point-slope form

Let m be a slope of line and $A(x_1, y_1)$ be a point lies on a line then equation of line is given by:

$$\boxed{y - y_1 = m(x - x_1)}$$

See proof on book at page 195

(iii) Symmetric form

Let α be an inclination of line and $A(x_1, y_1)$ be a point lies on a line then equation of line is given by:

$$\boxed{\frac{y - y_1}{\cos\alpha} = \frac{x - x_1}{\sin\alpha}}$$

See proof on book at page 195

(iv) Two-points form

Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be points lie on a line then it's equation is given by:

$$\boxed{y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)} \quad \text{or} \quad \boxed{y - y_2 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_2)} \quad \text{or} \quad \boxed{\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0}$$

See proof on book at page 196

(v) Two-intercept form

When a line intersect $x-$axis at $x = a$ and $y-$axis at $y = b$ i.e. $x-$intercept $= a$ and $y-$intercept $= b$, then equation of line is given by:

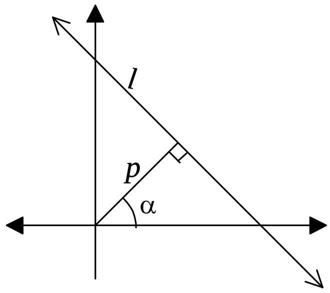
$$\boxed{\frac{x}{a} + \frac{y}{b} = 1}$$

See proof on book at page 197

(vi) Normal form

Let p denoted length of perpendicular from the origin to the line and α is the angle of the perpendicular from +ive *x*-axis then equation of line is given by:

$$\boxed{x\cos\alpha + y\sin\alpha = p}$$



See proof on book at page 198

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3 *General equation of the straight line*

A general equation of straight line (General linear equation) in two variable x and y is given by:

$$ax + by + c = 0$$

where a, b and c are constants and a and b are not simultaneously zero.

See proof on book at page: 199.

Note: Since $ax + by + c = 0 \quad \Rightarrow \quad by = -ax - c \quad \Rightarrow \quad y = -\frac{a}{b}x - \frac{c}{b}$

Which is an intercept form of equation of line with slope $m = -\frac{a}{b}$ and $c = -\frac{c}{b}$.

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3 *Position of the point with respect to line* *(Page 204)*

Consider $l:\ ax + by + c = 0$ with $b > 0$

Then point $P(x_1, y_1)$ lies

i) above the line l if $ax_1 + by_1 + c > 0$

ii) below the line l if $ax_1 + by_1 + c < 0$

3 *Corollary 1* *(Page 205)*

The point $P(x_1, y_1)$ lies above the line if $ax_1 + by_1 + c$ and b have the same sign and the point $P(x_1, y_1)$ lies below the line if $ax_1 + by_1 + c$ and b have opposite signs.

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³ *Point of intersection of lines*

Let $l_1:\ a_1x+b_1y+c_1=0$

$l_2:\ a_2x+b_2y+c_2=0$ be non-parallel lines.

Let $P(x_1,y_1)$ be the point of intersection of l_1 and l_2. Then

$$a_1x_1+b_1y_1+c_1=0 \text{ ....}(i)$$
$$a_2x_1+b_2y_1+c_2=0 \text{ ....}(ii)$$

Solving (i) and (ii) simultaneously, we have

$$\frac{x_1}{b_1c_2-b_2c_1}=\frac{-y_1}{a_1c_2-a_2c_1}=\frac{1}{a_1b_2-a_2b_1}$$

$$\Rightarrow\ x_1=\frac{b_1c_2-b_2c_1}{a_1b_2-a_2b_1} \quad \text{and} \quad y_1=-\frac{a_1c_2-a_2c_1}{a_1b_2-a_2b_1}$$

Hence $\left(\dfrac{b_1c_2-b_2c_1}{a_1b_2-a_2b_1},-\dfrac{a_1c_2-a_2c_1}{a_1b_2-a_2b_1}\right)$ is the point of intersection of l_1 and l_2.

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³ *Equation of line passing through the point of intersection.*

Let $l_1:\ a_1x+b_1y+c_1=0$

$l_2:\ a_2x+b_2y+c_2=0$

Then equation of line passing through the point of intersection of l_1 and l_2 is

$$l_1+k\,l_2=0\ ,\ \text{where } k \text{ is constant.}$$

i.e. $a_1x+b_1y+c_{11}+k\left(a_2x+b_2y+c_2\right)=0$.

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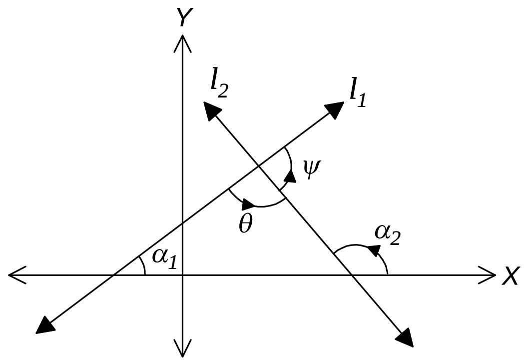
³ *Angle between lines*

Let l_1 and l_2 be two lines. If α_1 and α_2 be inclinations and m_1 and m_2 be slopes of lines l_1 and l_2 respectively, Let θ be a angle from line l_1 to l_2 then θ is given by

$$\tan\theta=\frac{m_2-m_1}{1+m_1m_2}$$

See proof on book at page 219



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³ *Homogenous 2nd Degree Equation*

Every homogenous second degree equation $ax^2+2hxy+by^2 = 0$ represents product of straight lines through the origin.

Let m_1 and m_2 be slopes of these lines. Then

$$\boxed{m_1m_2 = \frac{a}{b}} \quad \text{and} \quad \boxed{m_1+m_2 = -\frac{2h}{b}}$$

Let θ be the angles between the lines. Then

$$\boxed{\tan\theta = \frac{2\sqrt{h^2-ab}}{a+b}}$$

See proof on book at page 227.

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